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ON GENERALIZED MODEL OF NON-RECURSIVE TYPE SINGLE PARAMETER LOG PROBABILISTIC INFORMATION MEASURE WITH FUZZY INFERENCE

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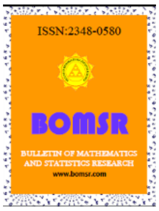

ABSTRACT

A new parametric function $C_a((P) = \sum_{i=1}^{n} \ln(a + p_i) - \sum_{i=1}^{n} \ln p_i - \sum_{i=1}^{n} p_i - \ln a$, $a > 0$ is put forth for the $P = (p_1, p_2, \ldots\ldots\ldots, p_n)$ probability distribution, and its characteristics are examined. This study uses logistic type growth models, the related measure of directed divergence, and its measurement in fuzzy sets. All functions are twice differentiable. We also look into the suggested function's monotonicity and the multivariate normal distribution that goes along with it.

Keywords: measure of entropy, directed divergence, multivariate normal distribution, logistic type growth model, innovation model, information theory

1. Introduction

We are inspired by R.K. Verma, C.L. Dewangan, P. Jha [12], Kapur [7], and Burg [1] to derive a novel parametric measure of entropy in this study, which is the joint effect of measures of information. C.E. Shannon [9] provided the measurement in 1948.

$$S(P) = -\sum_{i=1}^{n} p_i \ln p_i \tag{1.1}$$

to calculate the entropy or uncertainty of it. It is also possible to think of it as a gauge of how equal each individual is $p_{1,}, p_2, \ldots\ldots\ldots, p_n$ to the others. J.P. Burg [1] and J.N. Kapur [7] provided the measures later in 1972.

$$B(P) = \sum_{i=1}^{n} \ln p_i \tag{1.2}$$

And

$$K_a(P) = -\sum_{i=1}^{n} p_i \ln p_i + \frac{1}{a}\sum_{i=1}^{n}(1+ap_i)\ln(1+ap_i) - \frac{1}{a}(1+a)\ln(1+a) \tag{1.3}$$

While Kapur's measure has one parameter, Shannon's and Burg's measures have none. The meaures attributed to Shannon, Burg, and Kapur always provide non-negative probabilities when maximised using Lagrange's method, subject to linear limitations on probabilities. The most effective and most utilised metric is Shannon's measure. Burg's measure has also shown success, but because it is always negative, it is difficult to use it to gauge uncertainty. It has, nevertheless, been applied and is capable of being applied for entropy maximisation [5]. Its maximum value also diminishes with n, which is an undesirable characteristic for an entropy metric.

In the current discussion, we derive a new parametric entropy measure by modifying the Burg's and Kapur's measures. Along with examining the measure's characteristics, we will also look into the directed divergence that is connected to logistic type growth models, Kullback-Liebler [8], and its measures in fuzzy sets. It has also been possible to determine the equivalent multivariate normal distribution. The sections that follow display each of these.

2. Our Results

2.1. Some Properties of the New Measures of Information

The measure is defined by

$$C_a(P) = \sum_{i=1}^{n} \ln\left(\frac{1+ap_i}{p_i}\right) - \sum_{i=1}^{n} p_i - \ln a \tag{2.1.1}$$

It has the following properties:

1. It is a continuous function of $p_{1,}, p_2, \ldots\ldots\ldots, p_n$, so that it changes by a small amount when $p_{1,}, p_2, \ldots\ldots\ldots, p_n$ change by small amounts.

2. It is a permutationally symmetric function of $p_{1,}, p_2, \ldots\ldots\ldots, p_n$, i.e. the function does not change when $p_{1,}, p_2, \ldots\ldots\ldots, p_n$ are permuted among themselves.

3. It is maximum, subject to natural constraints $\sum_{i=1}^{n} p_i = 1$ when

$$p_1 = p_2 = \ldots\ldots\ldots = p_n = \frac{1}{n} \tag{2.1.2}$$

The maximum value is an increasing function of n. In fact the maximum value is given by

$$f(n) = \sum \ln\left(1+\frac{a}{n}\right) - \sum \ln\frac{1}{n} - 1 - \ln a \tag{2.1.3}$$

