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FIXED POINT THEOREMS IN STRONG GENERALIZED FUZZY METRIC SPACES

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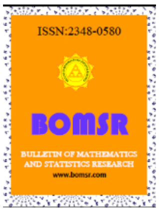

ABSTRACT

In this work, we carried over some fixed point results on strong fuzzy metric spaces into strong generalized fuzzy metric spaces along with the control function.

Keywords: Fuzzy metric, Strong fuzzy metric space, Continuous t-norm, Control function.

AMS Subject Classification: 54H25, 47H10, 47S40.

1. INTRODUCTION

Kramosil and Michalek [6] were the first to introduce the fuzzy metric space which was later modified by many authors among which we have taken the one that is given by George and Veeramani [1]. The control function, also known as altering distance function, was introduced by M.S. Khan et. al. [6] in the year 1984. In the year 2018, some fixed point theorems for generalized (ψ, ϕ)-contractive mapping were proved in strong *M*-fuzzy metric spaces. Here we proved some fixed point theorems in strong generalized fuzzy metric spaces using the control function.

2. PRELIMINARIES

Definition 2.1 [9]

A binary operation$*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is said to be a continuous t-norm if it satisfies the following conditions:

i) $*$ is associative and commutative,

ii) $*$ is continuous,

iii) $a * 1 = a$ for all $a \in [0, 1]$,

iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2 [10]

A 3-tuple $(X, M, *)$ is called an M-fuzzy metric space if M is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^3 \times (0,\infty)$, satisfying the following conditions for each $x, y, z, a \in X$ and $t, s > 0$:

(M1) $M(x, y, z, t) > 0$,

(M2) $M(x, y, z, t) = 1$ if and only if $x = y = z$,

(M3) $M(x, y, z, t) = M(p\{x, y, z\}, t)$, where p is the permutation function.

(M4) $M(x, y, a, t) * M(a, z, z, s) \leq M(x, y, z, t + s)$,

(M5) $M(x, y, z, .): (0, \infty) \rightarrow [0,1]$ is continuous,

(M6) $\lim_{t\to\infty} M(x, y, z, t) = 1$ for all $x, y, z \in X$.

Definition 2.3 [11]

Let $(X, M, *)$ be a M-fuzzy metric space. The M-fuzzy metric is said to be strong (non-Archimedean) if it satisfies

(M4'): $M(x, y, z, t) \geq M(x, y, a, t) * M(a, z, z, t)$, for each $x, y, z \in X$ and each $t > 0$.

Remark 2.4

Axiom (M4') cannot replace axiom (M4) in the above definition of fuzzy metric, since in that case, M could not be a fuzzy metric on X.

Note that it is possible to define a strong fuzzy metric by replacing (M4) by (M4') and demanding in (M5) that the function $M(x, y, z, .)$ be an increasing continuous function on t, for each $x, y, z \in X$. In fact, in such a case we have that, $M(x, y, z, t + s) \geq M(x, y, a, t + s) * M(a, z, z, t + s) \geq M(x, y, a, t) * M(a, z, z, s)$).

Remark 2.5

Not every M-fuzzy metric space is a strong fuzzy metric space.

Definition 2.6

Let $(X, M, *)$ be a M-fuzzy metric space.

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if

$\lim_{n\to\infty} M(x_n, x, x, t) = 1$ for all $t > 0$.

(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if, for each $0 < \epsilon < 1$ and $t > 0$, there exits $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, x_m, t) > 1-\epsilon$ for each $n, m \geq n_0$.

(iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

(iv) A fuzzy metric space in which every sequence has a convergent subsequence is said to be compact.

Lemma 2.7

Let $(X, M, *)$ be a fuzzy metric space. For all $a, v \in X$, $M(a, b, c, .)$ is a non-decreasing function.

Proof:

If $M(a, b, c, t) > M(a, b, c, s)$ for some $0 < t < s$.

Then $M(a, b, c, t) * M(c, c, c, s - t) \leq M(a, b, c, s) < M(a, b, c, t)$.

Since $M(c, c, c, s - t) = 1$, $M(a, b, c, t) < M(a, b, c, t)$, a contradiction.

Definition 3.1 [6]

A function $\psi: [0, 1] \rightarrow [0, 1]$ is called control function or an altering distance function if it satisfies the following properties:

(CF1) ψ is strictly decreasing and continuous,

(CF2) $\psi(\gamma) \geq 0$, for all $\gamma \neq 1$ and $\psi(\gamma) = 0$ if and only if $\gamma = 1$.

It is obvious that $\lim_{\gamma \to 1^-} \psi(\gamma) = \psi(1) = 0$.

