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RESEARCH ARTICLE

t- BIFUZZY IDEALS OF PS-ALGEBRAS

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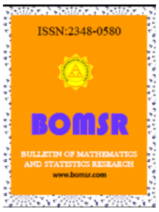

ABSTRACT

In this paper introduced the concept of t- Bifuzzy Ideals of PS-Algebras ,while their some properties and proposition are discussed.

Keywords: PS Algebra, Bifuzzy Ideals, t-Bifuzzy Ideals, Distributive Lattice.

1. INTRODUCTION

BCK algebra and BCI algebra are two classes of abstract algebras introduced by Iseki and Tanaka[4,5]. It is known that the class BCI algebra is a proper superclass of the class BCK algebra. Neggers and Kim HS[7] introduced d-algebra and Q algebra as the generalisation of BCK/BCI algebras. A new algebraic structure called KU algebra was introduced by Prabpayak and Leerawat[8] along with some properties. By generalising BCK/BCI/Q/d/KU algebras, Priya and Ramachandran[9,10] introduced a new algebraic structure called PS algebra.

Zadeh introduced a new concept called fuzzy set in 1965[15] and it is now a rigorous area of research with manifold applications ranging from engineering and computer science to medical diagnosis and social behaviour studies. In 1983, Atanassov generalised fuzzy sets and introduced a notion called Intuitionistic fuzzy sets which in 1995 renamed as bifuzzy sets by Gerstenkorn and Manko[3]. The elements of the bifuzzy sets are featured by an additional degree which is called the degree of uncertainity. Bifuzzy sets have also been defined by Takeuti and Titanti[14]. They considered bifuzzy logic in the narrow sense and derived a set theory from logic which they called bifuzzy set theory. The bifuzzy sets have drawn the attention of many researchers in the last decades. This is manily due to the fact that bifuzzy sets are consistent with human behaviour, by reflecting and modelling the hesitancy present in real-life situations. These kind of fuzzy sets have gained a wide recognition as a useful tool in the modelling of some uncertain phenomena. These have numerous applications in various areas of sciences, for instance, computer science, mathematics, medicine, chemistry, economics, astronomy, etc.

2. PRELIMINARIES

2.1 Definition

Let Ψ be a non-empty set with binary operation $*$ and constant. Then Ψ is a **PS-algebra** if it satisfies

i) $\upsilon * \upsilon = 0$

ii) $\upsilon * 0 = 0$

iii) $\forall\, \upsilon, \delta \in \mathcal{P}, \upsilon \leq \delta\ is\ defined\ by\ \delta * \upsilon = 0 \upsilon * \delta = 0\ \&\ \delta * \upsilon = 0 \ \Rightarrow \upsilon = \delta \ \ \forall\, \upsilon, \delta \in \mathcal{P}$

2.2 Definition

Let $\mathcal{L}$ be a non-empty subset of PS-algebra, then $\mathcal{L}$ is called a **PS subalgebra** of Ψ if

$\upsilon * \delta \in \mathcal{L} \forall\, \upsilon, \delta \in \mathcal{L}$. Also every sub-algebra contains the constant 0 of PS-algebra.

2.3 Definition

The subset $\mathcal{J}$ of $\Psi$is called a **PS-Ideal** if it satisfies

i) $0 \in \mathcal{J}$

ii) $\delta * \upsilon \in \mathcal{J}\ \& \delta \in \mathcal{J} \ \Rightarrow \upsilon \in \mathcal{J}$

2.4 Definition

A mapΩ: $\mathcal{P} \rightarrow\ [0,1]$, where $\Omega$is a fuzzy set is called a **fuzzy ideal** of PS-algebra if

i) $\rho(0) \geq \rho(\upsilon)(\forall \upsilon \in \mathcal{P})$

ii) $\rho(\upsilon) \geq min\{\rho_\Omega(\delta * \upsilon), \rho_\Omega(\delta)\}(\forall\, \upsilon, \delta \in \mathcal{P})$

2.5 Definition

Let$\Omega = (\rho_\Omega, \tau_\Omega)$: $\mathcal{P} \rightarrow\ [0,1] \times [0,1]$, Ω is called as **bifuzzy set** in $\mathcal{P}$ if$\rho_\Omega(\upsilon) + \tau_\Omega(\upsilon) \leq 1$ for all $\upsilon \in \mathcal{P}$, where ρ_Ω: $\mathcal{P} \rightarrow\ [0,1]$ is the degree of membership and τ_Ω: $\mathcal{P} \rightarrow\ [0,1]$ is the degree of non-membership.

