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RESEARCH ARTICLE

APPLICATION OF FLUID DYNAMICS IN CALCULATION OF FLOW VELOCITY AND OIL RECOVERY IN RESERVOIRS

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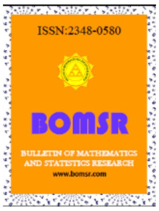

ABSTRACT

Reservoir rocks contain pores where water, oil, and natural gas are stored within the Earth. Calculating the reservoir's oil extraction volume is a crucial task in the oil industry. This calculation entails determining the velocity of fluid movement by applying the principles and equations derived from $Fluid\ Dynamics$. This formula is typically used to analyse the flow of fluid through a control volume with a well-defined shape. Various examples of cylindrical objects include cylinders, curved pipes, and straight pipes with varying diameters at the input and output ends. However, the calculation of fluid flow velocity becomes challenging due to the diverse forms of control volume in reservoir rock, which serves as a medium for fluid flow. This paper presents a method that utilises laboratory permeability measurements and various theories in Fluid Dynamics to calculate the velocity of fluid flow from a control volume. This method has demonstrated its capability to calculate fluid flow velocity and oil recovery in reservoir rocks, with a reasonably high level of accuracy. Permeability refers to the physical capacity of rocks to allow the flow of fluids. Alternatively, rock permeability can be regarded as a fixed value that represents the ratio between theoretical calculations based on fluid dynamics formula and simplified calculations assuming the rock sample as a medium or cylindrical control volume. The calculated flow velocity is the fractional flow velocity, which refers to the velocity of oil or water relative to the total fluid flow velocity ($oil + water$) within the rock.

Keywords: fluid dynamics, velocity of fluid flow, Darcy Law

INTRODUCTION

In the oil industry, calculating the oil yield that can be produced is an important thing to do. Because oil flows with water in reservoir rocks towards production wells, the calculation must start from calculating the fluid flow rate in the oil-water system. Fluid Dynamics, which is a branch of Physics, has provided tools in the form of principles and formulas that can be used for the above calculations. However, for the case of fluid flow in reservoir rocks, adaptations need to be made so that the tools can be applied. This is because water and oil flow in reservoir rocks through various grooves, as control volumes, the shape and diameter of which cannot be known in advance. These grooves or control volumes are formed by the arrangement of pores that form them. Even if the shape of each control volume can be known, the calculation will still be difficult to do, without adaptation steps, because it will involve many calculations for each fluid flow rate in each control volume.

The adaptation that can be done is by involving laboratory measurements of rock samples taken from the reservoir. From this measurement, we can obtain the value of a typical rock parameter called permeability which, when combined with the principles and formulas in Fluid Dynamics, can then calculate the fluid flow and oil recovery as mentioned above.

In this paper, the author will present a description of the application of Fluid Dynamics, supported by the results of laboratory permeability measurements on rock samples, for calculating oil and water flow in reservoir rocks and calculating the oil recovery that can be produced from the reservoir in question.

Fluid Dynamics and Darcy's Law

In Fluid Dynamics, the velocity of fluid flowing irrotationally is known, which can be written as:

$u = \frac{q}{A} = \frac{d\emptyset}{dL}$ … Eq. (2.1)

u is the fluid flow velocity; q is the fluid flow rate and A is the cross-sectional area penetrated by the fluid flow. $\emptyset$ is the potential of the fluid flow and L is the distance from the flow path. Schematically, this can be depicted as in Figure 1 as follows:

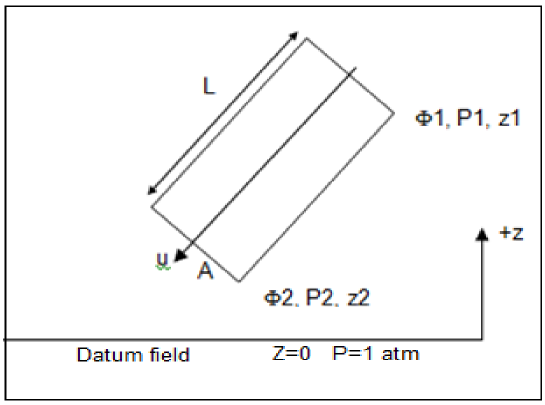

Figure 1 Schematic of irrotational flow in homogeneous cylindrical tube media

The scheme given in Figure 1 above illustrates the fluid flow in a cylindrical tube-shaped media or control volume. Different forms of control volumes will produce different u values. This difference concerns the difference in the cross-sectional diameter A or even differences in the shape of the media (for example, curved tubes and so on). The flow that occurs in rocks can be viewed as an immiscible fluid flow that flows irrotationally and laminarly so that the use of equation (2.1) above is quite relevant for calculating the fluid flow velocity in rocks. However, as stated in the introduction above, adaptation is still needed so that equation (2.1) above can be used. This is because in rocks there are various shapes and sizes of flow media or control volumes in the form of grooves formed by the arrangement of rock pores. The adaptation referred to above is reflected by Darcy's experiment which can be described as in Figure 2. Figure 2 schematically depicts the flow grooves or control volumes that may exist in rocks.

In his experiment, Darcy found that the speed of water flow through sandstone, where all the pore spaces have been 100% saturated by water, is always proportional to the difference in height of the water column in the manometer and inversely proportional to the length of the rock. Mathematically, this can be written as:

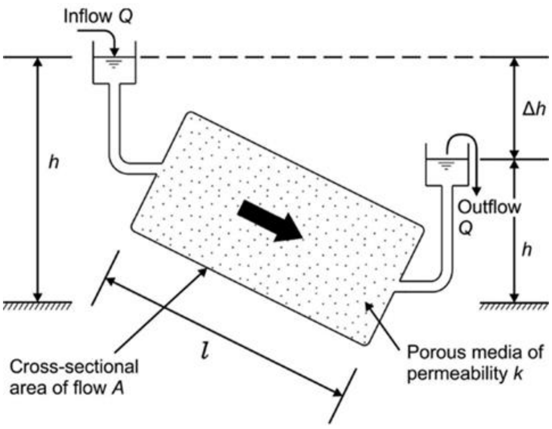

Figure 2 Schematic of Darcy's experiment

u is the water flow rate in cm/sec, A is the cross-sectional area of the rock, $\triangle h$ is the difference in height of the water column in the manometer, L is the length of the rock and K is the constant of proportionality.

$u = \frac{q}{A} = K \frac{h_1 - h_2}{L} = \text{K} \frac{\Delta h}{L}$ … Eq. (2.2)

From figure 1 above it can be seen that at each point of position on the rock there will be a pressure P of:

$P = \rho g(h - z)$ … Eq. (2.3)

or

$h = \frac{1}{g}(\frac{p}{\rho} + gz)$ … Eq. (2.4)

and

$\frac{dh}{dl} = \frac{1}{g}\frac{d}{dl}(\frac{p}{\rho} + gz)$ … Eq. (2.5)

If we write equation (2.2) in differential form:

$u = \text{K}\frac{dh}{dL}$ … Eq.(2.6)

So, by substituting equation (2.5) into equation (2.6) we will get:

$u = \text{K}\frac{dh}{dL} = \frac{K}{g}\frac{d}{dl}(\frac{p}{\rho} + gz)$ … Eq. (2.7)

where g is the acceleration due to gravity and ρ is the density of the fluid. The constant K/g is only valid for water flow, as used by Darcy. has the dimensions of length times force per unit mass or the dimensions of potential energy per unit mass $\frac{d}{dl}(\frac{p}{\rho} + gz)$ unit of mass. In the context of the discussion in this paper, this quantity has the same value as the potential flow of the fluid. Thus, equation (2.7) above can be written as:

$u = \text{K}\frac{dh}{dL} = \frac{K}{g}\frac{d\emptyset}{dl}$ … Eq. (2.8)