(CF3) $\psi(\gamma * \mu) \leq \psi(\gamma) + \psi(\mu)$, $\gamma, \mu \in \{M(a, Ta, Ta, t): a \in X, t > 0\}$.

3. Main Results

Theorem 3.2

Let $(X, M, *)$ be a complete strong generalized fuzzy metric space with continuous t-norm $*$ and let T be a self-mapping on X. If there exists a control function ψ and $\gamma_i = \gamma_i(t)$, $i = 1, 2, 3, \ldots, 6$, $\gamma_i \geq 0$ with $\gamma_1 + \gamma_2 + \gamma_3 + 2\gamma_4 + \gamma_5 + \gamma_6 < 1$ such that

$$\psi(M(Ta, Tb, Tc, t)) \leq \gamma_1\psi(M(a, b, c, t)) + \gamma_2\psi(M(Tb, Tb, c, t)) + \gamma_3\psi(M(Ta, b, c, t)) + \gamma_4\psi(M(a, Tb, Tc, t)) + \gamma_5\psi(M(Ta, Ta, a, t)) + \gamma_6\psi\{\max M(Tb, Tb, c, t), M(Ta, Ta, a, t))\}, \quad (3.2.1)$$

then T has a unique fixed point in X.

Proof:

Let a be any arbitrary point in X and define a sequence $\{u_n\} \in X$ such that $a_{n+1} = Ta_n$.

Assume that $a_{n+1} = Ta_n = a_n$ for some $n \in N$, then a_n is a fixed point of T.

Suppose $a_{n+1} \neq a_n$. Putting $a = a_{n-1}$, $b = a_n$ and $c = a_n$ in equation (3.2.1), we get

$$\psi(M(Ta_{n-1}, Ta_n, Ta_n, t)) \leq \gamma_1\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_2\psi(M(Ta_n, Ta_n, a_n, t)) + \gamma_3\psi(M(Ta_{n-1}, a_n, a_n, t)) + \gamma_4\psi(M(a_{n-1}, Ta_n, Ta_n, t)) + \gamma_5\psi(M(Ta_{n-1}, Ta_{n-1}, a_{n-1}, t)) + \gamma_6\psi\{\max (M(Ta_n, Ta_n, a_n, t), M(Ta_{n-1}, Ta_{n-1}, a_{n-1}, t))\},$$

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \gamma_1\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_2\psi(M(a_{n+1}, a_{n+1}, a_n, t)) +$

$\gamma_3\psi(M(a_n, a_n, a_n, t)) + \gamma_4\psi(M(a_{n-1}, a_{n+1}, a_{n+1}, t)) +$

$\gamma_5\psi(M(a_n, a_n, a_{n-1}, t)) + \gamma_6\psi\{\max(M(a_{n+1}, a_{n+1}, a_n, t), M(a_n, a_n, a_{n-1}, t))\},$

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \gamma_1\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_2\psi(M(a_{n+1}, a_{n+1}, a_n, t)) + \gamma_3\psi(1) +$

$\gamma_4\psi(M(a_{n-1}, a_{n+1}, a_{n+1}, t)) + \gamma_5\psi(M(a_n, a_n, a_{n-1}, t)) +$

$\gamma_6\psi\{\max(M(a_{n+1}, a_{n+1}, a_n, t), M(a_n, a_n, a_{n-1}, t))\},$

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \gamma_1\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_2\psi(M(a_{n+1}, a_{n+1}, a_n, t)) +$

$\gamma_4\psi(M(a_{n-1}, a_{n+1}, a_{n+1}, t)) + \gamma_5\psi(M(a_n, a_n, a_{n-1}, t)) +$

$\gamma_6\psi\{\max(M(a_{n+1}, a_{n+1}, a_n, t), M(a_n, a_n, a_{n-1}, t))\}.$ (3.2.2)

Here (X, M, $*$) is a strong generalized fuzzy metric space, and we have that

$M(a_{n-1}, a_{n+1}, a_{n+1}, t) \geq M(a_{n-1}, a_n, a_n, t) * M(a_n, a_{n+1}, a_{n+1}, t)$, by using (M4'),

$\psi(M(a_{n-1}, a_{n+1}, a_{n+1}, t)) \geq \psi((M(a_{n-1}, a_n, a_n, t)) * (M(a_n, a_{n+1}, a_{n+1}, t)))$, by using (CF3),

$\psi(M(a_{n-1}, a_{n+1}, a_{n+1}, t)) \geq \psi((M(a_{n-1}, a_n, a_n, t)) + (M(a_n, a_{n+1}, u_{n+1}, t)))$. (3.2.3)

