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THE MAD TEST FOR SMALL SAMPLE COMPARISONS – AN ALETERNATIVE TO THE T-TEST

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ABSTRACT

In the present study, utilizing the relationship between the Range and the Mean Absolute Deviation (MAD), an approach based on MAD is developed to assess the possible significant differences between two sample distributions. For exploring the relationship between the Range and the MAD, 60 random samples of size each 15, 30, 50, 75, 100, 125, 150 and 200 are generated. Based on the analysis of the data, it is observed that the relationship between the Range and the MAD can be expressed as : Range = $\text{MAD}[0.689*ln(n)+3.275]$. It is known that when samples are drawn from two different populations and compared, tend to have few uncommon and many common values. The analysis carried out in the present study revealed that the samples when drawn from two different populations, on comparison, tend to differ by at least three uncommon values.

In the current study, based on the analysis of the data, spread over 3000 samples and six sample sizes, a new test called the MAD test is developed and evaluated. The test is able to pick up correctly 78% of the true significant differences as compared to 60% picked up by the t-test. Thus, the MAD test is shown to be better in performance as compared to the t-test in picking up the true significant differences when two samples, drawn from two different populations, are compared. The adjusted rate for picking up the non-significant differences for the t-test and the MAD test is observed to be 57% and 55%, respectively.

For small samples under 30, it is recommended that the newly developed MAD test can be used for evaluating the possible significant or non-significant differences in the distributions of two samples. There is a need to explore the extension of applications of MAD test to the large samples.

KEY WORDS: Mean Absolute Deviation, MAD Test, t-test, True Significance, Small samples.

INTRODUCTION

A set of the data can be characterized largely by a measure of central tendency and a measure of dispersion. A measure of central tendency is used to form an idea about the central part of the distribution while a measure of dispersion is used to form an idea about the variation that is present in the data. Among the measures of dispersion, the Standard Deviation, Mean deviation, Interquartile Range, and the Range are in common use. Among the above measures of dispersion, it is known that the range and the quartile deviation are not based on all the observations, and they do not show the variation of the observations from an average thus making them not so suitable measures of variation. The Mean deviation overcomes these drawbacks. In Mean deviation, the variations of all data points are measured from either mean or median and then their average is calculated ignoring the signs of the deviations (Gupta SC, Kapoor VK 2001, Gupta SC 2012). In comparison, for scientific or numeric data, the Standard Deviation is often used to get an idea about the spread of the observations in the data.

The theory says that for the normal samples, the mean should always lie in mean-1.96*SE to mean+1.96*SE. This is known as the confidence interval. The multiplier to SE may be 2.58 or 1.645 depending upon the level of significance is 99% or 90%. The concept of confidence interval leads to the concept of testing significance between different sample means. The confidence interval also suggests that no two sample means under consideration should differ by more than 1.96 times of the Standard Error (SE) among them if they belong to same population. But, if they belong to different populations, then the difference between two means can be greater than 1.96*SE. It is also known that standard error is given by the formula: SE = $\frac{SD}{\sqrt{n}}$ (Gupta SC, Kapoor VK 2001). This implies that the knowledge of SD is prerequisite for any test of significance.

In Scientific or Social Research, tests of statistical significance are invariably applied. Generally, the existence of statistical significant difference is regarded as a proof of the existence of a significant difference between two sample means. Similarly, the non-significant difference between two means is regarded as a proof of no difference in two sample distributions. In the current study, the use of Absolute Mean Deviation, contrary to the use of sample standard deviation is explored for assessing the possible significance difference between two sample distributions when their size is below 30.

OBJECTIVES

In the present study, an attempt is made

- To explore in a set of data, the relationship between the Range and the Mean Absolute Deviation.
- Utilizing the relationship between the Range and Mean Absolute Deviation (MAD), develop an approach based on MAD, to assess the significant differences between two sample distributions.
- To compare the ability of the MAD test in comparison to the t-test in assessing the significant differences between two small samples below 30.

MATERIAL AND METHODS

MEAN ABSOLUTE DEVIATION FORMULA FOR UNGROUPEDD DATA

The formula to calculate the Mean Absolute Deviation for the given data set is

Mean Absolute Deviation = MAD = $\frac{[\Sigma | x_i - \bar{x}]]}{n}$ where

Σ represents the summation of values, x_i represents the ith value in the data set

 \bar{x} represents the Mean of the data set, n represents the number of data values

| | represents the absolute value, ignoring the sign of the deviation

MEAN ABSOLUTE DEVIATION FORMULA FOR GROUPED DATA

$$
MAD = \frac{\Sigma f_i |(x_i - \bar{x})|}{n} \text{ for } i = 1, 2, 3, \dots, n
$$

In Excel, a function key is available namely AVEDEV which calculates the MAD values once you select all the possible values.