$$= n\ln\left(\frac{n+a}{n}\right) - n\ln\frac{1}{n} - 1 - \ln a \tag{2.1.4}$$

$$f'(n) = \frac{n}{n+a} + \ln(n+a) \tag{2.1.5}$$

$$f''(n) = \frac{2}{n+a} - \frac{n}{(n+a)^2} \quad (2.1.6)$$

$$= \frac{1}{n+a} + \frac{a}{(n+a)^2} > 0 \quad (2.1.7)$$

So that f(n) is a convex of n and f′(n) is a monotonic increasing function of n. Now

$$f'(n) = \frac{n}{n+a} + \ln(n+a)$$

$$= 1 - \frac{a}{n+a} + \ln(n+a) \quad (2.1.8)$$

$$= 1 - \frac{a}{y}\ln y = \frac{y-a+y\ln y}{y}; \quad y = n+a > 0 \quad (2.1.9)$$

Since

$$y > 0 \text{and } y - a + \ln y > 0 \text{ when } y > 0 \quad (2.1.10)$$

We get

$$f'(n) > 0 \quad (2.1.11)$$

So that f(n) is always increasing.

4. Now, $\frac{\partial}{\partial p_i} C_a(P) = \frac{a}{1+ap_i} - \frac{1}{p_i};$ $\frac{\partial^2}{\partial p_i^2} C_a(P) = \frac{1+2ap_i}{p_i^2(1+ap_i)^2}$ (2.1.12)

and

$$\frac{\partial}{\partial p_i \partial p_j} C_a(P) = 0 \quad (2.1.13)$$

So that $C_a(P)$ is a convex function of $p_{1,}, p_2, \ldots\ldots\ldots, p_n$.

5. Since $C_a(P)$ is a convex function and its domain is

$p_1 \geq 0$, $p_2 \geq 0, \ldots\ldots\ldots p_n \geq 0$, $\sum_{i=1}^{n} p_i = 1$

Its minimum value occurs at each of the degenerate distributions

$$\Delta_i = (0, 0, \ldots, 1, 0, 0, \ldots, 0), \ i = 1, 2, \ldots, n, \quad (2.1.14)$$

where in Δ_i , unity occurs in the i^{th} place and for each of these, its value is zero.

Thus

$$C_a(P) \geq 0, \quad (2.1.15)$$

and it only disappears when P crosses over into a degenerate distribution. i.e., when all ambiguity is eliminated and there is complete confidence. Thus, $C_a(P)$ except for additivity and recursivity, meets all significant qualities satisfied by Shannon's measure of entropy. But these characteristics don't matter for the goal of maximising entropy, hence $C_a(P)$ entropy exists.

2.2. Generating Functions for the Measure of Information

Let us define

$$f_a(t) = \sum_{i=1}^{n} \left(\frac{1+ap_i}{p_i}\right)^t - \sum_{i=1}^{n} p_i^t - \sum_{i=1}^{n} a^t p_i \quad (2.2.1)$$

Then

$$f_a'(t) = \sum_{i=1}^{n}\left(\frac{1+ap_i}{p_i}\right)^t \ln\left(\frac{1+ap_i}{p_i}\right) - \sum_{i=1}^{n} p_i^t\, p_i - \sum_{i=1}^{n} a^t \ln a\ \ p_i \qquad (2.2.2)$$

Therefore

$$f_a'(0) = \sum_{i=1}^{n} \ln\left(\frac{1+ap_i}{p_i}\right) - \sum_{i=1}^{n} p_i - \ln a \qquad (2.2.3)$$

Which is same as (2.1.1)

Again we define

$$f_{a,b}(t) = \sum_{i=1}^{n}\left(\frac{b+ap_i}{bp_i}\right)^t - \sum_{i=1}^{n}(bp_i)^t - \sum_{i=1}^{n}(ba)^t\, p_i \qquad (2.2.4)$$

Then

$$f_{a,b}'(t) = \sum_{i=1}^{n}\left(\frac{b+ap_i}{p_i}\right)^t \ln\left(\frac{b+ap_i}{bp_i}\right) - \sum_{i=1}^{n}(bp_i)^t\, p_i - \sum_{i=1}^{n}(ba)^t \ln ba\ \ p_i \qquad (2.2.5)$$

So that

$$f_{a,1}'(0) = \sum_{i=1}^{n} \ln\left(\frac{1+ap_i}{p_i}\right) - \sum_{i=1}^{n} p_i - \ln a \qquad (2.2.6)$$

Which is same as (2.1.1)

2.3. Corresponding Measure of Fuzzy Information

Corresponding to new parametric measure of entropy, i.e.

