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RESEARCH ARTICLE

Maximum Likelihood Estimation for the parameters of the Novel Exponent Power Rayleigh (NovEPR) distribution

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ABSTRACT

In this research paper, we discuss about the estimation procedure for the unknown parameters for NovEPR distribution. Among the statistical inference methods, the maximum likelihood method is widely used due its desirable properties including consistency, asymptotic efficiency, and invariance. We present MLE of the unknown parameters of NovEPR distribution using Monte Carlo simulation procedure to simulate the data. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) for both the parameters under sample based on 10,000 simulations to assess the performance of the estimators. Also we derive the asymptotic confidence bounds for unknown parameters. For the sake of illustration, we apply our proposed methodology in two important data sets, demonstrating that the NovEPR distribution is a simple alternative to be used for lifetime data.

Keywords: : MLE, Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE).

1. INTRODUCTION

Statistics is a very important science in studying and modelling new and old natural phenomena and a lot of industrial and biological data. So as life continues a lot challenges arise and appear in real life, these challenges need a lot of studies. So in this manner we are in a deep need always new distributions to model this new phenomena. Here comes the importance of introducing new distribution, that has attracted great interest in the last years, because statistical modelling for any real data makes researchers understand this phenomena and can study this phenomena properly and make predictions for the forthcoming data, which may reduce the cost and the time on the researcher. There are many types of data that we can deal with when we are introducing any new distribution.

In the last few years, the literature of distribution theory has become rich due to the induction of additional parameters in the existing distribution. The inclusion of an extra parameter has shown greater flexibility compared to competitive models. The inclusion of a new parameter can be performed either using the available generator or by developing a new technique for generating new improved distribution compared to classical baseline distribution. Alshanbari et al. (2022) proposed the A novel extension of Fre´chet distribution: Application on real data and simulation. Mudholkar and Srivastava (1993) introduced exponentiated Weibull distribution by introducing a shape parameter in two-parameter Weibull distribution. Korkmaz and Genc (2017) presented a generalized two-sided class of probability distributions. Alzaghal et al. (2013) worked on the T-X class of distributions. Aldeni et al. (2017) used the quantile function of generalized lambda distribution and introduced a new family. Abd-Elfattah (2006) studied the Efficiency of Maximum Likelihood estimators under different censored sampling schemes for Rayleigh distribution. Francisco Louzada et al. (2016) studied the Different Estimation Procedures for the Parameters of the Extended Exponential Geometric Distribution for Medical Data. Vijaya lakshmi et al. (2020) studied the Estimation of parameters of Alpha Logarithm Transformed Rayleigh distribution by using Maximum Likelihood Estimation method. Ramamohan and Anjaneyulu (2011) studied how the least square method be good for estimating the parameters to Two Parameter Weibull Distribution from an optimally constructed grouped sample. Ramamohan and Anjaneyulu (2013) studied the estimation of Scale (σ) when Shape (β) parameter is known using least squares method from an optimally constructed grouped sample. Ramamohan and Anjaneyulu (2014) studied Estimation of Scale parameter (σ) when Shape parameter (β) is known in Log Logistic Distribution using Minimum Spacing Square Distance Estimation Method from an optimally constructed grouped sample. Vijaya lakshmi, Raja Sekharam and Anjaneyulu (2018) studied Estimation of Scale (λ) and Location (μ) of Two-Parameter Rayleigh Distribution Using Median Ranks method. Vijaya lakshmi, Raja Sekharam and Anjaneyulu (2019) studied Estimation of Scale (θ) and Shape (α) parameters of Power Function Distribution by Least Squares Method Using Optimally Constructed Grouped Data. Vijaya lakshmi and Anjaneyulu (2019) studied Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Least Square Regression Method. Vijaya lakshmi and Anjaneyulu (2019) studied Estimation of Location (μ) and Scale (λ) for Two-Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method.