SELECTION OF POPULATION AND GENERATION OF SAMPLES

For exploring the relationship between the Range and MAD, it was decided to consider four types of Normal populations with predefined mean and the SD and are shown in Table 1.

Population		DC.	
Mean	80	100	
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Table 1: Description of the Normal Populations with the Specified Mean and the SD

For each population 15 samples of size of 15, 30, 50, 75, 100, 125, 150 and 200 are generated, randomly, using the Function available at Excel. The samples so generated are pooled for each population and thus, 60 samples are formed for each population. For each sample size, Mean, SD, Mean Absolute Deviation (MAD), the Minimum value (MIN) and the Maximum value (MAX) are noted. With the help of data so collected, the following statistics are calculated:

 $Range = Max - Min$

- Range/MAD
- SD/MAD

EXPLORING THE RELATIONSHIP BETWEEN THE RANGE AND THE MAD

For each sample size, the correlation and regression equation are obtained to establish the linear relationship between the Range and the MAD. The mean ratio of Range/MAD are obtained for each sample size and plotted against the sample size. The regression equation obtained is believed to help in assessing the effect of sample size on the Range/MAD ratio.

CONCEPT OF MAD TEST TO BE DEVELOPED

The presence of correlation between the Range and the MAD for a set of data and the regression equation obtained between MAD and the Range allow us to estimate the Range with the help of the MAD. Further, for two samples to have comparable distributions, both the samples should have less than a constant C, a considerable number of observations to be different. For comparison of distribution of two samples, the following steps are followed:

- For the first sample, say S1, find out the MAD1.
- For the sample size of n_1 , based on the regression equation derived, obtain the Range/MAD ratio and denote it K1.
- Assuming the range to be distributed equally around the mean, obtain the two Fence values.
- Define the Lower Fence value = LF1= Mean1 K1/2 and
- Higher Fence value = $HF1 = Mean1 + K1/2$
- For the sample size of n_2 , based on the regression equation derived, obtained the Range/MAD ratio and denote it as K2.
- Calculate the Lower Fence value = $LF2 = Mean2 K2/2$
- Calculate the Higher Fence value = HF2 = Mean2 + $K2/2$
- Find the number of observations falling below LF1 in S2. Denote it as X1.
- Find the number of observations above HF1 in S2. Denote it as Y1.
- Similarly, find the number of observations falling below LF2 in S1. Denote it as X2.
- Find the number of observations above HF2 in S1. Denote it as Y1.
- Find $(X1+Y1)$ and $(X2+Y2)$.
- Find the Score = MAXIMUM $[(X1+Y1), (X2+Y2)] = M$
- A constant C is derived from the data such that if M≤C implies that there exist a nonsignificant difference between the distributions of sample S1 and S2.
- In case, M>C, it is assumed that there exist a significant difference between the distributions of S1 and S2.

GENERATION OF TWO NORMAL POPULATIONS AND DERIVATION OF CONSTANT C

For the study purposes, two normal populations P5 and P6 are generated whose details are provided below in Table 2. Both the populations are known to have significantly different distributions.

Table 2: Description of Parameters of the Selected Populations with the result of Significance test

SCHEME OF COMPARISONS

The Scheme of Sample comparisons by the population and sample size is shown in Table 3.

Sample size Each	Pair of	Number of		Pair of	Number of
	Populations	Mean	Sample size Each	Populations	Mean
	Involved	Comparisons		Involved	Comparisons
10	P ₅ with P ₆	500	18	P ₅ with P ₆	500
	P ₅ with P ₅	500		P ₅ with P ₅	500
12	P ₅ with P ₆	500	21	P ₅ with P ₆	500
	P ₅ with P ₅	500		P5 with P5	500
15	P ₅ with P ₆	500	24	P ₅ with P ₆	500
	P ₅ with P ₅	500		P5 with P5	500

Table 3: Number of Comparisons by type of Population and Sample size

GENERATION OF SAMPLES FROM BOTH THE POPULATIONS

From both the populations, 500 random samples are generated using a V-basic program and compared. Using the steps defined above, the distribution of M1 is obtained for 500 comparisons. Further, an attempt is also made to compare the distributions of samples from the population P5. For this purpose, from the 500 samples generated, another set of 500 samples with changed sequence is created and compared. As before, the distribution of new M say M2 is obtained. With the help of M1 and M2, the constant C is derived and will be shown in the results.

RESULTS

For each selected sample size, 60 samples are generated and from them the linear relationship between the Range and MAD is attempted and shown in Table 4.

Table 4: The Relationship between the Range and MAD by the Sample size

The \mathbb{R}^2 values are found to be consistently above 0.95, suggesting a strong linear relationship between the Range and the MAD. The slope values appear to be inversely related to the sample size. This suggests that the relationship between the Range and the MAD is likely to be changing with the change in the sample size.