Using above inequalities in (3.2.2), we get

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \gamma_1\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_2\psi(M(a_{n+1}, a_{n+1}, a_n, t)) +$

$\gamma_4[\psi(M(a_{n-1}, a_n, a_n, t)) + \psi(M(a_n, a_{n+1}, a_{n+1}, t))] +$

$\gamma_5\psi(M(a_n, a_n, a_{n-1}, t)) + \gamma_6\psi\{\max(M(a_{n+1}, a_{n+1}, u_n, t), M(a_n, a_n, a_{n-1}, t))\},$

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \gamma_1\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_2\psi(M(a_{n+1}, a_{n+1}, a_n, t)) +$

$\gamma_4\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_4\psi(M(a_n, a_{n+1}, a_{n+1}, t)) +$

$\gamma_5\psi(M(a_n, a_n, a_{n-1}, t)) + \gamma_6\psi\{\max(M(a_{n+1}, a_{n+1}, a_n, t), M(a_n, a_n, a_{n-1}, t))\}.$ (3.2.4)

If $\max(M(a_{n+1}, a_{n+1}, a_n, t), M(a_n, a_n, a_{n-1}, t)) = M(a_{n+1}, a_{n+1}, a_n, t)$, (3.2.5)

then the above inequality (3.2.3) becomes

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \gamma_1\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_2\psi(M(a_{n+1}, a_{n+1}, a_n, t)) +$

$\gamma_4\psi(M(a_{n-1}, a_n, a_n, t)) + \gamma_4\psi(M(a_n, a_{n+1}, a_{n+1}, t)) +$

$\gamma_5\psi(M(a_n, a_n, a_{n-1}, t)) + \gamma_6\psi(M(a_{n+1}, a_{n+1}, a_n, t)).$

We then obtain that,

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \frac{\gamma_1+\gamma_4+\gamma_5}{1-(\gamma_2+\gamma_4+\gamma_6)}\psi(M(a_{n-1}, a_n, a_n, t)) < \psi(M(a_{n-1}, a_n, a_n, t)).$ (3.2.6)

Similarly, if $\max(M(a_{n+1}, a_{n+1}, a_n, t), M(a_n, a_n, a_{n-1}, t)) = M(a_n, a_n, a_{n-1}, t)$, (3.2.7)

the inequality (3.2.3) becomes

$\psi(M(a_n, a_{n+1}, a_{n+1}, t)) \leq \frac{\gamma_1+\gamma_4+\gamma_5+\gamma_6}{(1-(\gamma_2+\gamma_4))}\psi(M(a_{n-1}, a_n, a_n, t)) < \psi(M(a_{n-1}, a_n, a_n, t)).$ (3.2.8)

Hence $\psi(M(a_n, a_{n+1}, a_{n+1}, t)) < \psi(M(a_{n-1}, a_n, a_n, t))$.

This gives $(M(a_n, a_{n+1}, a_{n+1}, t)) > (M(a_{n-1}, a_n, a_n, t))$.

Since the sequence $\{M(a_n, a_{n+1}, a_{n+1}, t)\}$ is non decreasing, taking limit $n \to \infty$, we get $\lim_{n\to\infty} M(a_n, a_{n+1}, a_{n+1}, t) = q(r)$, for $q:(0, \infty) \to [0, 1]$. (3.2.9)

Suppose that $q(r) \neq 1$ for some $r > 0$ as $n \to \infty$.

Now (3.2.8) becomes

$$\psi(q(r)) \leq \frac{\gamma_1 + \gamma_4 + \gamma_5 + \gamma_6}{(1-(\gamma_2+\gamma_4))}\psi(q(r)) < \psi(q(r)) \quad (3.2.10)$$

which is a contradiction. Hence $\lim_{n\to\infty} M(a_n, a_{n+1}, a_{n+1}, t) = 1, t > 0$.

Next, we prove that the sequence $\{a_n\}$ is a Cauchy's sequence. Suppose not, then for any $0< \epsilon<1$, $t > 0$, there exist sequences $\{a_{n_k}\}$ and $\{a_{m_k}\}$, where $n_k, m_k \geq n$, $n_k > m_k$, $n_k, m_k \in N$ such that

$$M(a_{n_k}, a_{m_k}, a_{m_k}, t) \leq 1 - \epsilon. \quad (3.2.11)$$

Let n_k be least integer exceeding m_k satisfying the above property.