In this paper, we discuss about the estimation procedure for the unknown parameters for NoVEPR distribution. There is several estimation procedures exist in the literature, however, the most popular estimation procedure is Maximum Likelihood Estimation procedure (MLE). The idea behind the Maximum Likelihood parameter estimation is to determine the parameters that maximize the likelihood function given the sample data. . We have taken two real time data sets and fitted our introduced model. We present MLE of the unknown parameters of NoVEPR distribution using Monte Carlo simulation procedure to simulate the data. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative

Absolute Bias (RAB) and Relative Error (RE) for both the parameters under sample based on 10,000 simulations to assess the performance of the estimators. Also we derive the asymptotic confidence bounds for unknown parameters. Probability density function, Distribution function, Quantiles and Random Generating numbers of NoVEPR is given by

A random variable say X is said to follow a NovEP-Y family, if it's DF (distribution function) K(x;σ^2, α) is given by

$$K(x;\sigma^2,\alpha) = 1-\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha} \quad \text{... (1.1)}$$

A random variable say X is said to follow a NovEP-Y family, if its PDF (Probability distribution function) k(x;σ^2, α) is given by

$$k(x;\sigma^2,\alpha) = \frac{\alpha(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha-1} \quad \text{...(1.2)}$$

A random variable X ~ NovEP-R (σ^2, α) has Quantile function and is in the form the pth quantile x_p of NovEP-R distribution is the root of the equation

y = Q(u) = $K^{-1}(u) = W^{-1}(z)$,

Where p = $e^{-x^2/2\sigma^2}$ is the solution of

$$y = Q(p) = 1-\left(1-\frac{1-e^{p}}{e^{e^{p}}}\right)^{\alpha} \quad \text{... (1.4)}$$

Let U~ U (0, 1), then equation (2.2) can be used to simulate a random sample of size n from the NovEP-R distribution as follows

$$x_i = 1-\left(1-\frac{1-e^{u_i}}{e^{e^{u_i}}}\right)^{\alpha} \quad \text{... (1.5)}$$

Where x ∈ $[0,\infty)$, W(x; σ^2,α) and α > 0 is baseline distribution function.

2. ESTIMATION OF PARAMETERS OF NovEP-R DISTRIBUTION MAXIMUM LIKELIHOOD METHOD

Let $x_1, x_2, \ldots x_n$ be a random sample of size 'n' from NovPER(σ^2, α) then the likelihood function L of this sample is defined as

$$l_n = ln(k(x;\ \sigma^2,\alpha)\ )$$

$$= ln\left(\frac{\alpha(1-e^{-x^2/2\sigma^2})\left[2-e^{-x^2/2\sigma^2}\right]}{e^{e^{-x^2/2\sigma^2}}}\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right)^{\alpha-1}\right)$$

$$= nln(\alpha) + \sum_{i=1}^{n} \ln(1-e^{-x^2/2\sigma^2}) + \sum_{i=1}^{n} \ln(n-e^{-x^2/2\sigma^2}) + e^{\sum_{i=1}^{n} x^2/2\sigma^2}$$

$$+ (\alpha-1)\sum_{i=1}^{n} \ln\left(1-\frac{1-e^{-x^2/2\sigma^2}}{e^{e^{-x^2/2\sigma^2}}}\right) \quad \text{...(2.1)}$$

Calculating the 1st and 2nd order partial derivative of (2.1) with respective to (σ^2, α)and then 1st order partial derivatives equating to zero we get the following equations

$$\frac{dln}{d\sigma^2} = \sum_{i=1}^{n} \frac{x^2 e^{-x^2/2\sigma^2}}{\sigma^3(e^{-x^2/2\sigma^2} - 1)} + \alpha \sum_{i=1}^{n} \frac{x^2 e^{-x^2/2\sigma^2}}{\sigma^3(e^{-x^2/2\sigma^2} - 2)} - \sum_{i=1}^{n} \frac{x^2 e^{-x^2/2\sigma^2}}{\sigma^3}$$

$$=> \sum_{i=1}^{n} \frac{x^2 e^{-x^2/2\sigma^2}}{\sigma^3(e^{-x^2/2\sigma^2}-1)(e^{-x^2/2\sigma^2}-2)} \left(3e^{-x^2/2\sigma^2} - 4 - (e^{-x^2/2\sigma^2} - 1)(e^{-x^2/2\sigma^2} - 2)\right) = 0 \quad ... (2.2)$$