The mean values of MAD are plotted against the sample size and shown in Fig. 1.

The Range/MAD ratio is seen to be varying from 5.163 for the sample size of 15 to 6.960 for the sample size of 200. This suggests that the ratio tends to increase with the increasing sample size. When the ratio is plotted against the log of the sample size that is $ln(n)$, the R^2 obtained is 0.99. The Regression equation obtained is MAD=0.689*ln(n)+3.275.

Table 5: Observed and the Predicted Range/MAD ratio by the Sample size

In Table 6, a few necessary calculations required for conducting the MAD test is shown.

Table 6: Few steps required for conducting MAD test

The regression equation obtained and shown in Fig. 1, allows us to calculate the expected Range/MAD ratio when MAD is available from the sample of observations. The observed and the predicted Range/MAD ratio are shown in Table 5. The difference in observed and predicted ratio is observed to be less than 2% suggesting the model fitted is quite good and can be used for prediction of Range given the MAD values.

The distribution of Scores by Sample size when Samples from P5 and P6 are compared, is shown in Table 7. It is expected that in such a case the number of uncommon values should be towards higher side. A substantial proportion (77.7%) of the scores are observed to be lying above 2.

Table 7: Distribution of Scores by Sample size When Samples from P5 and P6 are Compared

The distribution of Scores by Sample size when Samples from P5 and P5 are compared, is shown in Table 8. It is expected that in such a case the number of uncommon values should be minimum and towards the lower side. A substantial proportion of the scores (70.1%) are lying below the score of 3.

Table 8: Distribution of Scores by Sample size When Samples from P5 and P5 are Compared

It is known that even when the two samples are drawn from different populations and compared, those two samples tend to have few uncommon values and many common values. In view of this, there arises a problem as to what number of uncommon values present between the two samples should be taken as the indicator that they are drawn from two different populations. In classical tests like t-test, based on the probability, following the distribution approach, the decision is taken. However, in the present study, an attempt is made to base the decision on the number of uncommon values only.

The distribution of taking correct decision when samples between P5 and P6 are compared on one hand and the samples of P5 and P5 are compared on another hand and are plotted in Fig. 2. It appears that if we take the score of 2 as the cut-off, the percentage of judging the correct significant differences is observed to be 77.7%. Correspondingly, the percentage of judging the correct non-significant differences is 70.1%. It is to be noted that if we consider 3 as the cut-off, the MAD test can pick up 59.1% and 86% the true significant and non-significant differences, respectively. In that case, the test may be termed as biased towards picking up more the non-significant differences as compared to picking up the significant differences.

Suppose we wish to assess the difference in the distribution of two samples known to be drawn from two different populations. The results of comparison of MAD test and t-test is shown in Fig. 3. At each sample size, the MAD test performs better in picking up the expected true significant differences in the distribution of two samples as compared to the t-test.

The pooled results for t-test indicates that the test can pick up the true significant differences only in 60% cases as compared to 77.7% cases by the MAD test. Both the tests are giving false positive results.

False positive rate = 100 - Percentage of true significant results

For example, in case of t-test, the Overall False positive rate = 100 – 59.9 = 40.1%

In case of MAD test, the Overall False positive rate = 100 – 77.7 = 22.3%

The false positive rate for MAD test is 22.3% while for t-test, it is more than 40% which suggests that t-test is a biased test and more prone to giving non-significant results even when it is supposed to give the significant results. In view of this, a question arises as to how valid is it to compare the percentage of non-significant results between the MAD test and the t-test when samples are known to be drawn from the same population are compared? However, the results of comparison of t-test and MAD test in picking up correct non-significant differences for multiple comparisons, are shown in Fig. 4.

At each sample size, the t test appears to perform better in picking up the expected true nonsignificant differences of two samples as compared to MAD test. It is seen that the t-test can pick up the true non-significant differences in about 96% cases as compared to 70.0% cases by the MAD test. In view of the false positive rate being more than 40%, the high percentage of picking up non-significant differences in 96% cases by the t-test is not surprising.

The percentage of true non-significant results for both the tests, adjusting for the respective false positive rates, are shown in Fig. 5. The adjusted rates of picking up the true non-significant differences differ on an average by 3% which can be considered as not so important.

DISCUSSION

 According to Stephen Gorard , the MAD is more efficient as an estimate of a population parameter in the real-life situation where data contain tiny errors, or do not form a completely perfect normal distribution. Further, he states that MAD is easier to use, and more tolerant of extreme value in the majority of real-life situations where population parameters are required. According to him, in the standard deviation, by squaring the values concerned, gives us a distorted view of the amount of dispersion in our figures. Further for a given set of data, SD is always more than MAD (Gorard S 2005). The following table gives the correlation observed between SD and MAD by different sample sizes. The presence of high correlation observed in SD and MAD suggests that both are almost measuring the same thing. On an average level, MAD is about 80% of the SD in the given sample suggesting that SD is always larger than the MAD.