$$C(P) = \sum_{i=1}^{n} \ln\left(\frac{1+ap_i}{p_i}\right) - \sum_{i=1}^{n} p_i - \ln a$$

$$= \sum_{i=1}^{n} \ln(1+ap_i) - \sum_{i=1}^{n} \ln p_i - \sum_{i=1}^{n} p_i - \sum_{i=1}^{n} \ln a\ \ p_i \qquad a > 0$$

We get the measure

$$C(A) = \sum_{i=1}^{n} \ln(1 + a\mu_A(x_i)) + \sum_{i=1}^{n} \ln(1 + a - a\mu_A(x_i)) - \sum_{i=1}^{n} \ln \mu_A(x_i) - \sum_{i=1}^{n} \ln(1-\mu_A(x_i)) - \sum_{i=1}^{n} \mu_A(x_i) - \sum_{i=1}^{n}(1-\mu_A(x_i)) - \ln a \qquad (2.3.1)$$

2.4. Generating Function for Corresponding Measure of Fuzzy Information

Let us define

$$\sum_{i=1}^{n}\left(\frac{1+a+a^2\mu_A(x_i)-a^2\mu_A^2(x_i)}{\mu_A(x_i)-\mu_A^2(x_i)}\right)^t - \sum_{i=1}^{n}(\mu_A(x_i)(1-\mu_A(x_i)))^t - a^t$$

$$= \sum_{i=1}^{n}\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{\mu_A(x_i)(1-\mu_A(x_i))}\right)^t - \sum_{i=1}^{n}(\mu_A(x_i)(1-\mu_A(x_i)))^t - a^t \qquad (2.4.1)$$

Now , differentiating with respect to t we get

$$\sum_{i=1}^{n}\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{\mu_A(x_i)(1-\mu_A(x_i))}\right)^t \ln\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{\mu_A(x_i)(1-\mu_A(x_i))}\right) - \sum_{i=1}^{n}(\mu_A(x_i)(1-\mu_A(x_i)))^t \ln(\mu_A(x_i)(1-\mu_A(x_i)) - a^t \ln a$$

Taking $t = 0$, we obtain

$$\sum_{i=1}^{n} \ln\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{\mu_A(x_i)(1-\mu_A(x_i))}\right) - \sum_{i=1}^{n} \ln(\mu_A(x_i)(1-\mu_A(x_i)) - \ln a$$

$$= \sum_{i=1}^{n} \ln(1+a\mu_A(x_i)) + \sum_{i=1}^{n} \ln(1+a-a\mu_A(x_i)) - \sum_{i=1}^{n} \ln(\mu_A(x_i) - \sum_{i=1}^{n} \ln(1-\mu_A(x_i)) - \sum_{i=1}^{n} \mu_A(x_i) - \sum_{i=1}^{n}(1-\mu_A(x_i)) - \ln a \qquad (2.4.2)$$

Which is the same as (2.3.1).

Again, let us define

$$\sum_{i=1}^{n}(\frac{b^2+ab^2+a^2b^2\mu_A(x_i)-a^2b^2\mu_A^2(x_i)}{b\mu_A(x_i)-b\mu_A^2(x_i)})^t-\sum_{i=1}^{n}p_i^t-(ba)^t$$

$$=\sum_{i=1}^{n}(\frac{(b+ab\mu_A(x_i))(b+ab-ab\mu_A(x_i))}{\mu_A(x_i)(b-b\mu_A(x_i))})^t-\sum_{i=1}^{n}(\mathrm{b}\mu_A(\mathrm{x_i})(\mathrm{bb}\mu_A(\mathrm{x_i})))^t-(ba)^t \quad (2.4.3)$$

Now, differentiating with respect to t we get

$$\sum_{i=1}^{n}(\frac{(b+ab\mu_A(x_i))(b+ab-ab\mu_A(x_i))}{\mu_A(x_i)(b-b\mu_A(x_i))})^t\ln(\frac{(b+ab\mu_A(x_i))(b+ab-ab\mu_A(x_i))}{\mu_A(x_i)(b-b\mu_A(x_i))})-\sum_{i=1}^{n}(\mathrm{b}\mu_A(\mathrm{x_i})(\mathrm{b}-\mathrm{b}\mu_A(\mathrm{x_i})))^t\ln(\mathrm{b}\mu_A(\mathrm{x_i})(\mathrm{b}-\mathrm{b}\mu_A(\mathrm{x_i}))-(\mathrm{ba})^t\ln(\mathrm{ba}) \quad (2.4.4)$$