Experiments conducted by experts later proved that the equation above can be changed into a general form that applies to the flow of water, oil and gas in all directions, to:

$u = \frac{k\rho}{\mu}\frac{d\emptyset}{dl} = -\frac{k\rho}{\mu}\frac{d}{dl}\left(\frac{p}{\rho} + gz\right)$ … Eq. (2.9)

ρ is the density of the fluid and μ is the viscosity of the fluid. The minus sign indicates that the flow occurs from a low point with high potential to a high point with low potential. Equation (2.1) is a formula that has been provided by Fluid Dynamics and equation (2.9) is an adaptation of equation (2.1). Equation (2.9) has a similar form to equation (2.1) with the addition of an element k known as permeability, ρ and μ. Each rock has a different value of Permeability k. Physically, k is translated as the ability of the rock to flow fluids.

The assumption used is that the fluid flow is considered as a laminar and irrotational flow. The water and oil fluids used must be incompressible fluids. In Darcy units, equation (2.9) above can be written as:

$u = \frac{q}{A} == -\frac{k}{\mu}\left(\frac{dp}{dl} + \frac{\rho g}{1.0133x10^6}\frac{dZ}{dl}\right)$ … Eq. (2.10)

$q = -\frac{kA}{\mu}\left(\frac{dp}{dl} + \frac{\rho g}{1.0133x10^6}\frac{dz}{dl}\right)$ … Eq. (2.11)

Relative permeability

The permeability in equations (2.10) and (2.11) above is called absolute permeability. This means that the permeability is obtained when the rock is fully saturated with water (S_w= 100% or S_o = 0%) or when the rock is fully saturated with oil (S_o = 100% or S_w = 0%). In reality, the pores of reservoir rocks are usually filled with oil fluids together with water. Therefore, the calculation of fluid flow in the reservoir must involve relative permeability which is defined as:

$K_{ro(Sw)=\frac{K_0(Sw)}{k}\, dan\, k_{rw}(Sw)=\frac{K_w(Sw)}{k}}$… Eq. (3.1)

until obtained:

$K_{ro(Sw)=k.k_{ro}(Sw) dan\, k_w(Sw)=k.K_{rw}(Sw)}$ … Eq. (3.2)

k_o is the oil permeability at a certain water saturation S_w and k_w is the water permeability at the water saturation in question. K_{ro} and k_{rw} are the relative permeabilities of oil and water respectively at the respective water saturations. Because the rock pores are saturated with oil and water, the following applies:

$S_w = \frac{V_w}{V_f}$ *and* $S_w = \frac{V_o}{V_f}$… Eq. (3.3)

$S_w = 1 - S_o$… Eq. (3.4)

where V_w is the volume of water in the pore space, V_o is the volume of oil in the pore space and V_f is the volume of the pore space. So is the oil saturation in the rock pores. Determination of k_{ro} and k_{rw} can be done in the laboratory. These methods include: steady state, unsteady state and determination of relative permeability through capillary pressure data.

Fractional Flow

Schematically, the oil reservoir can be depicted as in Figure 2 below:

Figure 2 (a) depicts a scheme of a reservoir located below the earth's surface along with production wells that deliver oil to the ground surface. Figure 2 (b) depicts the scheme in Figure 2 (a) in more detail.