$$\frac{dln}{d\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} \frac{x^2 e^{-x^2/2\sigma^2}}{\sigma^3(e^{-x^2/2\sigma^2}-2)} = 0 \quad ... (2.3)$$

$$\frac{d^2ln}{d\sigma^4} = \sum_{i=1}^{n} \frac{x^2((3-x^2)e^{-x^2/2\sigma^2}-3\sigma^2)}{\sigma^6(e^{-x^2/2\sigma^2}-1)^2} + \sum_{i=1}^{n} \frac{x^2((3-x^2)e^{-x^2/2\sigma^2}-3\sigma^2)}{\sigma^6(e^{-x^2/2\sigma^2}-2)^2}$$

$$+ \sum_{i=1}^{n} \frac{(3x^2\sigma^2+x^4)e^{x^2/2\sigma^2})}{\sigma^6} + (\alpha - 1)\sum_{i=1}^{n} \frac{x^2(3\sigma^2 e^{-x^2/2\sigma^2}+(-3\sigma^2-x^2)e^{-x^2/2\sigma^2})}{\sigma^6(e^{-x^2/2\sigma^2}-1)^2}$$

... (2.4)

$$\frac{d^2ln}{d\alpha^2} = -\frac{n}{\alpha^2} \quad ... (2.5)$$

$$\frac{d^2ln}{d\sigma^2 d\alpha} = \sum_{i=1}^{n} \frac{x^2(3\sigma^2 e^{-x^2/2\sigma^2}+(-3\sigma^2-x^2)e^{-x^2/2\sigma^2})}{\sigma^6(e^{-x^2/2\sigma^2}-1)^2} \quad ... (2.6)$$

3. ASYMPTOTIC CONFIDENCE BOUNDS

Here we derive the asymptotic confidence bounds for unknown parameters Scale (σ^2), Shape (α) when σ^2> 0, α > 0 The simplest large sample approach is to assume that the MLEs (σ^2,α) are approximately normal with mean (σ^2,α) and covariance matrix I_0^{-1}, where I_0^{-1} is the inverse of the observed information matrix which defined as follows

$$I_0^{-1} = \begin{bmatrix} -E(\frac{d^2ln}{d\sigma^4}) & -E(\frac{d^2ln}{d\sigma^2 d\alpha}) \\ -E(\frac{d^2ln}{d\sigma^2 d\alpha}) & -E(\frac{d^2ln}{d\alpha^2}) \end{bmatrix}$$

The Asymptotic (1-r)100% Confident intervals for estimated parameters are as follows

$$\widehat{\sigma^2} + z_{\frac{r}{2}}[var(\widehat{\sigma^2})],$$

$$\hat{\alpha} + z_{\frac{r}{2}}[var(\hat{\alpha})]$$

4. SIMULATION STUDY

In this section, we develop a simulation study. The main goal of these simulations is to evaluate the efficiency of the Maximum likelihood estimation method for the parameters of the NovEP-R distribution. The following procedure was adopted:

Step 1: Set the sample size n and the vector of parameter values $\Psi = (\sigma^2, \alpha)$.

Step 2: Using the values obtained in step (2), compute $\hat{\sigma}^2_{MLE}$ via Maximum Likelihood estimation procedure.

Step 3: Repeat steps (2) and (3) N times

Step 4: Using $\hat{\varphi}$ of φ, compute the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE). If $\hat{\varphi}_{lm}$ is Maximum likelihood Method estimate of φ_m, m=1, 2 where $\Psi_m$is a general notation that can be replaced by $\Psi_1 = \sigma^2, \Psi_2 =$α based on sample l, (l =1,2,…,k), then the Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) and Relative Error (RE) are given respectively by

Average Estimate $(\hat{\varphi}_m) = \frac{\sum_{i=1}^{k} \hat{\varphi}_{lm}}{k}$

Variance$(\hat{\varphi}_m)$ = $\frac{\sum_{i=1}^{k} (\hat{\varphi}_{lm} - \overline{\hat{\varphi}_{lm}})^2}{k}$

SD $(\hat{\varphi}_m$= $\sqrt{\frac{\sum_{i=1}^{r} (\hat{\varphi}_{lm} - \overline{\hat{\varphi}_{lm}})^2}{r}}$

