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RESEARCH ARTICLE

# SINGLE SAMPLING PLAN FOR VARIABLES INDEXED BY AQL AND AOQL UNDER YULE'S MODEL

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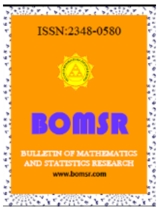


**ABSTRACT**

In this paper, we have studied the effect of Yule's model on single sampling plan for variables indexed by AQL and AOQL. Procedures and Tables are given for the selection of single sampling plans for variables for given AQL and AOQL, based on the assumption of normality and independence are affected when the characteristic of an item posses a normal distribution with Yule's model. The value of n and k are also calculated under the Yule's model.

**Keywords:** Single sampling plan, AQL, AOQL, Yule's model..

## 1. INTRODUCTION

Acceptance sampling is an inspecting procedure applied in statistical quality control. It is a method of measuring random samples of populations called "lots" of materials or products against predetermined standards. Acceptance sampling is a part of operations management or of accounting auditing and services quality supervision. It is important for industrial, but also for business purposes helping decision-making process for the purpose of quality management. Sampling plans are hypothesis tests regarding product that has been submitted for an appraisal and subsequent acceptance or rejection. A single sampling plan defined by AQL and AOQL is much suitable plan for industry in comparison to LTPD, AQL plan because it considers AOQL which is the worst average quality the consumer will receive in the long run, no matter what the incoming quality is. Rejected lots are often a nuisance to the producer as they result in extra work and extra cost. If too many lots are rejected , that will damage the reputation of the producer or supplier. Single sampling by attributes with relaxed requirements were discussed by Ohta and Ichihashi (1988) kanagawa and Ohta (1990), Tamaki et al. (1991), and Grzegorzewski (1998, 2001b). Sampling plan by attributes for vague data were considered by Hrniewicz (1992). Grzegrozewski (2000 b, 2002) also considered sampling plan by variables with fuzzy requirements

## 2. MODEL DESCRIPTION

For processes in which there is serial correlation between observations, a more reasonable model may be

$$x_t = \mu + \xi_t \qquad t = 1,2,...n, \tag{1}$$

where $\mu$ is the mean at time t. The assumption here is that the mean is not a fixed constant but rather continually wanders over time.

Suppose that a correlation test revealed the presence of data dependence and the identification technique suggested Yule's model, then we can express $\xi_t$ of equation (1) as

$$\xi_t = \alpha_1 \xi_{t-1} + \alpha_2 \xi_{t-2} + \varepsilon_t, \quad t = 1,2,...,n, \tag{2}$$

where

$$(i) \quad \varepsilon_t \sim N(O, \sigma_\varepsilon^2)$$

$$(ii) \quad Cov(\varepsilon_t, \varepsilon_\tau) = \begin{cases} \sigma_\varepsilon^2 & t = \tau \\ 0 & t \neq \tau \end{cases} . \tag{3}$$

For stationary, the roots of the characteristic equation of the process

$$\phi(B) = 1 - \alpha_1 B - \alpha_2 B^2 = 0 \tag{4}$$

must lie outside the unit circle, which implies that the parameters $\alpha_1$ and $\alpha_2$ must lie in the triangular region, i.e.,

$$\left. \begin{array}{l} \alpha_2 + \alpha_1 < 1, \\ \alpha_2 - \alpha_1 < 1 \ and \\ -1 < \alpha_2 < 1 \end{array} \right\} . \tag{5}$$

And $G_1^{-1}$ and $G_2^{-1}$ are the roots of the characteristic equation of the process given by equation (4). When the correlation is present in the data, Singh and Singh (1982) have shown the distribution of the sample mean $\overline{X}$, its mean and variance given by

$$E(\bar{x}) = \mu$$

$$Var(\bar{x}) = \frac{\sigma^2}{n} \lambda_{ap}(\alpha_1, \alpha_2, n)$$

$$= \frac{\sigma^2}{n} T^2 , \tag{6}$$

where $T^2 = \lambda_{ap}(\alpha_1, \alpha_2, n)$ depends on the nature of the roots $G_1$ and $G_2$ and for different situations is given as follows :