$q_o = -\frac{k.k_{ro}A}{\mu_O}\left[\frac{\partial P_O}{\partial X} + \frac{\rho g.sin\theta}{1.0133X10^6}\right]$ … Eq. (4.1)

$q_W = -\frac{k.k_{rw}A}{\mu_w}\left[\frac{\partial P_w}{\partial X} + \frac{\rho g.sin\theta}{1.0133X10^6}\right]$ … Eq. (4.2)

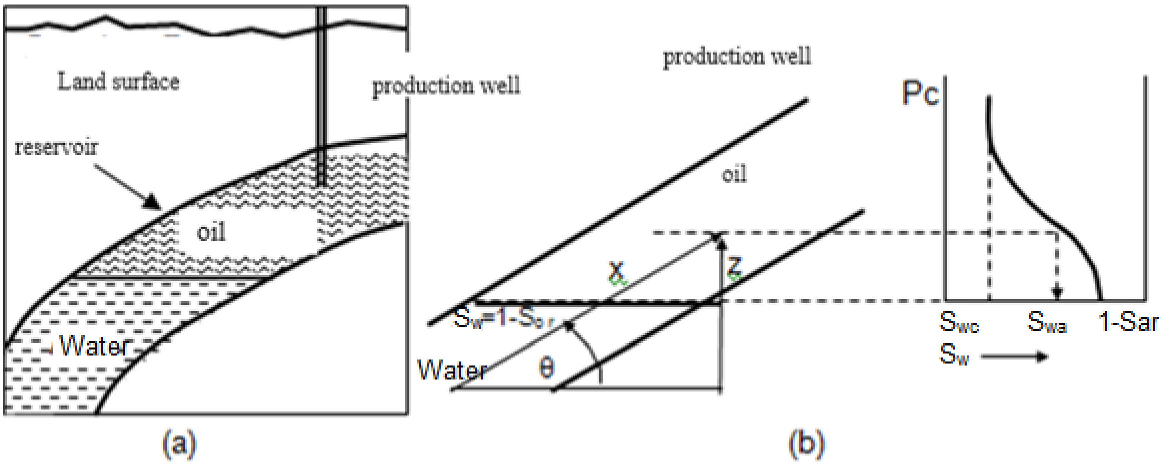

Figure 2: (a) Idealized scheme of the reservoir; (b) Relationship between pressure Pc and water saturation.

The flow of oil and water in reservoir rocks occurs due to the pressure that pushes the fluid. At the beginning of oil extraction, oil usually flows to the surface of the earth, through production wells, due to natural pressure (overburden pressure) from the earth. However, if the oil cannot be fully produced optimally, a further process is required, known as secondary recovery. In this process, water is injected through injection wells with the aim that the remaining oil in the reservoir can be pushed to the surface of the earth through the pressure of the injected water. This pressure (for example in the secondary recovery process) will cause pressure on the oil, symbolized as po, and on the water, symbolized as pw. From equation (2.11), for immiscible water and oil fluids (not mixed with each other), the oil and water flow rate can be written as:

$f_w = \frac{q_w}{q_t} = \frac{q_w}{q_o+q_w}$ … Eq. (4.3)

For the calculation of oil recovery, we need to know first the fractional flow that occurs in the reservoir. Fractional flow is the ratio of the flow rate of a type of fluid (eg: water) to the total flow rate of fluid that occurs in the reservoir. The fractional flow of water f_w is defined as:

q_t is the total flow rate which is the sum of the oil flow rate q_0 and water flow q_w.

By substituting equations (4.1) and (4.2) into equation (4.3), $f_w$can be written as:

$f_w = \frac{1+k.k_{rw}.A(\frac{\partial P_C}{\partial X} - \frac{\Delta\rho g.\sin\theta}{1.0133\times10^6})}{1+\frac{\mu_w\, k_{ro}}{k_{rw}\mu_{ro}}}$ … Eq. (4.4)

with notes $P_C = P_O - P_W$ OR $\frac{\partial P_C}{\partial X} = \frac{\partial P_O}{\partial X} - \frac{\partial P_W}{\partial X}$ dan: $\Delta\rho = \rho_O - \rho_W$ … Eq. (4.5)

p_c is the pressure that occurs at the interface between oil and water fluids. as depicted in Figure 2 (b).

