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Solving Triopoly Game with Triangular Intuitionistic Fuzzy Numbers as Payoff and its Application in Market Share

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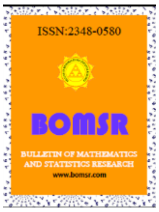


ABSTRACT

This paper proposes a technique for solving a triopoly game with Triangular Intuitionistic Fuzzy Numbers as payoffs. The players in the game interact strategically without cooperating with one another. The suggested method is applied in a numerical example of internet service providers, which is helpful for the suppliers to make decisions on their costs effectively.

Keywords: Triopoly game, Non- cooperative games, Intuitionistic Fuzzy set, Triangular Intuitionistic Fuzzy Number, $(\alpha, \beta)-$cut values, Nash Equilibrium.

1.Introduction

Game theory is a prominent approach for evaluating the strategic interactions between the decision makers. In the decision making process there is always a competition among the players and it deals with the conditions like the number of strategies, their level of importance, collecting information about their problem and the opponents. In classical game theory the relation between two players is widely discussed[3]. When the game pattern involves three firms selling a same product competing over the market share the triopoly structure become apparent[14]. In this situation the decision maker not only affected by their decisions, but the choices of their competitors as well. The triopoly game is the oligopoly game with three suppliers on a market. The triopoly game structure is commonly used to evaluate the pricing of the products.

L.A.Zadeh introduced the theory of fuzzy sets 1965 to deal with the uncertainty caused by dealing the real life circumstances with classical set theory in decision making[7]. The fuzzy set uses a membership function to denote the degree of belongingness or acceptance[2]. In the process of decision making it is insufficient where in reality the level of belongingness and non-belongingness both are involved. Then the concept of Intuitionistic Fuzzy Sets evolved which has a membership as well as a non-membership function[6]. Intuitionistic Fuzzy sets are the extension of Fuzzy sets with two indices(membership and non membership functions) instead of one index and delas with the acceptance and hesitance levels.

The decision criteria of 3D- Matrix games under uncertainty are derived by Ozkaya.M[10] in which the laplace, wald, Hurwicz and savage criteria are demonstrated for three player games. The triopoly structure for non-cooperative games is discussed in[11] and the strategic interaction of the game is represented by a model to solve a three suppliers game. The behaviour of heterogeneous players in the discrete triopoly form is given in[1], the players in the game are considered aa rational. Matrix games with Trapezoidal Intuitionistic Fuzzy(TrIF) payoff are solved[12] by converting the TrIF payoff values into crisp matrix. The famous methods of dominance, graphical and algebraic are used to solve the payoff matrix. The triopoly structure considered in this paper makes it impossible to solve the payoff matrix by classical method which are useful only in the two dimensional form. Bi-Matrix game with Triangular Intuitionistic Fuzzy(TIF) payoff values are solved by using the α, β −cut values of the Intuitionistic fuzzy sets and their mean values[9]. A non-linear Intuitionistic fuzzy approach is applied to find the equilibrium in $m \times n$ games[4]. Various methods to find the Nash Equilibrium in $m \times n$ is discussed in [13]. In this paper the Gambit software tool[8] is used to find the equilibrium solution of the triopoly game. The stability and allocation of three player game introduced to deal with the cooperative players[14]. In this paper a technique is introduced which can be used in the triopoly structure of the game matrix with payoffs in the form of Triangular Intuitionistic Fuzzy Numbers. In the strategic situation considered here, the payoff matrix is calculated by conducting a survey for the population. The survey can be affected by the acceptance or the hesitance of the people participating regarding the selection of particular supplier. So the Intuitionistic Fuzzy set structure is considered to represent the membership and non-membership levels of the payoff in each outcome. A ranking function is used to defuzzify the Triangular Intuitionistic numbers into crisp numbers. This method is a first attempt on approaching the triopoly game structure with Intuitionistic Fuzzy sets to make the decision making process more realistic.

This paper is organized as follows, In section 2 the definitions and arithmetic operations of TIFNs are given, section 3 contains the model for solving the triopoly game with TIFN payoffs and the numerical example of applying the proposed method in the game of Internet service providers is explained in section 4, section 5 concludes the paper.

2. Preliminaries

Definition 2.1:

A Triangular Intuitionistic Fuzzy Number (TIFN) is defined as $\tilde{l} = \langle (\underline{l}, l_1, \overline{l}); w_{\tilde{l}}, u_{\tilde{l}} \rangle$ in $\mathbb{R}$ with membership function $\mu_{\tilde{l}}(x)$ and non-membership function $v_{\tilde{l}}(x)$ which are defined as

$$\mu_{\tilde{l}}(x) = \begin{cases} \dfrac{x-\underline{l}}{l_1-\underline{l}} w_{\tilde{l}}, & \underline{l} \le x < l_1 \\ w_{\tilde{l}}, & x = l_1 \\ \dfrac{\overline{l}-x}{\overline{l}-l_1} w_{\tilde{l}}, & l_1 < x \le \overline{l} \\ 0, & x < l \ or \ x > \overline{l} \end{cases}$$

and

$$v_{\tilde{l}}(x) = \begin{cases} \dfrac{(\underline{l}-x) + u_{\tilde{l}}(x-\underline{l})}{(l_1-\underline{l})}, & \underline{l} \le x < l_1 \\ u_{\tilde{l}}, & x = l_1 \\ \dfrac{(x-l_1) + u_{\tilde{l}}(\overline{l}-x)}{(\overline{l}-l_1)}, & l_1 < x \le \overline{l} \\ 1, & x < \underline{l} \ or \ x > \overline{l} \end{cases}$$

The values $w_{\tilde{l}}, u_{\tilde{l}}$ represents the maximum degree of membership and minimum degree of non-membership, respectively. And they satisfy the condition $0 \le w_{\tilde{l}} \le 1$,$0 \le u_{\tilde{l}} \le 1$ and $0 \le w_{\tilde{l}} + u_{\tilde{l}} \le 1$.