(i) If $G_1$ and $G_2$ are real and distinct,

$$\lambda_{ap}(\alpha_1,\alpha_2,n) = \left[ \frac{G_1(1-G_2^2)}{(G_1-G_2)(1+G_1G_2)}\ \lambda(G_1,n) - \frac{G_2(1-G_1^2)}{(G_1-G_2)(1+G_1G_2)}\ \lambda(G_2,n) \right]$$

$$= \lambda_{rd}(\alpha_1,\alpha_2,n), \qquad (7)$$

where $\lambda(G,n) = \left[ \frac{1+G}{1-G} - \frac{2G}{n}\ \frac{(1-G^n)}{(1-G)^2} \right]$

(ii) If $G_1$ and $G_2$ are real and equal

$$\lambda_{ap}(\alpha_1,\alpha_2,n) = \left( \frac{1+G}{1-G} \right) - \frac{2G}{n}\ \frac{(1-G^n)}{(1-G)^2} \left[ 1 + \frac{(1+G)^2(1-G^n) - n\,(1-G^2)(1+G^2)}{(1+G^2)(1-G^n)} \right]$$

$$= \lambda_{re}(\alpha_1,\alpha_2,n). \qquad (8)$$

(iii) If $G_1$ and $G_2$ are complex conjugate

$$\lambda_{ap}(\alpha_1,\alpha_2,n) = \left[ Y(d,u) + \frac{2d}{n}(W(d,u,n) + z(d,u,n)) \right]$$

$$= \lambda_{cc}(\alpha_1,\alpha_2,n), \qquad (9)$$

where

$$Y(d,u) = \frac{1-d^4+2d(1-d^2)\cos u}{(1+d^2)(1+d^2-2d\ \cos u)}$$

$$W(d,u,n) = \frac{2d(1+d^2)\sin u - (1+d^4)\sin 2u - d^{n+4}\sin(n-2)u}{(1+d^2)(1+d^2-2d\cos u)^2\ \sin u}$$

$$Z(d,u,n) = \frac{2d^{n+3}\ \sin\,(n-1)u - 2d^{n+1}\ \sin(n+1)u + d^n\sin(n+2)u}{(1+d^2)(1+d^2-2d\ \cos u)^2\ \sin u}$$

$$d^2 = -\alpha_2$$

and $u = \cos^{-1}\left(\frac{\alpha_1}{2d}\right)$.

## 3.SAMPLING PLANS FOR VARIABLES INDEXED BY AQL AND AOQL UNDER YULE'S MODEL

In connection with a single sampling variables plan when $\sigma$ is known the following symbols will be used:

| | | |
|---|---|---|
| L | = | Lower specification Limit |
| U | = | Upper specification Limit |
| n | = | Sample size |
| k | = | Acceptance parameter |
| $\bar{x}$ | = | Sample mean |

$$\Phi(y) = \int_{-\infty}^{y}\left(\frac{1}{\sqrt{2\pi}}\right)\exp\left(-\frac{1}{2}z^2\right)dz, \tag{10}$$

where $z \sim N(0,1)$. The acceptance criterion for the single sampling plan is

Accept the lot if $$\bar{x} + k\sigma \leq U, \tag{11}$$

for the upper specification limit, when use with L, the acceptance criterion is

Accept the lot if $$\bar{x} + k\sigma \geq L, \tag{12}$$

The fraction nonconforming in a given lot will be

$$\Phi(-K_p) = p, \tag{13}$$

with

$$K_p = \frac{U-\mu}{\sigma}, \tag{14}$$

where $K_p$ is the p percent point of the standard normal distribution. If p is the proportion in defective in the lot we know that

$$U = \mu + K_p\sigma \tag{15}$$

and its probability of acceptance under Yule's model will be

$$P_a(p) = \Phi(w), \tag{16}$$

with

$$w = (K_p - k)\frac{\sqrt{n}}{T}. \tag{17}$$

If the quality of the accepted lot is p and all nonconforming units found in the rejected lots are replaced by conforming units in a rectify inspection scheme, the AOQ can be approximated as

$$AOQ = p.P_a\,(p) \tag{18}$$

If $P_m$ is the proportion nonconforming at which AOQ is maximum, one has

$$AOQL = p_m \;\; P_a\,(p_m) \tag{19}$$

If AQL ($P_1$) is prescribed, then corresponding value of $K_{AQL}$ or$K_1$ will be fixed and if $P_a(p_1)$ is fixed at 95% then $w_{AQL}= w_1 = 1.645$. Hence, we have

$$1.645 = (\,K_1 \text{ - } k)\,\frac{\sqrt{n}}{T}\,. \tag{20}$$

So that for a given AQL, k is determined by the sample size n.