If p_c is assumed to be unchanged along the flow path ($\frac{\partial P_C}{\partial X} = 0$)then for a horizontal reservoir (sin θ =0):

$f_w = \frac{1}{1+\frac{\mu_w\, k_{ro}}{k_{rw}\mu_{ro}}}$… Eq. (4.6)

METHODOLOGY

As explained above, relative permeability can be determined in the laboratory. In this article, we will describe one of the commonly used methods, namely the unsteady state method. In this method, the fluid flow process that occurs in the reservoir is simulated by injecting water into a rock sample that has been filled (100% saturated) by oil. This rock sample is taken from the reservoir. The water and oil used are usually artificial water and oil whose properties have been adjusted to be the same as the water and oil from the reservoir. Schematically, the unsteady state method above can be described as in Figure 3 below. This method is usually carried out in the Special Core Analyses laboratory.

Determination of relative permeability using the unsteady state method, as illustrated in Figure 3, is carried out using a constant water flow pressure. This pressure is adjusted so that the water flow that occurs does not exceed 20 cc/second. This is done so that the fluid flow that occurs in the rock does not experience turbulence. In other words, the flow that occurs must be a laminar, irrotational flow. The water and oil fluids used in this method are assumed to be incompressible (density = constant) and immiscible (do not mix with each other). Oil and water that come out of the rock are collected by a collection glass. The volume of water and oil is recorded at each specified time period. As a note, before carrying out the steps above, the rock sample must first be washed, which is continued with the measurement of the pore volume and porosity of the rock sample. This measurement is usually carried out in the Routine Core Analyses laboratory. The density and viscosity of water and oil must also be determined first. This measurement is usually carried out in the Special Core Analyses laboratory according to the specified temperature. The above quantities are then used for the calculation process of relative permeability.

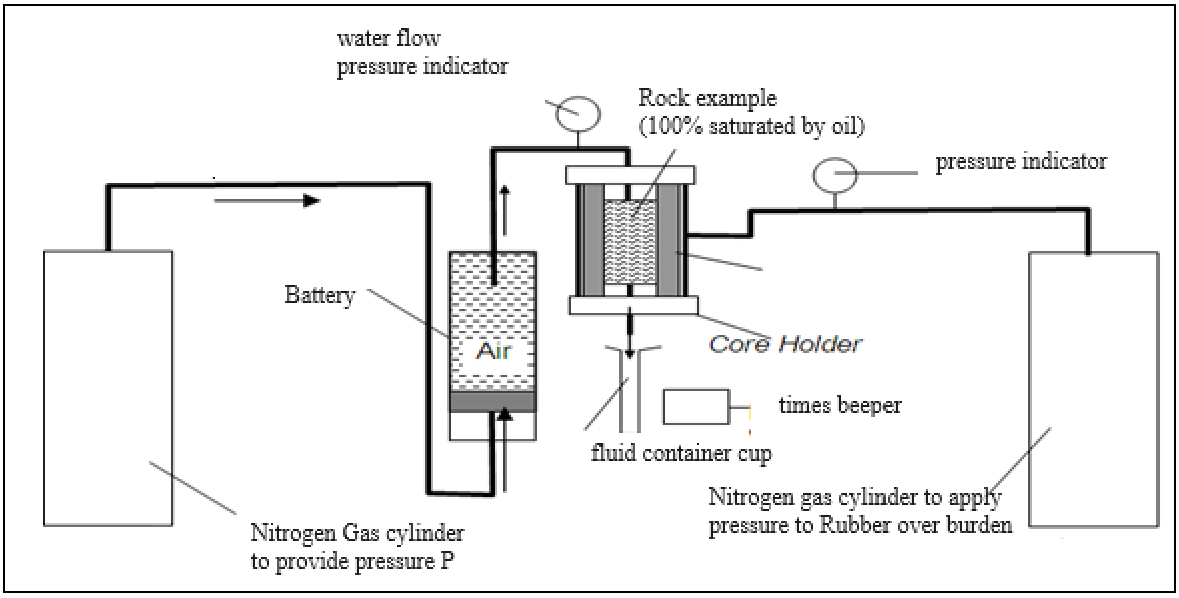

Figure 3: Schematic of laboratory equipment for measuring relative permeability using the unsteady state method

At the initial moments of water and oil flow in the rock sample, no water has been detected in the container glass. This is because the water is still behind the oil. Naturally, this situation does not always last like this. At a certain time, the water surface will penetrate the oil column and the flow that occurs is a mixture of water and oil. The initial appearance of water from the rock sample is called break through.