That is $M(a_{n_k-1}, a_{m_k}, a_{m_k}, t) > 1 - \epsilon$, $n_k, m_k \in N$ and $t > 0$. (3.2.12)

Putting $a = a_{n_k-1}$ and $b = a_{m_k-1}$, $c = a_{m_k-1}$. (3.2.13)

$$\psi(M(Ta_{n_k-1}, Ta_{m_k-1}, Ta_{m_k-1}, t)) \leq \gamma_1\psi(M(a_{n_k-1}, a_{m_k-1}, a_{m_k-1}, t)) +$$
$$\gamma_2\psi(M(Ta_{m_k-1}, Ta_{m_k-1}, a_{m_k-1}, t)) + \gamma_3\psi(M(Ta_{n_k-1}, a_{m_k-1}, a_{m_k-1}, t)) +$$
$$\gamma_4\psi(M(a_{n_k-1}, Ta_{m_k-1}, Ta_{m_k-1}, t)) + \gamma_5\psi(M(Ta_{n_k-1}, Ta_{n_k-1}, a_{n_k-1}, t)) +$$
$$\gamma_6\psi\{\max(M(Ta_{m_k-1}, Ta_{m_k-1}, a_{m_k-1} t), M(Ta_{n_k-1}, Ta_{n_k-1}, a_{n_k-1}, t))\},$$

$$\psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) \leq \gamma_1\psi(M(a_{n_k-1}, a_{m_k-1}, a_{m_k-1}, t)) + \gamma_2\psi(M(a_{m_k}, a_{m_k}, a_{m_k-1}, t)) +$$
$$\gamma_3\psi(M(a_{n_k}, a_{m_k-1}, a_{m_k-1}, t)) + \gamma_4\psi(M(a_{n_k-1}, a_{m_k}, a_{m_k}, t)) +$$
$$\gamma_5\psi(M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)) + \gamma_6\psi\{\max(M(a_{m_k}, a_{m_k}, a_{m_k-1}, t), M(a_{n_k}, a_{n_k}, a_{n_k-1}, t))\}. \quad (3.2.13)$$

If $\max(M(a_{m_k}, a_{m_k}, a_{m_k-1}, t), M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)) = M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)$

$$\psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) \leq \gamma_1\psi(M(a_{n_k-1}, a_{m_k-1}, a_{m_k-1}, t)) + \gamma_2\psi(M(a_{m_k}, a_{m_k}, a_{m_k-1}, t)) +$$
$$\gamma_3\psi(M(a_{n_k}, a_{m_k-1}, a_{m_k-1}, t)) + \gamma_4\psi(M(a_{n_k-1}, a_{m_k}, a_{m_k}, t)) +$$
$$\gamma_5(M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)) + \gamma_6\psi(M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)). \quad (3.2.14)$$

By (M4'), (CF3) and (CF1) it follows that

$$\psi(M(a_{n_k}, a_{m_k-1}, a_{m_k-1}, t)) \leq \psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) + \psi(M(a_{m_k}, a_{m_k-1}, a_{m_k-1}, t)), \quad (3.2.15)$$

and $\psi(M(a_{n_k-1}, a_{m_k-1}, a_{m_k-1}, t)) \leq \psi(M(a_{n_k-1}, a_{n_k}, a_{n_k}, t)) + \psi(M(a_{n_k}, a_{m_k-1}, a_{m_k-1}, t))$.

Applying the previous inequalities, we get

$$\psi(M(a_{n_k-1}, a_{m_k-1}, a_{m_k-1}, t)) \leq \psi(M(a_{n_k-1}, a_{n_k}, a_{n_k}, t)) + \psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) +$$
$$\psi(M(a_{m_k}, a_{m_k-1}, a_{m_k-1}, t)). \quad (3.2.16)$$

From (3.2.12) and (CF1), we get

$$\psi(M(a_{n_k-1}, a_{m_k}, a_{m_k}, t)) \leq \psi(1-\epsilon) \quad (3.2.17)$$

Substituting (3.2.15), (3.2.16), and (3.2.17) in (3.2.14), we have

$$\psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) \leq \gamma_1\psi(M(a_{n_k-1}, a_{n_k}, a_{n_k}, t)) + \gamma_1\psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t))$$
$$+ \gamma_1\psi(M(a_{n_k}, a_{m_k-1}, a_{m_k-1}, t)) + \gamma_2\psi(M(a_{m_k}, a_{m_k}, a_{m_k-1}, t))$$
$$+ \gamma_3\psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) + \gamma_3\psi(M(a_{m_k}, a_{m_k-1}, a_{m_k-1}, t))$$
$$+ \gamma_4\psi(1-\epsilon) + \gamma_5\psi(M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)) + \gamma_6\psi(M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)),$$

$$(1-\gamma_1-\gamma_3)\,\psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) \leq (\gamma_1+\gamma_5+\gamma_6)\,\psi(M(a_{n_k}, a_{n_k}, a_{n_k-1}, t))$$
$$+ (\gamma_1+\gamma_3)\,\psi(M(a_{m_k}, a_{m_k-1}, a_{m_k-1}, t))$$
$$+ \gamma_2\,\psi(M(a_{m_k}, a_{m_k}, a_{m_k-1}, t) + \gamma_4\,\psi(1-\epsilon). \quad (3.2.18)$$