Putting t=0 and b=1, we obtain

$$\sum_{i=1}^{n}\ln(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{\mu_A(x_i)(1-\mu_A(x_i))})-\sum_{i=1}^{n}\ln(\mu_A(\mathrm{x_i})(1-\mu_A(\mathrm{x_i}))-\ln a=\sum_{i=1}^{n}\ln(1+a\mu_A(x_i))+\sum_{i=1}^{n}\ln(1+a-a\mu_A(x_i))-\sum_{i=1}^{n}\ln(\mu_A(\mathrm{x_i})-\sum_{i=1}^{n}\ln(1-\mu_A(\mathrm{x_i}))-\sum_{i=1}^{n}\mu_A(\mathrm{x_i})-\sum_{i=1}^{n}(1-\mu_A(\mathrm{x_i}))-\ln a \quad (2.4.5)$$

Which is the same as (2.3.1).

2.5. Corresponding Measures of Directed Divergence

Motivated by Kullback and Libeler [8] we get the corresponding measure of directed divergence,

$$D_a(P:Q)=\sum_{i=1}^{n}q_i\ln\frac{p_i}{q_i}-\sum_{i=1}^{n}q_i\ln\left(\frac{q_i+ap_i}{q_i}\right)+\sum_{i=1}^{n}q_ip_i+\ln a \quad (2.5.1)$$

Here $D_a(P:Q)$ satisfies the properties $D_a(P:Q)\geq 0$, vanishes if Q=P and is a concave function of both $p_1,p_2,\ldots\ldots p_n$ and $q_1,q_2,\ldots\ldots q_n$.

2.6. Generating Function for Corresponding Measure of Directed Divergence

Let

$$g_a(t)=\sum_{i=1}^{n}q_i(\frac{p_i}{q_i+ap_i})^t+\sum_{i=1}^{n}q_ip_i^t+a^t, \qquad a>0 \quad (2.6.1)$$

So that

$$g_a'(t)=\sum_{i=1}^{n}q_i(\frac{p_i}{q_i+ap_i})^t\ln(\frac{p_i}{q_i+ap_i})+\sum_{i=1}^{n}q_ip_i^t\ln p_i+a^t\ln a, \qquad a>0 \quad (2.6.2)$$

Therefore

$$g_a'(0)=\sum_{i=1}^{n}q_i\ln\frac{p_i}{q_i}-\sum_{i=1}^{n}q_i\ln(\frac{q_i+ap_i}{q_i})+\sum_{i=1}^{n}q_ip_i+\ln a, \qquad a>0 \quad (2.6.3)$$

or

$$g_a'(0)=\sum_{i=1}^{n}q_i\ln(\frac{p_i}{q_i+ap_i})+\sum_{i=1}^{n}q_i\ln p_i+\ln a, \qquad a>0 \quad (2.6.4)$$

Which is the same as (2.5.4).

Again we define

$$g_{a,b}(t)=\sum_{i=1}^{n}q_i\left(\frac{bq_i}{b+ap_i}\right)^t+\sum_{i=1}^{n}q_i\left(\frac{p_i}{q_i}\right)^t+\sum_{i=1}^{n}q_ip_i^t+a^t, \qquad ab>0 \quad (2.6.5)$$

So that

$$g'_{a,b}(t) = \sum_{i=1}^{n} q_i \left(\frac{bq_i}{b+ap_i}\right)^t \ln\left(\frac{bq_i}{b+ap_i}\right) + \sum_{i=1}^{n} q_i \left(\frac{p_i}{q_i}\right)^t \ln\left(\frac{p_i}{q_i}\right) + \sum_{i=1}^{n} q_i\, p_i^t \ln p_i \quad + a^t \ln a\,, ab > 0 \qquad (2.6.6)$$

Hence

$$g'_{a,1}(0) = \sum_{i=1}^{n} q_i \ln\frac{p_i}{q_i} - \sum_{i=1}^{n} q_i \ln(\frac{q_i+ap_i}{q_i}) + \sum_{i=1}^{n} q_i p_i + \ln a \qquad a > 0 \qquad (2.6.7)$$

Which is the same as (2.5.1)

2.7. Corresponding Measure of Fuzzy Directed Divergence

Corresponding to new parametric measure of directed divergence is

$$\mathrm{D(P:Q)} = \sum_{i=1}^{n} q_i \ln(\frac{p_i}{q_i+ap_i}) + \sum_{i=1}^{n} q_i \ \ln p_i + \ln a \ , \qquad a > 0$$