Based on the data that has been obtained, the next step is to calculate the relative permeability of water and oil. This calculation requires a long step, but, basically, this step is intended to obtain a calculation of the fractional flow of water at each water saturation price, calculation of relative permeability of water, calculation of relative permeability ratio of oil and water and calculation of relative permeability ratio of oil.

Calculation of Fractional Flow of Water at Each Water Saturation Price

Along with the passage of time, the volume of water entering the rock will also increase. Before break through, the volume of water that occurs in the rock is the same as the volume of oil contained in the reservoir glass. After break through, the volume of water is the same as the volume of oil above and added to the additional volume of water contained. While the volume of oil is the same as the volume of oil above plus the additional volume of oil contained. The oil flow rate q_o and water q_w can be calculated as the additional volume of water or oil divided by the additional time △t. The fractional flow f_w for each certain period of time is calculated by entering q_o and q_w into equation (4.3). Water saturation can be calculated using equation (3.3). The relationship between fractional flow of water and water saturation can be exemplified by Figure 4 below:

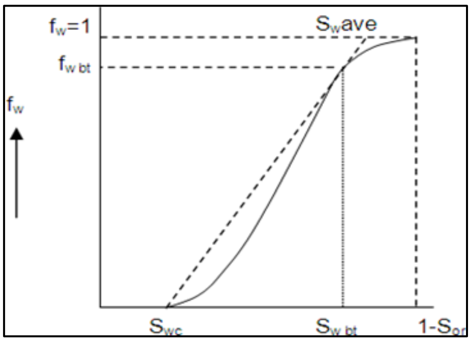

Figure 4: Example of a graph of the relationship between fractional water flow and water saturation obtained at laboratory conditions

S_{we} ave is the average water saturation until break through occurs, S_{wc} is the connate water saturation (minimum water saturation in rocks), Sor is the irreducible oil saturation (minimum oil saturation in rocks), $S_{w\,bt}$ is the water saturation at break through and $f_w$bt is the fractional flow of water at break through.

Calculation of Relative Permeability of Water

For each q_w obtained in step a above, the effective permeability value of the oil can be found in Equation (4.2) as:

$$k_w = -\frac{\mu_o . q_w . \Delta L . c}{c . \Delta p_{w.} A + \rho_w . g}$$

c = 1.0133x10^6, △L= has a negative value = length of the rock sample, sin θ=0. The relative permeability of water can be calculated using equation (3.1):

$$K_{rw}\,(Sw) = \frac{K_w(Sw)}{k}$$

Calculation of Relative Permeability Comparison of Oil and Water

Because the fractional flow of water has been known, while the viscosity of water and oil has also been known previously, then by using equation (4.6) the relative permeability comparison of oil and water can be calculated:

$$\frac{k_{ro}}{k_{rw}} = \frac{\mu_o}{\mu_w}\left(\frac{1}{f_w} - 1\right)$$

Calculation of Relative Permeability Comparison of Oil

The calculation to determine the relative permeability of oil can be done by multiplying the calculation result in step c with k_{rw} obtained in step b. From the steps above, the k_{ro} values of k_{rw} will be obtained as a function of water saturation as exemplified in Figure 5 below:

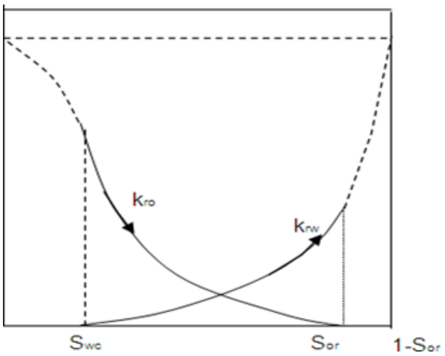

Figure 5: Example of a graph of the relationship between relative permeability and water saturation obtained in the laboratory conditions

$N_{pd\ bt}$= (S_{we} ave $-S_{wc}$) … Eq. (5.1)

RESULTS AND DISCUSSION

Calculation of Oil Recovery

At the initial moments when water and oil flow in the reservoir rock, no water has been detected in the production well. This is because the water surface is still below (or behind) the oil column. Naturally, this condition does not always last. At a certain time, the water surface will penetrate the oil column and the flow that occurs is a mixture of water and oil. The initial appearance of water in the production well is called break through.

Oil recovery before and during breakthrough in the production well is:

$N_{pd\ bt}$ is the oil recovery before and at break through, Swe ave is the average water saturation before and until break through occurs, S_{wc} is the connate water saturation (minimum water saturation in the rock) and $S_{w\ bt}$ is the water saturation at break through. After break through occurs, the calculation of oil recovery in the production well can be done with the formula in equations (5.2) and (5.2) below. A complete description of the derivation of equations (5.1), (5.2) and (5.3) can be seen for example in the reference according to Dusseault, Maurice. (2011).

$N_{pd\ bt}$= (S_{we} ave $-S_{wc}$)+ (1-f_{we})W_{id}… Eq. (5.2)

$W_{id}= \frac{1}{\left.\frac{df_w}{dS_w}\right|_{S_{we}}}$… Eq. (5.3)

N_{pd} is the oil recovery in the production well, Swe is the water saturation that occurs in the production well, S_{wc} is the connate water saturation (minimum water saturation in the rock) and f_{we} is the fractional flow of water in the production well. Wi_d is the amount of water that has been injected.

Example of Oil Recovery Calculation

Example data and calculations for oil recovery here are taken from Dake (2010), practice question number 10.1 concerning fractional flow and question 10.2 concerning the calculation of oil recovery. As a note, in question 10.1, data from relative permeability measurements are given. This measurement can be done in a laboratory using standard methods. From question 5.1, relative permeability data is obtained in the case of oil pushing by water that occurs in a horizontal reservoir, as in Table 1 below:

Table 1 Relative Permeability Data

s_w	k_{rw}	k_{ro}	$\frac{k_{ro}}{k_{rw}}$	f_w
0.2	0.000	0.792	0.000	0.000
0.25	0.002	0.604	301.950	0.032
0.3	0.009	0.465	51.700	0.159
0.35	0.020	0.366	18.315	0.347
0.4	0.033	0.282	8.550	0.532
0.45	0.050	0.218	4.272	0.692
0.5	0.074	0.161	2.151	0.813
0.55	0.099	0.119	1.188	0.884
0.6	0.131	0.080	0.608	0.933
0.65	0.168	0.050	0.291	0.961
0.7	0.206	0.027	0.129	0.977
0.75	0.248	0.010	0.040	0.986
0.8	0.297	0.000	0.000	0.990

The relationship between f_w and S_w can be described in Figure 6 below:

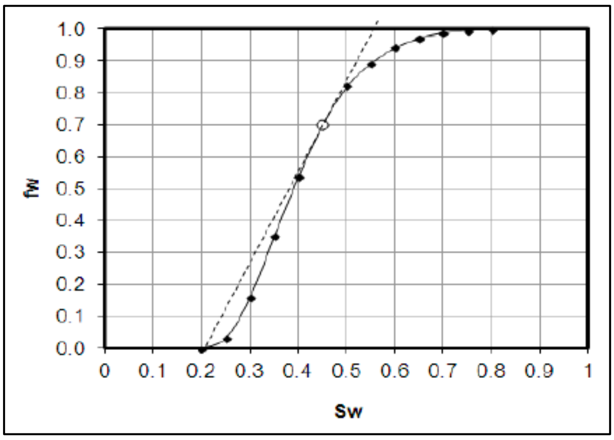

Figure 6 Relationship between f_w and S_w

From figure 6 (as in figure 4), it can be obtained:

S_{wc} = 0.2, S_{or} = 1 - 0.8 = 0 .2, S_{we} ave = 0.55, $S_{w\ bt}$ = 0.45, $f_{w\ bt}$ = 0.7

Oil recovery before and up to break through can be calculated using equation (5.1), which is: $N_{pd\ bt}$ = 0.35. The unit for $N_{pd\ bt}$ is pore volume (PV). Therefore:

$N_{pd\ bt}$ = 0.35 x pore volume of the rock in the reservoir.

The calculation of oil recovery after break through, this calculation begins with the calculation of N_{pd} and W_{id} based on equations (5.2) and (5.3), namely (see also Table 2 below):

$$N_{pd} = (S^*_{we} - S_{wc}) + (1 - f^*_{we})W_{id}$$

$$W_{id} = \frac{1}{\frac{\Delta f_w}{\Delta s_w}\Big|_{S^*_{we}}}$$

Table 2 Calculation of W_{id} and N_{pd}

S_{we}	f_{we}	F^*_{we}	ΔS_{we}	Δf_{we}	$\Delta f_{we}/\Delta S_{we}$	S^*_{we}	W_{id} (PV)	N_{pd} (PV)
0.45	0.699	0.762	0.05	0.122	2.44	0.471	0.41	0.371
0.5	0.821	0.846	0.05	0.072	1.44	0.521	0.694	0.415
0.55	0.893	0.952	0.05	0.049	0.98	0.575	1.024	0.452
0.6	0.954	0.962	0.05	0.029	0.58	0.625	1.724	0.491
0.65	0.972	0.984	0.05	0.016	0.32	0.675	3.124	0.532
0.7	0.981	0.997	0.05	0.006	0.12	0.725	5.562	0.564

From the previous data it is known that S_{wc} = 0.2 and S_{or} = 0 .2, which also means that the oil available for production is: 1- S_{wc} - S_{or} = 0 .6 of the pore volume. Meanwhile, from Table 2 above it can be seen that the oil drained through the production well is 0.564 of the reservoir rock pore volume. This means that almost all available oil has been obtained (0.564 PV). The remaining 0.6 - 0.564 = 0.036 PV. This condition is achieved when the water saturation filling the production well has reached 5,556 times the pore volume of the reservoir rock.

CONCLUSION

The principles and formulas provided by the field of Fluid Dynamics play a role in calculating the flow rate of oil and water fluids in reservoir rocks which in turn will also play a role in calculating oil recovery in production wells. Reservoir rocks can be described as a fluid flow medium that has a control volume in the form of flow channels formed by rock pores with very varied shapes and diameter sizes. Calculation of fluid flow velocity using the principles and formulas of Fluid Dynamics requires adjustments if it is to be applied to cases of fluid flow in reservoir rocks. The adjustments above in their embodiment are a combination of the principles and formulas of Fluid Dynamics supported by empirical data from laboratory permeability measurements in reservoir rocks. In this paper, a method has been applied to calculate the fluid flow velocity from this control volume, which is a combination of laboratory permeability measurements and the use of several theories in Fluid Dynamics. This method has been proven to be used to calculate fluid flow velocity and oil recovery in reservoir rocks, with fairly good accuracy. Permeability is physically translated as the ability of rocks to flow fluids. However, from another perspective, rock permeability can be viewed as a constant which is a comparison between ideal calculations, using the Fluid Dynamics formula, to calculations with simplifications if the rock sample is considered as a cylindrical media or control volume. The calculated flow velocity is the fractional flow velocity, namely the flow velocity of oil or water relative to the total fluid flow velocity (oil + water) that occurs in the rock.

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