Mean Absolute Deviation$(\hat{\varphi}_m)$ = $\frac{\sum_{i=1}^{k} Med(|\hat{\varphi}_{lm} - \overline{\hat{\varphi}_{lm}}|)}{k}$

Mean Square Error $(\hat{\varphi}_m)$ = $\frac{\sum_{i=1}^{k} (\widehat{\varphi_{lm}} - \varphi_m)^2}{k}$

Relative Absolute Bias$(\hat{\varphi}_m)$ =$\frac{\sum_{i=1}^{k} |(\widehat{\varphi_{lm}} - \varphi_m)|}{k\varphi_m}$

Relative Error$(\hat{\varphi}_m)$ = $\frac{1}{k} \left(\frac{\sum_{i=1}^{k} MSE \sqrt{(\widehat{\varphi_{lm}})}}{\varphi_m} \right)^2$

The results were computed using the software R (R Core Development Team). The seed used to generate the random values. The chosen values to perform this procedure

N = 10,000, and n = (20, 40, 60, …, 200) for various population parameter values are considered.

5 APPLICATIONS

In this section, we considered two data sets from the sports and health sciences are considered. The first data set represents the time-to-even data collected from different football matches during the period 1964–2018. Whereas, the second data set is taken from the health sector, representing the survival times of the COVID-19 infected patients. Based on some well-known statistical tests, it is observed that the NovEP- Rayleigh model is a very competitive distribution for modeling the data sets in the sports and health sectors.

In Section 1.4, our simulation study indicated that the ML estimators should be used for estimating the parameters of the NovEP-R distribution. Initially, we compared the estimates obtained from the different procedures with the ML estimator. Then, we compared the results obtained from the NovEP-R distribution fitted by the ML estimators with some common lifetime models, such as Rayleigh, Exponential, Weibull, and Generalized Exponential distributions.

The Kolmogorov-Smirnov (KS) test is considered to check the goodness of fit. This procedure is based on the KS statistic $D_n = sup_x |K_n(x) - K(x; \sigma^2, \alpha)|$

Where sup_x is the supremum of the set of distances $K_n(x)$ is the empirical distribution function and $K(x; \sigma^2, \alpha)$ is cumulative distribution function of NovEP-R. In this case, we test the null hypothesis that the data comes from $K(x; \sigma^2, \alpha)$, and, with significance level of 5%, we will reject the null hypothesis if p value is smaller than 0.05. As discrimination criterion method, we considered the AIC (Akaike Information Criteria) computed, respectively, by

AIC = $-2l(\widehat{\varphi}, x) + 2k$

Where 'k' is the number of parameters fitted and $\widehat{\varphi}$ is estimate of φ.

The first data set represents the time-to-even data collected from different football matches during the period 1964–2018. We obtained

$\hat{\sigma}^2_{MLE}$ = 1.3064 and $\hat{\alpha}_{MLE}$ = 2.9431

Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	NovEP-R	Rayleigh	Exponential	Weibul	Generalised Exponential
KS	0.8653	0.2478	0.3593	0.1658	0.4687
AIC	1325.72	2530.25	20356.06	3215.87	1543.87

Data Set 2

Second data set is taken from the health sector, representing the survival times of the COVID-19 infected patients.

We obtained

$\hat{\sigma}^2_{MLE}$ = 2.872 and $\hat{\alpha}_{MLE}$ = 4.3254

Results of the KS test (p value), AIC for the different probability distributions considering the above data set

Test	NovPER	Rayleigh	Exponential	Weibull	Generalised Exponential
KS	0.4121	0.9875	0.7659	0.8653	0.5236
AIC	998.36	2014.63	1562.87	1766.77	1136.54

Comparing the empirical function with the adjusted distributions, a better fit for the NovEP-R distribution among the chosen models can be observed. This result is confirmed from AIC, since NovEP-R distribution has the minimum values among the chosen models.