We get, the measure in fuzzy set

$$\mathrm{D(A:B)} = \sum_{i=1}^{n} \mu_B(x_i) \ln(\frac{\mu_A(x_i)}{a\mu_A(x_i)+\mu_B(x_i)}) + \sum_{i=1}^{n}(1-\mu_B(x_i)) \ln(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}) + \sum_{i=1}^{n} \mu_B(x_i)\mu_A(x_i) + \sum_{i=1}^{n}(1-\mu_B(x_i))\,(1-\mu_A(x_i)) + \ln a \qquad a > 0 \qquad (2.7.1)$$

2.8. Generating Function for Corresponding Measure of Fuzzy Directed Divergence

Let

$$\sum_{i=1}^{n} \mu_B(x_i) \left(\frac{\mu_A(x_i)+a\mu_A(x_i)-a\mu_A^2(x_i)-\mu_A(x_i)\mu_B(x_i)}{a\mu_A(x_i)+\mu_B(x_i)-a\mu_A^2(x_i)-\mu_A(x_i)\mu_B(x_i)}\right)^t + \sum_{i=1}^{n} \left(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)+\mu_B(x_i)}\right)^T + \sum_{i=1}^{n}\left(\mu_A(x_i) - \mu_A(x_i)\mu_B(x_i)\right)^t + a^t \qquad a > 0$$

Now , differentiating with respect to t we obtain

$$\sum_{i=1}^{n} \mu_B(x_i) (\frac{\mu_A(x_i)+a\mu_A(x_i)-a\mu_A^2(x_i)-\mu_A(x_i)\mu_B(x_i)}{a\mu_A(x_i)+\mu_B(x_i)-a\mu_A^2(x_i)-\mu_A(x_i)\mu_B(x_i)})^t \ln(\frac{\mu_A(x_i)+a\mu_A(x_i)-a\mu_A^2(x_i)-\mu_A(x_i)\mu_B(x_i)}{a\mu_A(x_i)+\mu_B(x_i)-a\mu_A^2(x_i)-\mu_A(x_i)\mu_B(x_i)}) + \sum_{i=1}^{n}(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)})^t \ln(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}) + \sum_{i=1}^{n}(\mu_A(x_i) - \mu_A(x_i)\mu_B(x_i))^t\, (\mu_A(x_i) - \mu_A(x_i)\mu_B(x_i)) + a^t \ln a$$

Putting t=0, we obtain

$$\sum_{i=1}^{n} \mu_B(x_i) \ln(\frac{\mu_A(x_i)(1+a-a\mu_A(x_i)-\mu_B(x_i))}{a\mu_A(x_i)+\mu_B(x_i)-\mu_A(x_i)(a\mu_A(x_i)+\mu_B(x_i))}) + \sum_{i=1}^{n} \ln(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}) + \sum_{i=1}^{n} (\mu_A(x_i) - \mu_A(x_i)\mu_B(x_i)) + \ln a$$

$$= \sum_{i=1}^{n} \mu_B(x_i) \ln(\frac{\mu_A(x_i)(1+a-a\mu_A(x_i)-\mu_B(x_i))}{(a\mu_A(x_i)+\mu_B(x_i))(1-\mu_A(x_i))}) + \sum_{i=1}^{n} \ln(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}) + \sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)(1-\mu_A(x_i)\mu_B(x_i)) + \ln a$$

$$= \sum_{i=1}^{n} \mu_B(x_i) \ln(\frac{\mu_A(x_i)}{(a\mu_A(x_i)+\mu_B(x_i))}) + \sum_{i=1}^{n}(1-\mu_B(x_i)) \ln(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}) + \sum_{i=1}^{n}(1-\mu_A(x_i))(1-\mu_B(x_i))(1-\mu_A(x_i)\mu_B(x_i)) + \ln a \qquad (2.8.1)$$

The last is the same as (2.7.1). again we re-write

$$\sum_{i=1}^{n}\mu_B(x_i)\left(\frac{b\mu_A(x_i)+ab\mu_A(x_i)-ab\mu_A^2(x_i)-b\mu_A(x_i)\mu_B(x_i)}{ba\mu_A(x_i)+b\mu_B(x_i)-ab\mu_A^2(x_i)-b\mu_A(x_i)\mu_B(x_i)}\right)^t + \sum_{i=1}^{n}\left(\frac{b-b\mu_A(x_i)}{b+ab-ab\mu_A(x_i)+b\mu_B(x_i)}\right)^t + \sum_{i=1}^{n}(b\mu_A(x_i)-\mu_A(x_i)b\mu_B(x_i))^t + (ba)^t$$