### 4. NUMERICAL ILLUSTRATION AND RESULT

For visual comparison the curves for OC and AOQ functions are shown in Fig.-1 and Fig.-2 for different values of $\alpha_1$ and $\alpha_2$. If $\alpha_1 = \alpha_2 = 0$ then $\lambda(0,0,n) = 1$. Table-1 is used for the selection of known $\sigma$ single sampling variable plan under Yule's model. The value of k in complex conjugate roots are approximately same as in independent case but value of n is widely varied. From Table-2, it is seen that the value of OC function shows higher values for the lot of poor quality in all three different cases as compared to the independent observations. Table-3 gives $P_a(p_m)$ values of the plan given in Table-1.

**Table-1: Single Sampling Plans for Variables Indexed by AQLand AOQL under Yule's Model**

| | AOQL | AQL(%) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ($\alpha_1$,$\alpha_2$) | (%) | 0.040 | 0.065 | 0.100 | 0.150 | 0.250 | 0.400 | 0.650 | 1.000 | 1.500 | 2.500 | 4.000 | 6.500 |
| | 0.050 | 47,3.112 | | | | | | | | | | | |
| | 0.125 | 9,2.817 | 16,2.807 | 40,2.831 | | | | | | | | | |
| | 0.320 | 9,2.558 | 6,2.523 | 8,2.500 | 12,2.489 | 31,2.513 | | | | | | | |
| (0, 0) | 0.800 | 9,2.293 | 3,2.249 | 4,2.213 | 4,2.183 | 6,2.154 | 11,2.145 | **29,2.178** | | | | | |
| | 2.000 | | | 2,2.006 | 2,1.860 | 3,1.817 | 4,1.783 | 5,1.748 | 8,1.741 | 18,1.788 | | | |
| | 5.000 | | | | | | | 2,1.337 | 3,1.308 | 3,1.288 | 6,1.284 | 15,1.332 | |
| | 8.000 | | | | | | | | 2,1.044 | 2,1.019 | 3,1.998 | 5,0.999 | 13,1.060 |
| | 0.050 | 225,3.146 | | | | | | | | | | | |
| | 0.125 | 12,2.680 | 25,2.724 | 194,2.868 | 104,2.688 | | | | | | | | |
| (0.8, -0.6) | 0.320 | 5,2.338 | 7,2.336 | 10,2.349 | 17,2.384 | 117,2.535 | | | | | | | |
| real | 0.800 | 3,1.996 | 3,1.979 | 4,1.971 | 5,1.971 | 8,1.988 | 16,2.039 | **184,2.243** | | | | | |
| & equal | 2.000 | | | 3,1.704 | 2,1.553 | 3,1.546 | 4,1.551 | 7,1.576 | 12,1.632 | 50,1.779 | | | |
| | 5.000 | | | | | | | 2,1.028 | 3,1.040 | 4,1.068 | 9,1.146 | 74,1.394 | |
| | 8.000 | | | | | | | | 2,0.701 | 2,0.718 | 4,0.761 | 7,0.847 | 66,1.350 |
| | 0.050 | 141,2.932 | | | | | | | | | | | |
| | 0.125 | 19,2.503 | 88,2.723 | 121,2.637 | | | | | | | | | |
| (0.3, 0.6) | 0.320 | 6,1.972 | 9,2.048 | 16,2.150 | 43,2.321 | 110,2.324 | | | | | | | |
| real & | 0.800 | 3,1.469 | 4,1.514 | 5,1.569 | 7,1.638 | 13,1.770 | 47,2.009 | **78,1.926** | | | | | |
| distinct | 2.000 | | | 3,1.181 | 3,1.019 | 4,1.094 | 6,1.191 | 11,1.343 | 37,1.600 | 79,1.587 | | | |
| | 5.000 | | | | | | | 3,0.512 | 4,0.622 | 7,0.768 | 26,1.109 | 45,1.011 | |
| | 8.000 | | | | | | | | 2,0.124 | 3,0.239 | 6,0.446 | 21,0.808 | 34,0.672 |
| | 0.050 | 231,3.050 | | | | | | | | | | | |
| | 0.125 | 16,2.562 | 46,2.692 | 199,2.764 | | | | | | | | | |
| (0.8, -0.16) | 0.320 | 6,2.112 | 8,2.154 | 13,2.217 | 27,2.322 | 187,2.462 | | | | | | | |
| complex & | 0.800 | 3,1.676 | 4,1.696 | 5,1.723 | 6,1.762 | 10,1.843 | 27,1.988 | **127,2.081** | | | | | |
| conjugate | 2.000 | | | 3,1.379 | 3,1.228 | 4,1.268 | 5,1.325 | 9,1.419 | 21,1.576 | 148,1.764 | | | |
| | 5.000 | | | | | | | 3,0.710 | 4,0.777 | 6,0.870 | 15,1.081 | 75,1.222 | |
| | 8.000 | | | | | | | | 2,0.346 | 3,0.418 | 5,0.552 | 12,0.776 | 56,0.912 |