2.6 Definition

A bifuzzy set $\Omega = (\rho_\Omega, \tau_\Omega)$:$\mathcal{P} \rightarrow\ [0,1] \times [0,1]$ is called a **bifuzzy ideal of PS-algebra**Ψ if the following conditions are true

i) $(\rho_\Omega(e) \geq \rho_\Omega(\upsilon), \tau_\Omega(e) \leq \tau_\Omega(\upsilon))(\forall\, \upsilon \in \mathcal{P})$

ii) $(\rho_\Omega(\upsilon) \geq min\{\rho_\Omega(\delta * \upsilon), \rho_\Omega(\delta)\})(\forall\, \upsilon, \delta \in \mathcal{P})$

iii) $(\tau_\Omega(\upsilon) \leq max\{\tau_\Omega(\delta * \upsilon), \tau_\Omega(\delta)\})(\forall\, \upsilon, \delta \in \mathcal{P})$

2.7 Theorem

Let $\Omega = (\rho_\Omega, \tau_\Omega)$be a bifuzzy ideal of PS algebra and let $(\alpha_1, \beta_1), (\alpha_2, \beta_2) \in I(\rho_\Omega) \times I(\tau_\Omega)$ with $\alpha_a + \beta_a \leq 1$, here a = 1,2 and $I(\rho_\Omega), I(\tau_\Omega)$denotes the images of ρ_Ω, τ_Ω respectively. Then $\mathcal{P}_\Omega^{(x_1,y_1)} = \mathcal{P}_\Omega^{(x_2,y_2)}$ iff $(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$.

Proof: Assume $(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$ then it is obvious that $\mathcal{P}_\Omega^{(\alpha_1,\beta_1)} = \mathcal{P}_\Omega^{(\alpha_2,\beta_2)}$.

Conversely, If $\mathcal{P}_\Omega^{(\alpha_1,\beta_1)} = \mathcal{P}_\Omega^{(\alpha_2,\beta_2)}$, then to prove $(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$.

Given $(\alpha_1, \beta_1) \& (\alpha_2, \beta_2) \in I(\rho_\Omega) \times I(\tau_\Omega)$, then $\exists\, \upsilon \in \mathcal{P} \ni$

$$\rho_\Omega(\upsilon) = \alpha_1 \& \tau_\Omega(\upsilon) = \beta_1$$

$$\Rightarrow \upsilon \in \mathcal{P}_\Omega^{(\alpha_1,\beta_1)} = \mathcal{P}_\Omega^{(\alpha_2,\beta_2)}$$

We get $\alpha_1 = \rho_\Omega(\upsilon) \geq \alpha_2 \& \beta_1 = \tau_\Omega(\upsilon) \leq \beta_2$

In the same way, $\alpha_1 \leq \alpha_2 \& \beta_1 \geq \beta_2$.

Thus $(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$

3. t-BIFUZZY IDEALS OF PS-ALGEBRAS

3.1 Definition

Let Ω be a bifuzzy set of PS algebra. Then $\Omega_t$is said to be a t-bifuzzy set of Ψ if

$\Omega_t = \{\upsilon, \rho_{\Omega_t}(\upsilon), \tau_{\Omega_t}(\upsilon) | \upsilon \in \mathcal{P}\}$, where $\rho_{\Omega_t}(\upsilon) = min\{\rho_\Omega(\upsilon), t\}$ and $\tau_{\Omega_t}(\upsilon) = max\{\tau_\Omega(\upsilon), t\} \forall\, \upsilon \in \mathcal{P}$.

3.2 Definition

A t-bifuzzy set $\Omega_t = (\rho_{\Omega_t}, \tau_{\Omega_t}) : \mathcal{P} \to [0,1] \times [0,1]$ is called a t-bifuzzy ideal of PS algebra Ψ if the below conditions are true

i) $(\rho_{\Omega_t}(e) \geq \rho_{\Omega_t}(\upsilon), \tau_{\Omega_t}(e) \leq \tau_{\Omega_t}(\upsilon))(\forall\, \upsilon \in \mathcal{P})$

ii) $(\rho_{\Omega_t}(\upsilon) \geq min\{\rho_{\Omega_t}(\delta * \upsilon), \rho_{\Omega_t}(\delta)\})(\forall\, \upsilon, \delta \in \mathcal{P})$

iii) $(\tau_{\Omega_t}(\upsilon) \leq max\{\tau_{\Omega_t}(\delta * \upsilon), \tau_{\Omega_t}(\delta)\})(\forall\, \upsilon, \delta \in \mathcal{P})$

3.3 Theorem

Let $\Omega_t = (\rho_{\Omega_t}, \tau_{\Omega_t})$be a t-bifuzzy set of PS algebra. Then prove that Ω_t is a t-bifuzzy ideal of PS algebra iff$\mathcal{P}_{\Omega_t}^{(\alpha,\beta)}$ is an ideal of PS algebra $\forall (\alpha, \beta) \in I(\rho_{\Omega_t}) \times I(\tau_{\Omega_t}) \& \alpha + \beta \leq 1$.

