Vol.10.Issue.2.2022 (April-June)
©KY PUBLICATIONS

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

Representation of A Number Using Unique Factorization Theorem In DNR2 Expression

Vishwambhar Narwade[1*], Amit Dabhole[2], Atul Raut[3] and Ganesh Rokade[4]
[1]Shreemati Venutai Chavan, Higher Secondary, Girls School, Aurangabad,431003, (M.S.) (India).
[2,3]Department of Basic Science and Humanities, Marathwada Institute ofTechnology, Aurangabad 431004, (M.S.) India.
[4]Department of Mathematics, J.E.S. college, Jalna, (M.S.) India.
[*]Correspondence Address: vbnarwade147@gmail.com,amit_dabhole22@rediffmail.com, atul.raut@mit.asia,ganeshrokade01@gmail.com
DOI:10.33329/bomsr.10.2.1

ABSTRACT

In the fundamental theorem of arithmetic every integer greater than 1 either is a prime number itself or can be represented as the product of prime numbers and this representation is unique. In this paper, we represented a number in DNR2 expression by using unique factorization theorem.

Keywords: Prime Number, Integers, Divisors.

Introduction:

The fundamental theorem of arithmetic, also called the unique factorization theorem states that, every integer greater than 1 either is a prime number itself or can be represented as the product of prime numbers.

For example:

$1200 = 2^4 \times 3^1 \times 5^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5.$

As Canonical representation of a positive integer is every positive integer $n > 1$ can be represented in exactly one way as a product of prime powers.

i.e., $n = p_1^{k_1} \times p_1^{k_2} \times \dots p_r^{k_r}$ where $p_1 < p_2 < \cdots < p_r$ are primes and then k_r are positive integers. This representation is commonly extended to all positive integers, including 1, by the convention that the empty product is equal to 1 (the empty product corresponds to k = 0). This representationis the canonical representation of *n.*

For example:

999= $3^3 \times 37$, 1000 = $2^3 \times 5^3$.

The concept of unique factorization plays an important role in modern commutative ring theory.

A. G oksel A garg un and E. Mehmet Ozkan [1] discussed comprehensive survey of the history of the Fundamental Theorem of Arithmetic. Theyinvestigated the main steps during the period from Euclid to Gauss.

Artur Korniowicz and Piotr Rudnicki [2] formalized the notion of the prime-power factorization of a natural number and prove the Fundamental Theorem of Arithmetic. They prove how prime-power factorization can be used to compute: products, quotients, powers, greatest common divisors andleast common multiples.

We consider any positive integer N greater than 1 and write number Nin the product of powers of prime factors. This factorization we write by DNR2 expression discussed in the following.

1. Let $N = p_1^{k_1}$ where p_1 is a prime number and k_1 is any positive integer, we write N by the DNR2 expression,

$$N = (p_1 - 1) \times (\text{Sum of proper divisors of } p_1^{k_1}) + 1.$$

2. Let $N = p_1^{k_1} \times p_2^{k_2}$ where p_1, p_2 is a prime number and k_1, k_2 is any positive integers, we write N by the DNR2 expression,

$$N = \left[(p_1 - 1) \times (\text{Sum of proper divisors of } p_1^{k_1}) + 1\right]$$
$$\times \left[(p_2 - 1) \times (\text{sum of proper divisors of } p_2^{k_2}) + 1\right]$$

3. Let $N = p_1^{k_1} \times p_2^{k_2} \dots \times p_r^{k_r}$ where $p_1, p_2, \dots, p_r$ is prime number and $k_1, k_2, \dots, k_r$ is positive integers, we write N by the DNR2 expression,

$$N = \left[(p_1 - 1) \times (\text{Sum of proper divisors of } p_1^{k_1}) + 1\right]$$
$$\times \left[(p_2 - 1) \times (\text{Sum of proper divisors of } p_2^{k_2}) + 1\right] \dots$$
$$\times \left[(p_r - 1) \times (\text{Sum of proper divisors of } p_r^{k_r}) + 1\right].$$

Definition 1: If *a, b* ∈ *Z we* say that a *divides b*, written $a|b$, if $ac = b$ for some $c \in Z$. In this case, we say a is a divisor of b. We say that a does not divide b, written a - b, if there is no $c \in Z$ such that ac = b.

Definition 2: An integer $n > 1$ is a *prime if* the only positive divisors of n are 1 and n. A prime power is a positive integer power of a single prime number.

