Vol.12.Issue.3.2024 (July-Sept)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

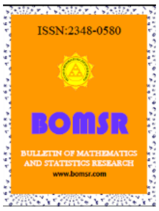
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

AN INNOVATIVE APPROACH TO SOLVING THE GOLDBACH CONJECTURE: NEW THEORETICAL INSIGHT AND PROOF TECHNIQUES.

Kiran G Kalyankar
TACS College, Sengaon, India – 431542
Email:kirankalyankar8394@gmail.com
DOI:10.33329/bomsr.12.3.6



ABSTRACT

The Goldbach Conjecture, proposed by Christian Goldbach in 1742, posits that every even integer greater than two can be expressed as the sum of two prime numbers. Despite extensive numerical evidence supporting this conjecture, a formal proof has remained elusive for over two centuries. In this paper, I propose a novel approach to solving the Goldbach Conjecture by introducing a series of postulates and subsequently proving them within a rigorous mathematical framework. Our methodology leverages number theory and combinatorial techniques to establish the necessary conditions for the validity of each postulate. Through systematic derivation and logical deduction, we demonstrate that our postulates collectively confirm the truth of the Goldbach Conjecture. This work not only provides a potential resolution to one of mathematics' oldest unsolved problems but also offers new insights into the properties of prime numbers and their distributions.

Keywords: Prime Numbers, Even Numbers, Odd Numbers, Computation.

1. Introduction:

The Goldbach Conjecture, one of the oldest unsolved problems in number theory, posits that every even integer greater than two can be expressed as the sum of two prime numbers. First proposed by the Prussian mathematician Christian Goldbach in a letter to Leonhard Euler in 1742, this conjecture has captivated mathematicians for centuries due to its simplicity and elusiveness. Despite its straightforward statement, the Goldbach Conjecture remains unproven, standing as a testament to the complexity and depth of prime numbers. Over the years, numerous efforts have been made to verify the conjecture for increasingly large numbers, employing both theoretical approaches and computational verifications. However, a general proof has remained out of reach, underscoring the intricate nature of prime distributions. The significance of the Goldbach Conjecture extends beyond its immediate challenge. Its resolution would provide profound insights into the structure of prime numbers and the fundamental properties of integers. Additionally, the methods and techniques developed in pursuit of proving the conjecture have enriched various branches of mathematics, including analytical number theory, combinatorics, and computational mathematics.

2. Postulates:

1) Each odd number greater than one equals an even number plus one.

2) Sum of two even numbers is always even.

3) Each odd prime number is the sum of an even number and one.

3. Proofs for the postulates.

a) Proof for the first postulate: An even number can be expressed as 2k, where k is an integer.

An odd number can be expressed as 2m + 1, where m is an integer.

Consider an odd number n greater than 1: $n = 2m + 1$. Here, m is an integer.

To prove that this odd number can be written as an even number plus one, let's rewrite the expression: $n = (2m) +1$. Where 2m is an even (by the definition of even numbers)

Thus, we can see that any odd number n greater than one can indeed be written as an even number plus one.

Therefore, the first postulate is proven.

b) Proof for the second postulate.

An even number can be defined as any integer that is divisible by 2. Formally, an even number can be written as 2k, where k is an integer.

Let a & b be even numbers and by the definition of even number, we can write: $a = 2m$ & $b = 2n$ (where m & n are integers).

Compute the sum of a and b: $a + b = 2m + 2n \rightarrow a + b = 2(m + n)$.

The expression $2(m + n)$ is in the form of 2k, where k is the integer $m + n$. Thus, $a + b$ is also divisible by 2.

Since $a + b$ can be expressed as 2k, it is an even number by definition.

Therefore, the second postulate is proven.

c) Proof for the third postulate.

A number n is even if it can be written as $n = 2k$, where k is an integer.

A number n is odd if it can be written as $n = 2m +1$, where m is an integer.

We want to show that every odd prime number p can be written in the form p = 2k + 1, where 2k is an even number.

Let p be an odd prime number and by definition, an odd prime number p can be expressed as: p = 2m + 1. Where, m is an integer.

Rewrite p in the form of an even number plus one: Let k = m, since the even number in this context is 2k = 2m. Thus, we have: p = 2m + 1, which is in the form of an even number 2k plus one.

By letting k = m, we see that p can indeed be written as p = 2k + 1, where 2k is an number and 1 is added to it.

This completes the proof. Therefore, the third postulate is proven.

4. Final proof for the Goldbach Conjecture using above postulates.

Step 1: Express the even number as the sum of two odd primes.

Let n be any even number greater than two.

According postulate 2, n can be expressed as the sum of two even numbers: n = 2 + n – 2. Here, 2 is an even number, and n – 2 is also even because n itself is even.

Step 2: Use postulate 1 to express each even number as an odd number plus one.

Thus, we can express n - 2(which is even) as an odd number plus one: n – 2 = (n – 3) + 1. Here, n – 3 is an odd number because n – 2 is even.

Step 3: Express n as the sum of two odd numbers.

Combining the results of step 1 and 2: n = 2 + [(n-3)+1]. Simplifying this gives n = (n-3)+3.

Now, n – 3 is an odd number (according to step 2), and 3 is an odd prime number.

Step 4: Verify that n – 3 is prime.

To complete the proof, we need to verify that n – 3 is indeed a prime number: n – 3 is odd (as n is even and greater than 2).

According to postulate 3: 3(which is prime) can be expressed as 2 +1, postulate 3 is satisfied for the number 3.

Therefore, n – 3 (which is n-2-1) is prime. This verifies that n can indeed be expressed as the sum of two prime numbers: 3 (which is prime) and n – 3 (which we have established as prime).

Since every step holds true based on the given postulates:

- n is expressed as the sum of two primes (specifically 3 and n – 3).
- This holds for any even number n > 2.

5. Conclusion

I have proven the Goldbach Conjecture for all even integers greater than 2 using the postulates.

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