Proof: Assume$\Omega_t = (\rho_{\Omega_t}, \tau_{\Omega_t})$is a t-bifuzzy ideal of PS algebra (Ψ).

then to prove $\mathcal{P}_{\Omega_t}^{(\alpha,\beta)}$ is also an ideal of Ψ

$\Rightarrow \zeta(\rho_{\Omega_t}, \alpha)$ & $\xi(\tau_{\Omega_t}, \alpha)$ are ideals.

Thus $\mathcal{P}_{\Omega_t}^{(\alpha,\beta)} = \zeta(\rho_{\Omega_t}, \alpha) \cap \xi(\tau_{\Omega_t}, \alpha)$ is an ideal of Ψ

Conversely, Assume $\mathcal{P}_{\Omega_t}^{(\alpha,\beta)}$is an ideal of Ψ& let $\Omega_t = (\rho_{\Omega_t}, \tau_{\Omega_t})$ be a t-bifuzzy set on Ψ.

obvious that $\Omega_t = (\rho_{\Omega_t}, \tau_{\Omega_t})$ is a t-bifuzzy ideal of Ψ.

Let $\upsilon, \delta \in \mathcal{P} \ni \Omega_t(\delta * \upsilon) = (\alpha_1, \beta_1) \& \Omega_t(\delta) = (\alpha_2, \beta_2)$

$$\rho_{\Omega_t}(\delta * \upsilon) = \alpha_1, \tau_{\Omega_t}(\delta * \upsilon) = \beta_1$$

$$\rho_{\Omega_t}(\delta) = \alpha_2, \quad \tau_{\Omega_t}(\delta) = \beta_2$$

We can assume$(\alpha_1, \beta_1) \leq (\alpha_2, \beta_2)$, that is $\alpha_1 \leq \alpha_2 \& \beta_2 \leq \beta_1$

$$\Rightarrow \mathcal{P}_{\Omega_t}^{(\alpha_2,\beta_2)} \subseteq \mathcal{P}_{\Omega_t}^{(\alpha_1,\beta_1)} \text{ Where } \upsilon, \delta \in \mathcal{P}_{\Omega_t}^{(\alpha_1,\beta_1)}$$

$$\Rightarrow \upsilon \in \mathcal{P}_{\Omega_t}^{(\alpha_1,\beta_1)} \text{ Therefore } \mathcal{P}_{\Omega_t}^{(\alpha_1,\beta_1)} \text{ is an ideal of } \Psi.$$

Hence,

$$\rho_{\Omega_t}(\upsilon) \geq \alpha_1 = min\{\rho_{\Omega_t}(\delta * \upsilon), \rho_{\Omega_t}(\delta)\}$$

$$\tau_{\Omega_t}(\upsilon) \leq \beta_1 = max\{\tau_{\Omega_t}(\delta * \upsilon), \tau_{\Omega_t}(\delta)\}$$

Thus $\Omega_t = (\rho_{\Omega_t}, \tau_{\Omega_t})$ is a t-bifuzzy ideal of PS algebra.

3.4 Theorem

Let $\Omega_t = (\rho_{\Omega_t}, \tau_{\Omega_t})$be a t-bifuzzy ideal of PS algebra and let $(\alpha_1, \beta_1), (\alpha_2, \beta_2) \in I(\rho_{\Omega_t}) \times I(\tau_{\Omega_t})$ with $\alpha_a + \beta_a \leq 1$, here a = 1,2 and $I(\rho_{\Omega_t}), I(\tau_{\Omega_t})$ denotes the images of ρ_Ω, τ_Ωrespectively. Then $\mathcal{P}_{\Omega_t}^{(x_1,y_1)} = \mathcal{P}_{\Omega_t}^{(x_2,y_2)}$ iff $(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$.

Proof: Assume $(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$

Then it is obvious that $\mathcal{P}_{\Omega_t}^{(\alpha_1,\beta_1)} = \mathcal{P}_{\Omega_t}^{(\alpha_2,\beta_2)}$.

Conversely,

If $\mathcal{P}_{\Omega_t}^{(\alpha_1,\beta_1)} = \mathcal{P}_{\Omega_t}^{(\alpha_2,\beta_2)}$, then to prove $(\alpha_1, \beta_1) = (\alpha_2, \beta_2)$.