Using (3.2.11), we obtain

$$\psi(M(a_{n_k}, a_{m_k}, a_{m_k}, t)) > (1-\epsilon) \quad (3.2.19)$$

$$(1-\gamma_1-\gamma_3)\,\psi(1-\epsilon) \leq (\gamma_1+\gamma_5+\gamma_6)\,\psi(M(a_{n_k}, a_{n_k}, a_{n_k-1}, t)) + \gamma_2\,\psi(M(a_{m_k}, a_{m_k}, a_{m_k-1}, t)$$
$$(\gamma_1+\gamma_3)\,\psi(M(a_{m_k}, a_{m_k-1}, a_{m_k-1}, t)) + \gamma_4\,\psi(1-\epsilon). \quad (3.2.20)$$

Taking $k \to \infty$ in the above inequality, we obtain

$$(1-\gamma_1-\gamma_3)\,\psi(1-\epsilon) \leq \gamma_4\psi(1-\epsilon) \quad (3.2.21)$$

That is, $(1-\gamma_1-\gamma_3-\gamma_4)\,\psi(1-\epsilon) \leq 0$, and which implies that $\epsilon = 0$ and we get a contradiction.

Hence $\{a_n\}$ is a Cauchy's sequence. Since X is complete there exists $x \in X$ such that $\lim_{n\to\infty} a_n = x$. That is $M(a_n, x, x, t) = 1$ as $n \to \infty$.

Putting $a = a_{n-1}$, $b = x$ and $c = x$ in equation (3.2.1), we get

$$\psi(M(a_n, Tx, Tx, t)) \leq \gamma_1\psi(M(a_{n-1}, x, x, t)) + \gamma_2\psi(M(Tx, Tx, x, t)) + \gamma_3\psi(M(a_n, x, x, t))$$
$$+ \gamma_4\psi(M(a_{n-1}, Tx, Tx, t)) + \gamma_5\psi(M(a_n, a_n, a_{n-1}, t))$$
$$+ \gamma_6\psi\{\max(M(Tx, Tx, x, t), M(a_n, a_n, a_{n-1}, t))\} \quad (3.2.22)$$

Taking $n \to \infty$ in (3.2.22), we get

$$(1-\gamma_2-\gamma_4-\gamma_6)\,\psi(M(x, Tx, Tx, t)) \leq 0,\ t > 0. \quad (3.2.23)$$

Therefore $M(x, Tx, Tx, t) = 1$, and $x = Tx$.

To prove the uniqueness, suppose that $Tx = x$ where $q \neq z$.

$$(1-\gamma_1-\gamma_3-\gamma_4)\,\psi(M(x, c, c, t)) \leq 0,\ t > 0.$$

Hence $x = c$ is the unique fixed point of T.

Corollary 3.3

Let (X, M, $*$) be a complete strong generalized fuzzy metric space with continuous t-norm $*$ and let T be a self-mapping in X. If there exists a control function ψ and $\gamma_i = \gamma_i(t)$, i = 1, 2, 3,..., 6, $\gamma_i \geq 0$ and $\gamma_1 + \gamma_2 + \gamma_3 + 2\gamma_4 + \gamma_5 < 1$ such that

$$\psi(M(Ta, Tb, Tc, t)) \leq \gamma_1\psi(M(a, b, c, t)) + \gamma_2\psi(M(Tb, Tb, c, t)) + \gamma_3\psi(M(Ta, b, c, t)) + \gamma_4\psi(M(a, Tb, Tc, t)) + \gamma_5\psi(M(Ta, Ta, a, t)) \qquad (3.3.1)$$

Then T has a unique fixed point in X.

Proof:

The proof follows by considering the fuzzy contraction on the generalized fuzzy metric space (X, M, $*$),

$\psi(M(Ta, Tb, Tc, t)) \leq \gamma_1\psi(M(a, b, c, t)) + \gamma_2\psi(M(Tb, Tb, c, t)) + \gamma_3\psi(M(Ta, b, c, t)) + \gamma_4\psi(M(a, Tb, Tc, t)) + \gamma_5\psi(M(Ta, Ta, a, t))$.

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