6. OBSERVATIONS FOR THE SIMULATION RESULT

1. Maximum likelihood estimators of scale (σ^2) and shape (α) parameters are less biased.
2. The Average estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error (RE) of the estimators are dependent on the sample sizes.
3. Here it may be noted the Maximum likelihood estimators are obtaining from complete sample. Thus Maximum likelihood method applying is yielding more efficient estimators especially when sample is a large. This is an interesting application of maximum likelihood method.

4. The Average estimate (AE), Variance (VAR), Standard deviation (SD), Mean Square Error (MSE), Relative Absolute Error (RAB), Relative Error (RE) of the estimators are independent on the population parameter values.

5. The Average estimate (AE) of the Maximum likelihood scale $(\hat{\sigma}^2)$ and shape $(\hat{\alpha})$ estimators are increased when sample size increased.

6. The Variance (VAR) of Maximum likelihood scale $(\hat{\sigma}^2)$ and shape $(\hat{\alpha})$ estimators are decreased when sample size increased.

7. The Standard Deviation of Maximum likelihood scale $(\hat{\sigma}^2)$ and shape $(\hat{\alpha})$ estimators are decreased when sample size increased.

8. The Mean square error (MSE) Maximum likelihood scale $(\hat{\sigma}^2)$ and shape $(\hat{\alpha})$ estimators are decreased when sample size increased.

9. The Relative absolute bias (RAB) Maximum likelihood scale $(\hat{\sigma}^2)$ and shape $(\hat{\alpha})$ estimators are decreased when sample size increased.

10. The Relative error (RE) Maximum likelihood scale $(\hat{\sigma}^2)$ and shape $(\hat{\alpha})$ estimators are decreased when sample size increased.

Maximum Likelihood process for estimating the NovEP-R $(\sigma^2$, α) Newton-Raphson iterative method for a two parameter combinations and the procedure is repeated 10,000 times for various sample sizes *n* = 20(20)200 are considered. The MLEs and their Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE) and Relative Absolute Bias (RAB), Relative Error (RE) of the scale and shape parameters are unknown population parameters of NovEP-R distribution. Population parameters Scale(σ^2)=4.5 and Shape (α) =3 in Table 1.

Table 1

Sample size	Para meters	AE	VAR	SD	MAD	MSE	RAB	RE
20	σ^2	2.3142	1.9964	1.9865	0.1352	0.3547	0.1555	0.2505
	α	0.9873	1.8701	1.8657	0.1785	0.2457	0.1998	0.2034
40	σ^2	2.4587	1.7428	1.6541	0.1269	0.3684	0.1633	0.2654
	α	0.9965	1.8104	1.7653	0.1533	0.2564	0.2005	0.2047
60	σ^2	2.2578	1.5879	0.9866	0.1198	0.3478	0.1577	0.2471
	α	0.8693	1.7993	1.5479	0.1487	0.2436	0.1989	0.1996
80	σ^2	2.5632	1.4872	0.9732	0.1065	0.2178	0.1486	0.2047
	α	1.3254	1.6559	1.3204	0.1366	0.2387	0.1874	0.1772
100	σ^2	2.7832	1.3247	0.8574	0.0987	0.1932	0.1330	0.1928
	α	1.5478	1.4388	1.2456	0.1299	0.2141	0.1652	0.1638
120	σ^2	2.7935	1.1089	0.8165	0.9763	0.1732	0.1257	0.1665
	α	1.6543	1.2880	1.0657	0.1163	0.2059	0.1542	0.1507
140	σ^2	2.9635	0.9987	0.7635	0.9478	0.1932	0.1175	0.1585
	α	1.7221	1.0054	0.9872	0.1099	0.1935	0.1487	0.1325
160	σ^2	3.0168	0.9735	0.6547	0.9132	0.1736	0.1086	0.1322
	α	1.8642	0.9983	0.9443	0.0968	0.1654	0.1244	0.1199

180	σ^2	3.1458	0.9553	0.4732	0.8936	0.1531	0.0987	0.1125
	α	2.3227	0.9235	0.7843	0.0901	0.1563	0.1098	0.1087
200	σ^2	3.3647	0.8736	0.2981	0.7936	0.1328	0.0635	0.1065
	α	2.5632	0.7981	0.6558	0.0873	0.1269	0.9652	0.0658

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