Given $(\alpha_1, \beta_1) \& (\alpha_2, \beta_2) \in I(\rho_{\Omega_t}) \times I(\tau_{\Omega_t})$, then $\exists\, \upsilon \in \mathcal{P} \ni \rho_{\Omega_t}(\upsilon) = \alpha_1 \& \tau_{\Omega_t}(\upsilon) = \beta_1 \Rightarrow \upsilon \in \mathcal{P}_{\Omega_t}^{(\alpha_1,\beta_1)} = \mathcal{P}_{\Omega_t}^{(\alpha_2,\beta_2)}$ We get $\alpha_1 = \rho_{\Omega_t}(\upsilon) \geq \alpha_2 \& \beta_1 = \tau_{\Omega_t}(\upsilon) \leq \beta_2$

$\alpha_1 \leq \alpha_2 \& \beta_1 \geq \beta_2 \;\; \Rightarrow (\alpha_1, \beta_1) = (\alpha_2, \beta_2)$

3.5 Theorem

Prove that a complete distributive lattice is formed by the family of t-bifuzzy ideals of under t-bifuzzy set inclusion ($\subset$) ordering.

Proof: Let $\{\Omega_{t_i} \ni i \in \mathcal{I}\}$ be a family of t-bifuzzy ideals of Ψ.

Since, we know that [0,1] is a lattice of complete distribution under usual ordering in [0,1]. It is enough to prove that $\cap\, \Omega_{t_i} = \left(\wedge \rho_{\Omega_{t_i}}, \vee \tau_{\Omega_{t_i}}\right)$ is a t-bifuzzy ideal of Ψ.

For any $\upsilon \in \mathcal{P}$, $\left(\vee_{i \in \mathcal{I}} \rho_{\Omega_{t_i}}\right)(e) = sup\; \rho_{\Omega_{t_i}}(e) \geq sup\; \rho_{\Omega_{t_i}}(\upsilon) = \left(\vee_{i \in \mathcal{I}} \rho_{\Omega_{t_i}}\right)(\upsilon) \&$

$$\left(\wedge_{i \in \mathcal{I}} \tau_{\Omega_{t_i}}\right)(e) = inf\; \tau_{\Omega_{t_i}}(e) \leq inf\; \tau_{\Omega_{t_i}}(\upsilon) = \left(\wedge_{i \in \mathcal{I}} \tau_{\Omega_{t_i}}\right)(\upsilon)$$

Let $\upsilon, \delta \in \mathcal{P}$, then $(\vee \rho_{\Omega_{t_i}})(\upsilon) = sup\left\{\rho_{\Omega_{t_i}}(\upsilon) | \; i \in \mathcal{I}\right\}$

$$\geq sup\left\{max\;(\rho_{\Omega_{t_i}}(\delta * \upsilon) \ni i \in \mathcal{I}, \rho_{\Omega_{t_i}}(\delta)) | \; i \in \mathcal{I}\right\}$$

$$= max\left(sup\left\{\rho_{\Omega_{t_i}}(\delta * \upsilon) \ni i \in \mathcal{I}\right\}, sup\left\{\rho_{\Omega_{t_i}}(\delta) \mid i \in \mathcal{I}\right\}\right)$$

$$= max\left(\left(\vee\rho_{\Omega_{t_i}}\right)(\delta * \upsilon), \left(\vee\rho_{\Omega_{t_i}}\right)(\delta)\right)$$

$$\left(\wedge\tau_{\Omega_{t_i}}\right)(\upsilon) = inf\ \left\{\tau_{\Omega_{t_i}}(\upsilon) \mid i \in \mathcal{I}\right\}$$

$$\leq inf\left\{min\left(\tau_{\Omega_{t_i}}(\delta * \upsilon), \tau_{\Omega_{t_i}}(\delta)\right) \mid i \in \mathcal{I}\right\}$$

$$= min\left(inf\left\{\tau_{\Omega_{t_i}}(\delta * \upsilon) \ni i \in \mathcal{I}\right\}, inf\left\{\tau_{\Omega_{t_i}}(\delta) \mid i \in \mathcal{I}\right\}\right)$$

$$= min\left(\left(\wedge\tau_{\Omega_{t_i}}\right)(\delta * \upsilon), \left(\wedge\tau_{\Omega_{t_i}}\right)(\delta)\right)$$

Thus $\cap\, \Omega_{t_i} = \left(\wedge\rho_{\Omega_{t_i}}, \vee\tau_{\Omega_{t_i}}\right)$ is a t-bifuzzy ideal of Ψ.

CONCLUSION

In this paper, an attempt has been made to study t – bifuzzy ideals of PS algebras and their properties are discussed. Further development can be made in these ideals by using rings and also in soft set, smooth set and floppy set.

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