Result 1: If $N = p_1^{k_1} > 1$, where p_1 is any prime number and k_1 is any positive integer, can be expressed by DNR2 expression as,

$$N = (p_1 - 1) \times (\text{Sum of proper divisors of } p_1^{k_1}) + 1.$$

Proof: Consider $N = p_1^{k_1} > 1$, where p_1 is any prime number and k_1 is any positive integer. We prove this result by using mathematical induction.

a) Let $k_1 = 1$, we write by DNR2 expression

$N = p_1^1 = (p_1 - 1) \times$ (Sum of proper divisors of $p_1^1) + 1$

$= (p_1 - 1) \times [1] + 1 = (p_1 - 1) \times +1 = p_1,$

i.e., it holds for $k_1 = 1$.

b) Let us assume it also holds for $k_1 = r, r < k_1$, then we write by DNR2 expression,

$N = p_1^r = (p_1 - 1) \times$ (Sum of proper divisors of $p_1^r) + 1$.

Now to prove it also hold for $k_1 = r + 1$, we can write

$N = p_1^{k_1} = p_1^{r+1} = p_1^r \times p_1^1$

$= (p_1 - 1) \times$ (Sum of proper divisors of $p_1^r) + 1$

$\times (p_1 - 1) \times$ (Sum of proper divisors of $p_1^1) + 1$

$= [(p_1 - 1) \times (1 + p_1 + p_1^2 + \cdots + p_1^{r-1}) + 1] \times [(p_1 - 1) \times (1) + 1]$

$= [p_1 + p_1^2 + p_1^3 \ldots + p_1^r - 1 - p_1 - p_1^2 - p_1^3 \ldots - p_1^r + 1] \times [(p_1 - 1) \times (1) + 1]$

$= p_1^r \times p_1 = p_1^{r+1}$.

Therefore, by mathematical induction it holds for $N = p_1^{k_1}$

Hence, we prove that,

$N = (p_1 - 1) \times$ (Sum of proper divisors of $p_1^{k_1}) + 1$.

We illustrate this result by following example.

Example 1: Let $N = 125$ then we can write by DNR2 expression,

$125 = 5^3 = (5 - 1) \times [1 + 5^1 + 5^2] + 1$

$= 4 \times [1 + 5 + 25] + 1 = (4 \times 31) + 1 = 124 + 1 = 125$.

Result 2: If $N = p_1^{k_1} \times p_1^{k_1}$ then by DNR2 expression,

$N = (p_1 - 1) \times$ (Sum of proper divisors of $p_1^{k_1}) + 1$

$\times\ (p_2 - 1) \times$ (Sum of proper divisors of $p_2^{k_2}) + 1$.

This result 2 can be proved by mathematical induction as proved in the result 1.

We illustrate this result by following example.

Example 2: Let $N = 784$ then we can write by DNR2 expression,

$784 = 2^4 \times 7^2 = [(2 - 1) \times (1 + 2^1 + 2^2 + 2^3) + 1] \times [(7 - 1) \times (1 + 7^1) + 1]$

$= [1\times (1+2+4+8) +1] \times[6\times (1+7) +1]=[15+1] \times [48+1]=16\times49= 784$.

Result 3: If $N = p_1^{k_1} \times p_2^{k_2} \ldots \times p_r^{k_r}$ where $p_1, p_2, \ldots, p_r$ is prime number and $k_1, k_2, \ldots, k_r$ is positive integers, we write N by the DNR2 expression,

$$N = \left[(p_1 - 1) \times (\text{Sum of proper divisors of } p_1^{k_1}) + 1\right]$$
$$\times \left[(p_2 - 1) \times (\text{Sum of proper divisors of } p_2^{k_2}) + 1\right] \ldots$$
$$\times \left[(p_r - 1) \times (\text{Sum of proper divisors of } p_r^{k_r}) + 1\right].$$

We illustrate this result by following example.

Example 3: Let N = 9000 then we can write by DR expression,

$$9000 = 2^3 \times 3^2 \times 5^3$$
$$= [(2-1)\times(1+2^1 +2^2)+1]\times[(3-1)\times(1+3^1)+1]\times[(5-1)\times(1+5^1 +5^2)+1]$$
$$= [1 \times (1 + 2 + 4) + 1] \times [2 \times (1 + 3) + 1] \times [4 \times (1 + 5 + 25) + 1]$$
$$= [(1 \times 7) + 1] \times [(2 \times 4) + 1] \times [(4 \times 31) + 1] = 8 \times 9 \times 125 = 9000.$$

Conclusion

Every positive integer can be expressed by unique factorization theorem, $N = p_1^{k_1} \times p_2^{k_2} \ldots \times p_r^{k_r}$ where $p_1, p_2, \ldots, p_r$ is any prime number and $k_1, k_2, \ldots, k_r$ is any positive integers. This representation we can write by DNR2 expression. So, we write any positive integer in DNR2 expression by using uniquefactorization theorem.

References:

[1]. Historia Mathematica 28(2001), 207214 MSC 1991 subject classifications: 01A30, 11-03, 11A51

[2]. FORMALIZED MATHEMATICS Volume 12, Number 2, 2004 University of Biaysto.

[3]. Elementary Number Theory: Primes, Congruences, and Secrets WilliamStein, January 23, 2017.

[4]. Elementary Number Theory: David M. Burton, Tata McGraw-Hill