$ab > 0$

Now , differentiating with respect to t we get:

$$\sum_{i=1}^{n}\mu_B(x_i)\left(\frac{b\mu_A(x_i)+ab\mu_A(x_i)-ab\mu_A^2(x_i)-b\mu_A(x_i)\mu_B(x_i)}{ba\mu_A(x_i)+b\mu_B(x_i)-ab\mu_A^2(x_i)-b\mu_A(x_i)\mu_B(x_i)}\right)^t$$

$$\ln\frac{b\mu_A(x_i)+ab\mu_A(x_i)-ab\mu_A^2(x_i)-b\mu_A(x_i)\mu_B(x_i)}{ba\mu_A(x_i)+b\mu_B(x_i)-ab\mu_A^2(x_i)-b\mu_A(x_i)\mu_B(x_i)}) + \sum_{i=1}^{n}\left(\frac{b-b\mu_A(x_i)}{b+ab-ab\mu_A(x_i)-b\mu_B(x_i)}\right)^t \ln\left(\frac{b-b\mu_A(x_i)}{b+ab-ab\mu_A(x_i)-b\mu_B(x_i)}\right) + \sum_{i=1}^{n}(b\mu_A(x_i)-\mu_A(x_i)b\mu_B(x_i))^t\,(b\mu_A(x_i)-\mu_A(x_i)b\mu_B(x_i)) + (ba)^t\ln(ba)$$

Therefore (if we set t=0)

$$\sum_{i=1}^{n}\mu_B(x_i)\,ln\left(\frac{b\mu_A(x_i)(1+a-a\mu_A(x_i)-\mu_B(x_i))}{(ab\mu_A(x_i)+b\mu_B(x_i))-\mu_A(x_i)(ab\mu_A(x_i)+b\mu_B(x_i))}\right) + \sum_{i=1}^{n}ln\left(\frac{b-\mu_A(x_i)}{b+ab-ab\mu_A(x_i)-b\mu_B(x_i)}\right) + \sum_{i=1}^{n}a\mu_A(x_i)b\mu_B(x_i)(1-ab\mu_A(x_i)\mu_B(x_i)) + ln\,a$$

or putting b=1,

$$\sum_{i=1}^{n}\mu_B(x_i)\ln\left(\frac{\mu_A(x_i)(1+a-a\mu_A(x_i)-\mu_B(x_i))}{(a\mu_A(x_i)+\mu_B(x_i))(1-\mu_A(x_i))}\right) + \sum_{i=1}^{n}\ln\left(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}\right) + \sum_{i=1}^{n}\mu_A(x_i)\mu_B(x_i)(1-\mu_A(x_i)\mu_B(x_i)) + \ln a$$

$$= \sum_{i=1}^{n}\mu_B(x_i)\ln\left(\frac{\mu_A(x_i)}{(a\mu_A(x_i)+\mu_B(x_i))}\right) + \sum_{i=1}^{n}(1-\mu_B(x_i))\ln\left(\frac{1-\mu_A(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}\right) + \sum_{i=1}^{n}(1-\mu_A(x_i))(1-\mu_B(x_i))(1-\mu_A(x_i)\mu_B(x_i)) + \ln a \quad (2.8.2)$$

Which is the same as (2.7.1).

2.9. Corresponding Logistic Type Growth Model

We can use the entropy

$C_a(P) = \sum_{i=1}^{n}\ln\left(\frac{1+ap_i}{p_i}\right) - \sum_{i=1}^{n}p_i - \sum_{i=1}^{n}\ln ap_i,$

$a > 0$

$= \sum_{i=1}^{n}\ln(1+ap_i) - \sum_{i=1}^{n}\ln p_i - \sum_{i=1}^{n}p_i - \sum_{i=1}^{n}\ln ap_i,\ a > 0$