**Table 2: Performance Characteristics of the variables plan for Yule's Model**

**For AQL=.065, AOQL=.008, U=10, S.D=2**

| $(\alpha_1,\alpha_2)$ | μ | v' | p(%) | w | $P_a$ | AOQ |
|---|---|---|---|---|---|---|
| (0, 0) | 5.0000 | 2.5000 | 0.62 | 1.6155 | 0.9469 | 0.5880 |
| | 5.4000 | 2.3000 | 1.07 | 0.5385 | 0.7049 | 0.7559 |
| | 5.8000 | 2.1000 | 1.79 | -0.5385 | 0.2951 | 0.5272 |
| | 6.0000 | 2.0000 | 2.28 | -1.0770 | 0.1407 | 0.3202 |
| (0.8, -0.6) real & equal | 5.0000 | 2.5000 | 0.62 | 3.2007 | 0.9993 | 0.6205 |
| | 5.4000 | 2.3000 | 1.07 | 1.0669 | 0.8570 | 0.9190 |
| | 5.8000 | 2.1000 | 1.79 | -1.0669 | 0.1430 | 0.2555 |
| | 6.0000 | 2.0000 | 2.28 | -2.1338 | 0.0164 | 0.0374 |
| (0.3, 0.6) real & distinct | 5.0000 | 2.5000 | 0.62 | 2.9177 | 0.9982 | 0.6199 |
| | 5.4000 | 2.3000 | 1.07 | 1.9451 | 0.9741 | 1.0447 |
| | 5.8000 | 2.1000 | 1.79 | 0.9726 | 0.8346 | 1.4910 |
| | 6.0000 | 2.0000 | 2.28 | 0.4863 | 0.6866 | 1.5621 |
| (0.8, -0.16) complex & conjugate | 5.0000 | 2.5000 | 0.62 | 2.8315 | 0.9977 | 0.6195 |
| | 5.4000 | 2.3000 | 1.07 | 1.4158 | 0.9216 | 0.9883 |
| | 5.8000 | 2.1000 | 1.79 | 0.0000 | 0.5000 | 0.8932 |
| | 6.0000 | 2.0000 | 2.28 | -0.7079 | 0.2395 | 0.5449 |