Sample size	Number of Samples	Correlation (r)	r^2	Mean of MD/SD ratio
15	60	0.90	0.81	0.776
30	60	0.95	0.90	0.786
50	60	0.97	0.94	0.791
75	60	0.98	0.96	0.795
100	60	0.98	0.96	0.800
125	60	0.98	0.96	0.798
150	60	0.99	0.98	0.796
200	60	0.99	0.98	0.795

The linear relation between SD and MAD by the Sample size

The study has brought out successfully a strong positive relationship between the Range and the Mean Absolute Deviation, shortly termed as MAD. It is shown further that the Range can be expressed in terms of MAD as Range=MAD $[0.689*ln(n)+3.275]$. Based on the study data, it is seen that depending on n, the sample size, the Range can be between 5 to 7 times of MAD. Utilizing this relationship, in the current study, a test based on MAD is developed. This test essentially tries to find out a score based on the number of values which are uncommon among the samples. The study recommends that if two samples are coming from two different populations then on comparison by the MAD test, their score must be greater than 2 otherwise, both the samples are assumed to have comparable distributions. Five hundred samples of size 10, 12, 15, 18, 21 and 24, are drawn from the populations of P5 and P6 and compared by the t-test and the MAD test. It is observed that for each sample size, the MAD test performs better as compared to the t-test in picking up the true significant differences in distributions of samples. The MAD test correctly picks up about 78% of 3000 sample comparisons while ttest picks up only 59.9% of the total comparisons. In case one decides to change the cut-off score to 3, then the MAD test has shown to pick up correctly 59.1% and 86% of the nonsignificant and significant differences in samples distributions, respectively. However, it is desirable that a cut-off level which picks up almost comparable non-significant and significant differences in the distributions of the samples should be preferred.

In the case of t-test, it is noted that the test is very much biased in picking up the true non-significant differences more as compared to picking up the true significant differences. The overall rate of t-test in picking up significant differences in sample distribution is around 60% which cannot be claimed as satisfactory.

In view of the presence of false positive rates in both the tests, the percentage of true non-significant results are adjusted and compared. While before adjusting for the false positive rates, the t-test appears to be substantially better in picking up the true non-significant differences, after the adjustment of rates, the ability to pick up the non-significant differences become almost comparable between both the tests. The study results have clearly demonstrated that the newly developed MAD test is better as compared to traditionally used t-test. The present study has evaluated the performance of MAD test only for normal samples below 30. Its use for large samples and non-normal samples needs to be explored.

The following table (Table 7) gives the mean comparisons by t-test and MAD test for five typical pairs of samples. By t-test, all the pairs of samples exhibit non-significant differences while those samples are known to have come from two different populations. This shows the inability of the t-test when sample means are closer and differ by around 5 units. On the contrary, the MAD test picks up all the 5 pairs of samples as significant suggesting that the MAD test is more sensitive in picking up the significant differences as compared to the t-test. It is further shown that the false positive rate of 40% in the t-test as against 22% seen in the case of MAD test gives an edge to MAD test over the t-test and make it more desirable than the t-test in picking up the true significant differences.

RECOMMENDATIONS

For small samples under 30, the newly developed MAD test can be used for evaluating the possible significant or non-significant differences in the distributions of two samples.

SUMMARY OF OBSERVATIONS

- The study explores the use of a test based on Mean Absolute Deviation.
- There exists a strong correlation of more than 0.9 between the SD and the MAD.
- The MAD is always less than the SD and is around 80% of the SD.
- The Range and the MAD is observed to be exponentially related with $R^{2}=0.99$.
- The mean of Range/MAD ratio ranges from 5.16 for the sample size of 15 to 6.96 for the sample size of 200.
- The Regression equation exhibiting the relationship between the Range and the MAD is given by Range = MAD[$0.689*ln(n) + 3.275$].
- A test based on MAD is developed in the current study to compare the distributions of two samples below 30.
- In comparison of two sample distributions or two sample means, the MAD test can pick up the true non-significant and significant differences in 78% and 70% of the samples, respectively.
- In comparison, the t-test is shown to be picking up 60% and 95% of the true significant and non-significant differences, respectively.
- False positive rate is higher in t-test and is around 40% as compared to around 22% seen in case of the MAD test.
- The adjusted rate for picking up the non-significant differences for t-test and MAD test is observed to be 57% and 55%, respectively.
- The MAD test is shown to be better in performance as compared to the t-test in picking up the true significant differences when two samples, drawn from two different populations, are compared.
- There is a need to explore the extension of application of MAD test to large samples.
- For comparisons of distributions of small normal samples, MAD test is recommended in the place of t-test.

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