For the continuous variate case, we get the entropy

$$\int_b^d(\ln(1+af(x)) - \ln f(x) - f(x) - \ln a\,f(x))\,dx \quad (2.9.1)$$

The corresponding LTGM is

$$\frac{1}{c}\frac{df}{dt} = \ln(1+af) - \ln f - f - \ln af \quad (2.9.2)$$

Now we consider the corresponding function

$$\emptyset(\mathrm{f}) = \ln(1+af) - \ln f - f - \ln af \quad (2.9.3)$$

So that;

$$\emptyset(0) = \infty, \qquad \emptyset(1) = 0 \quad (2.9.4)$$

Therefore

$$\emptyset'(f) = \frac{1}{1+af}a - \frac{1}{f} - 1 - \ln a \quad (2.9.5)$$

$$\emptyset''(f) = -\frac{1}{(1+af)^2}a^2 + \frac{1}{f^2} = \frac{1+2af}{(1+af)^2} > 0 \quad (2.9.6)$$

Hence , $\emptyset(\mathrm{f})$ is a convex function of f. From (2.9.5) there is no point of inflexion.

Since, the corresponding LTGM is

$$\frac{1}{c}\frac{df}{dt} = \ln(1+af) + \ln\frac{1}{f} - f - \ln af$$

Let us find the limiting model as a → 0, I.e.

$$\frac{1}{c}\frac{df}{dt} = [\ln(1+af) + \ln\frac{1}{f} - f - \ln af] \quad (2.9.7)$$

$$= \ln\frac{1}{\mathrm{f}} \quad (2.9.8)$$

Which is $\frac{1}{f}$ times Gompertz's model, see [3].

2.10. Corresponding Logistic Type Growth Model in a Fuzzy Set

In fact the corresponding LTGM for the new parametric measure of entropy in a fuzzy set is given by

$$\frac{1}{c}\frac{d}{dt}f(A) = \ln\big(1+a\mu_A(x_i)\big) + \ln\big(1+a-a\mu_A(x_i)\big) - \ln\mu_A(x_i) - \ln\big(1-\mu_A(x_i)\big) - \mu_A(x_i) - \big(1-\mu_A(x_i)\big) \quad (2.10.1)$$

For this model, when a → 0, we obtain

$$\frac{1}{c}\frac{d}{dt}f(A) = -\ln\mu_A(x_i) - \ln(1-\mu_A(x_i)) - \mu_A(x_i) - (1-\mu_A(x_i)) \quad (2.10.2)$$

$$= -\ln(\mu_A(x_i)(1-(\mu_A(x_i))) - \mu_A(x_i) - \big(1-\mu_A(x_i)\big) \quad (2.10.3)$$

$$= \ln[\frac{1}{2}(2(\mu_A(x_i) - 2\mu_A^2(\mathrm{x_i}))] - \mu_A(x_i) - \big(1-\mu_A(x_i)\big) \quad (2.10.4)$$

$$= -\ln\left[\frac{1}{2}\Big(\mu_A(x_i) + 1 - \mu_A(x_i) - \mu_A^2(\mathrm{x_i}) - 1 - \mu_A^2(\mathrm{x_i}) + 2(\mu_A(x_i)\Big)\right] - [\mu_A(x_i) + (1-\mu_A(x_i))] - [\mu_A^2(\mathrm{x_i}) + (1-\mu_A^2(\mathrm{x_i}))] \quad (2.10.5)$$

$$= -\ln[\frac{1}{2}((\mu_A(x_i) + (1-\mu_A(x_i))) - (\mu_A^2(\mathrm{x_i}) - (1-\mu_\mathrm{A}(\mathrm{x_i}))^2))] - [\mu_A(x_i) + (1-\mu_A(x_i))] - [\mu_A^2(\mathrm{x_i}) + (1-\mu_A^2(\mathrm{x_i}))] \quad (2.10.6)$$

Which is the anti- logarithm of corresponding model of innovation of diffusion due to Fisher-Pry (see[2]), or logistic model due to Mickendric - Pai (see[9]) in a fuzzy set. Of course, we can also modify the model (2.10.1)

$$\frac{1}{c}\frac{d}{dt}f(A)=\ln(1+a\mu_A(x_i))+\ln(1+a-a\mu_A(x_i))-\mu_A(x_i)\ln\mu_A(x_i)-(1-\mu_A(x_i)\ln(1-\mu_A(x_i))-\mu_A(x_i)-(1-\mu_A(x_i)) \quad (2.10.7)$$

For this model when a → 0, we get

$$\frac{1}{c}\frac{d}{dt}f(A)=\mu_A(x_i)\ln\frac{1}{\mu_A(x_i)}+(1-\mu_A(x_i))\ln\frac{1}{1-\mu_A(x_i)}-\mu_A(x_i)-(1-\mu_A(x_i)) \quad (2.10.8)$$

Which is the corresponding model of logistic of ompertz (see[3]) in a fuzzy set.