**Table-3 : $P_a(p_m)$ Values of Known Sigma Plans under Yule's Model**

| | | AQL(%) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(\alpha_1,\alpha_2)$ | AOQL(%) | 0.040 | 0.065 | 0.100 | 0.150 | 0.250 | 0.400 | 0.650 | 1.000 | 1.500 | 2.500 | 4.000 | 6.500 |
| (0, 0) | 0.050 | 0.700 | | | | | | | | | | | |
| | 0.125 | 0.389 | 0.515 | 0.700 | | | | | | | | | |
| | 0.320 | 0.258 | 0.318 | 0.392 | 0.489 | 0.696 | | | | | | | |
| | 0.800 | 0.203 | 0.236 | 0.274 | 0.321 | 0.402 | 0.514 | 0.719 | | | | | |
| | 2.000 | | | 0.104 | 0.255 | 0.297 | 0.349 | 0.425 | 0.524 | 0.672 | | | |
| | 5.000 | | | | | | | 0.336 | 0.379 | 0.435 | 0.540 | 0.714 | |
| | 8.000 | | | | | | | | 0.358 | 0.395 | 0.460 | 0.553 | 0.720 |
| (0.8, -0.6) real & equal | 0.050 | 0.878 | | | | | | | | | | | |
| | 0.125 | 0.509 | 0.714 | 0.876 | 0.822 | | | | | | | | |
| | 0.320 | 0.315 | 0.399 | 0.445 | 0.568 | 0.849 | | | | | | | |
| | 0.800 | 0.234 | 0.278 | 0.300 | 0.355 | 0.454 | 0.595 | 0.897 | | | | | |
| | 2.000 | | | 0.114 | 0.274 | 0.322 | 0.383 | 0.475 | 0.601 | 0.813 | | | |
| | 5.000 | | | | | | | 0.358 | 0.410 | 0.478 | 0.611 | 0.877 | |
| | 8.000 | | | | | | | | 0.379 | 0.422 | 0.501 | 0.619 | 0.987 |
| (0.3, 0.6) real & distinct | 0.050 | 0.836 | | | | | | | | | | | |
| | 0.125 | 0.553 | 0.805 | 0.836 | | | | | | | | | |
| | 0.320 | 0.335 | 0.427 | 0.549 | 0.732 | 0.843 | | | | | | | |
| | 0.800 | 0.245 | 0.292 | 0.348 | 0.420 | 0.554 | 0.775 | 0.832 | | | | | |
| | 2.000 | | | 0.132 | 0.306 | 0.366 | 0.447 | 0.575 | 0.777 | 0.856 | | | |
| | 5.000 | | | | | | | 0.400 | 0.468 | 0.563 | 0.779 | 0.836 | |
| | 8.000 | | | | | | | | 0.416 | 0.473 | 0.584 | 0.781 | 0.831 |
| (0.8, -0.16) complex & conjugate | 0.050 | 0.878 | | | | | | | | | | | |
| | 0.125 | 0.509 | 0.714 | 0.877 | | | | | | | | | |
| | 0.320 | 0.315 | 0.399 | 0.506 | 0.661 | 0.885 | | | | | | | |
| | 0.800 | 0.234 | 0.278 | 0.329 | 0.394 | 0.513 | 0.696 | 0.872 | | | | | |
| | 2.000 | | | 0.120 | 0.293 | 0.349 | 0.421 | 0.534 | 0.699 | 0.899 | | | |
| | 5.000 | | | | | | | 0.383 | 0.445 | 0.528 | 0.704 | 0.877 | |
| | 8.000 | | | | | | | | 0.401 | 0.453 | 0.550 | 0.708 | 0.872 |

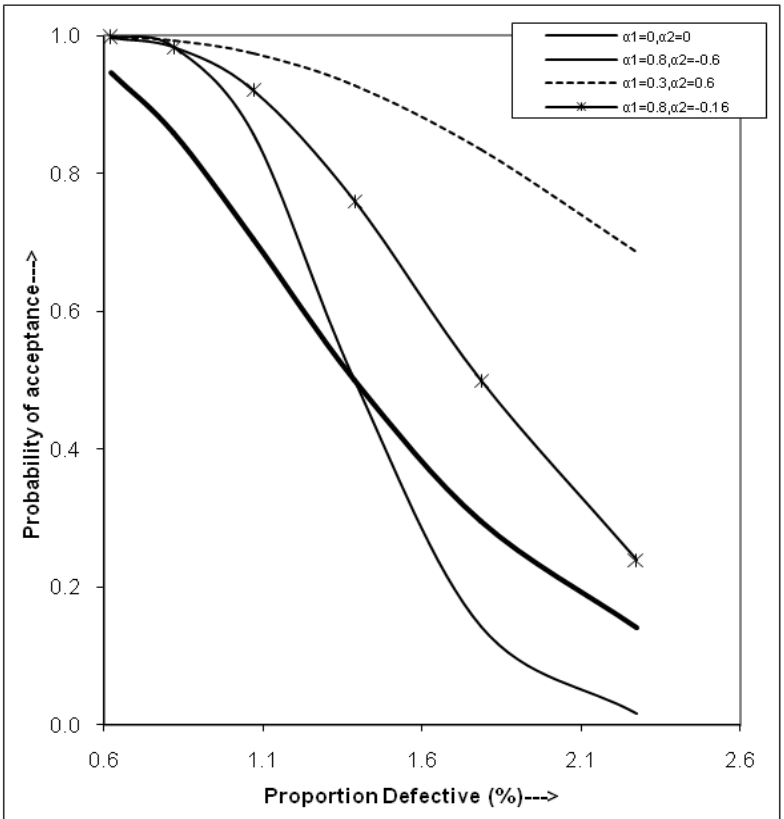


**Fig.-1: OC Curve Under Yule's Model For AQL=.065, AOQL=.008,**

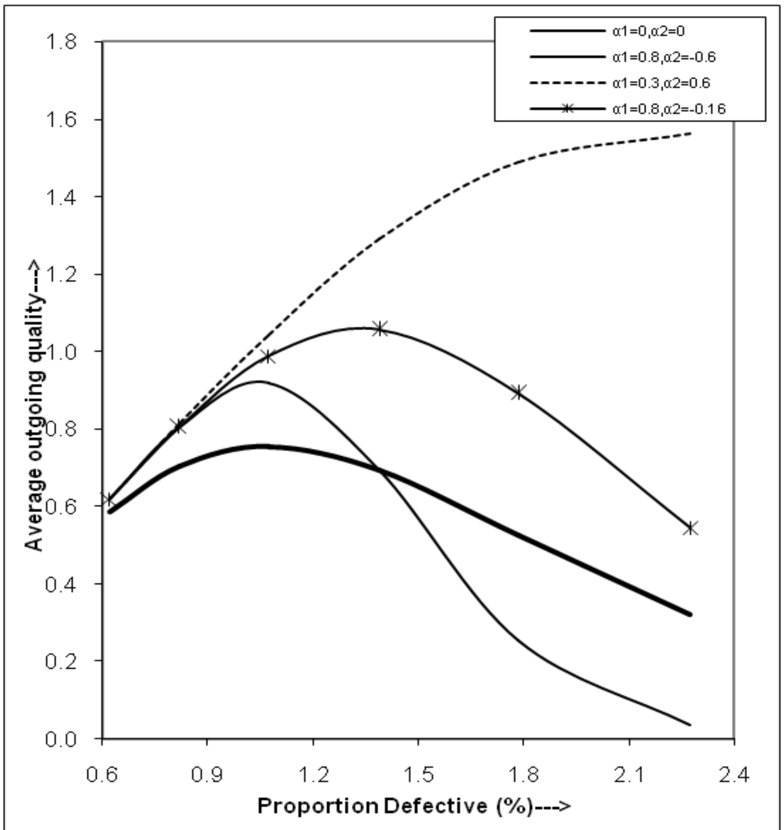


**Fig.-2: AOQ Curve Under Yule's Model For AQL=.065, AOQL=.008**

## 5. Conclusion

In order to avoid such inconvenience the producer should maintain the process quality more or less at the AQL. The high rate of rejection of lot at $p = p_m$ will also indirectly put pressure on the producer to improve the submitted quality. Through the visual comparison from the Figure-1 and Figure-2 shows that the effect of Yule's model is serious over the OC and ASN curves. Yule's model give the serious impact on the performance of acceptance sampling plans, causing a dramatic increase in the frequency of the false alarms. Auto-correlated process data renders most conventional acceptance sampling plans uninformative. The sampling plan for variables under Yule's model should be useful in correcting different types of upsets in a reasonably wide variety of industrial control.

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