2.11. Multivariate Normal Distribution

The probability density function of $X_1X_{2,}\ldots\ldots\ldots\ldots X_n$ is

$$\mathrm{f(x)}=\frac{1}{(\sqrt{2\pi})^n|K|^{\frac{1}{2}}}e^{-\frac{1}{2}(X-\mu)^TK^{-1}(X-\mu)} \quad (2.11.1)$$

We can use

$$\int_b^d\ln(1+af(x))dx-\int_b^d\ln f(x)dx \quad (2.11.2)$$

As an entropy of a continuous random variable. Now we consider the corresponding function

$$\mathrm{H(f)}=\int_b^d\ln(1+af(x))\,dx-\int_b^d\ln f(x)dx \quad (2.11.3)$$

$$=\int_b^d\ln\langle 1+a\frac{1}{((2\pi)^n|K|)^{\frac{1}{2}}}e^{-\frac{1}{2}(X-\mu)^TK^{-1}(X-\mu)}\rangle\,dx-\int_b^d\ln\langle\frac{1}{((2\pi)^n|K|)^{\frac{1}{2}}}e^{-\frac{1}{2}(X-\mu)^TK^{-1}(X-\mu)}\rangle\,dx \quad (2.11.4)$$

$$=\ln\langle\frac{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{1}{2}(X-\mu)^TK^{-1}(X-\mu)}}{((2\pi)^n|K|)^{\frac{1}{2}}}\rangle\,dx+\frac{1}{2}\int_b^d(X-\mu)^TK^{-1}(X-\mu)dx+\int_b^d\ln((2\pi)^n|K|)^{\frac{1}{2}}dx \quad (2.11.5)$$

$$=\int_b^d\ln\left\{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{1}{2}(X-\mu)^TK^{-1}(X-\mu)}\right\}dx+\frac{1}{2}\int_b^d(X-\mu)^TK^{-1}(X-\mu)dx \quad (2.11.6)$$

$$=\sum_{i,j}\ln\left\{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{1}{2}(X_i-\mu_i)^TK^{-1}(X_j-\mu_j)}\right\}dx+\frac{1}{2}\sum_{i,j}(X_i-\mu_i)^T(K^{-1})_{ij}(X_j-\mu_j) \quad (2.11.7)$$

$$=\sum_{i,j}\ln\left\{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{1}{2}(X_j-\mu_j)(X_i-\mu_i)^T((K^{-1})_{ij})}\right\}dx+\frac{1}{2}\sum_{ji}(X_j-\mu_j)\ (X_i-\mu_i)^T(K^{-1})_{ij} \quad (2.11.8)$$

$$=\sum_j\sum_i\ln\left\{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{1}{2}K_{ji}((K^{-1})_{ij})}\right\}dx+\frac{1}{2}\sum_j\sum_iK_{ji}((K^{-1})_{ij}) \quad (2.11.9)$$

$$=\sum_j\ln\left\{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{1}{2}((KK^{-1})_{jj})}\right\}dx+\frac{1}{2}\sum_j(KK^{-1})_{jj} \quad (2.11.10)$$

$$=\ln\left\{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{n}{2}}\right\}+\frac{n}{2} \quad (2.11.11)$$

$$=\ln\left\{((2\pi)^n|K|)^{\frac{1}{2}}+ae^{-\frac{n}{2}}\right\}+\ln e^{\frac{n}{2}} \quad (2.11.12)$$

$$=\ln\left\{((2\pi e)^n|K|)^{\frac{1}{2}}+a\right\} \quad (2.11.13)$$

$$=\log\left\{((2\pi e)^n|K|)^{\frac{1}{2}}+a\right\}$$

3. Conclusion

We conducted an analysis in this work and found that, with the exception of additivity and recursivity, the suggested function meets all significant qualities satisfied by Shannon's measure of entropy. These characteristics, however, have no bearing on maximising entropy because, anytime a measure is maximised, its monotonically growing functions are also maximised, even though the initial measure might be additive or recursive. We note that the presence of the parameter in the suggested function gives it more flexibility in its uses. Finally, a few more significant findings are also made that have applications in the information and statistical sciences.

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