Definition 2.2:

For a TIFN $\tilde{l} = \langle \underline{l}, l_1, \overline{l}; w_{\tilde{l}}, u_{\tilde{l}} \rangle$ the (α, β)- cut set is a subset of R that is

$\tilde{l}_{\alpha,\beta} = \{x : \mu_{\tilde{l}}(x) \ge \alpha, v_{\tilde{l}}(x) \le \beta\}$, where $0 \le \alpha \le w_{\tilde{l}}, u_{\tilde{l}} \le \beta \le 1$ and $0 \le \alpha + \beta \le 1$.

and $\tilde{l}_\alpha$ is defined by the closed interval $[L_{\tilde{l}}(\alpha), R_{\tilde{l}}(\alpha)]$,

$$L_{\tilde{l}}(\alpha) = \frac{(w_{\tilde{l}} - \alpha)\underline{l} + \alpha l_1}{w_{\tilde{l}}}, \qquad R_{\tilde{l}}(\alpha) = \frac{(w_{\tilde{l}} - \alpha)\overline{l} + \alpha l_1}{w_{\tilde{l}}}$$

Then,

$$\tilde{l}_\alpha = \left[\frac{(w_{\tilde{l}} - \alpha)\underline{l} + \alpha l_1}{w_{\tilde{l}}}, \quad \frac{(w_{\tilde{l}} - \alpha)\overline{l} + \alpha l_1}{w_{\tilde{l}}}\right]$$

Similarly the β −cut is defined as

$$\tilde{l}_\beta = \left[\frac{(1-\beta)l_1 + (\beta - u_{\tilde{l}})\underline{l}}{1-u_{\tilde{l}}}, \frac{(1-\beta)l_1 + (\beta - u_{\tilde{l}})\overline{l}}{1-u_{\tilde{l}}}\right]$$

2.1 Arithmetic operations on TIFN:

For two TIFNs $\tilde{l} = \langle (\underline{l}, l_1, \overline{l}); w_{\tilde{l}}, u_{\tilde{l}} \rangle$ and $\tilde{f} = \langle (\underline{f}, f_1, \overline{f}); w_{\tilde{f}}, u_{\tilde{f}} \rangle$ the arithmetic operations are of the form,

$$\tilde{l} + \tilde{f} = \langle (\underline{l} + \underline{f}, l_1 + f_1, \overline{l} + \overline{f},); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}} \rangle$$

$$\tilde{l} - \tilde{f} = \langle\left(\underline{l} - \overline{f}, l_1 - f_1, \overline{l} - \underline{f}\right); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}}\rangle$$

$$\tilde{l} \times \tilde{f} = \begin{cases} \langle\left(\underline{lf}, l_1 f_1, \overline{lf}\right); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}}\rangle \ if\ \tilde{l} > 0\ and\ \tilde{f} > 0 \\ \langle\left(\underline{l}\overline{f}, l_1 f_1, \overline{l}\underline{f}\right); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}}\rangle \ if\ \tilde{l} < 0\ and\ \tilde{f} > 0 \\ \langle\left(\overline{lf}, l_1 f_1, \underline{lf}\right); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}}\rangle \ if\ \tilde{l} < 0\ and\ \tilde{f} < 0 \end{cases}$$

$$\frac{\tilde{l}}{\tilde{f}} = \begin{cases} \langle\left(\underline{l}/\overline{f}, l_1/f_1, \overline{l}/\underline{f}\right); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}}\rangle \ if\ \tilde{l} > 0\ and\ \tilde{f} > 0 \\ \langle\left(\overline{l}/\overline{f}, l_1/f_1, \underline{l}/\underline{f}\right); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}}\rangle \ if\ \tilde{l} < 0\ and\ \tilde{f} > 0 \\ \langle\left(\overline{l}/\underline{f}, l_1/f_1, \underline{l}/\overline{f}\right); w_{\tilde{l}} \wedge w_{\tilde{f}}, u_{\tilde{l}} \vee u_{\tilde{f}}\rangle \ if\ \tilde{l} < 0\ and\ \tilde{f} < 0 \end{cases}$$

For any real number λ,

$$\lambda\tilde{l} = \begin{cases} \langle(\lambda\underline{l}, \lambda l_1, \lambda\overline{l}); w_{\tilde{l}}, u_{\tilde{l}}\rangle, if\ \lambda > 0 \\ \langle(\lambda\overline{l}, \lambda l_1, \lambda\underline{l}); w_{\tilde{l}}, u_{\tilde{l}}\rangle, if \lambda < 0 \end{cases}$$

2.2 Value and Ambiguity of TIFN:

For the TIFN $\tilde{l} = \langle(\underline{l}, l_1, \overline{l}); w_{\tilde{l}}, u_{\tilde{l}}\rangle$ the value of membership and non-membership functions is denoted as $S_\mu(\tilde{l})$ and $S_v(\tilde{l})$, $f(\alpha) = \frac{\alpha}{2w_{\tilde{l}}}$ and $g(\beta) = \frac{1-\beta}{2(1-u_{\tilde{l}})}$ then

$$S_\mu(\tilde{l}) = \int_0^{w_{\tilde{l}}} \left[\frac{(w_{\tilde{l}} - \alpha)\underline{l} + \alpha l_1 + (w_{\tilde{l}} - \alpha)\overline{l} + \alpha l_1}{w_{\tilde{l}}}\right] \frac{\alpha}{2w_{\tilde{l}}} d\alpha$$

$$= \frac{w_{\tilde{l}}\left(\underline{l} + \overline{l} + 4l_1\right)}{12}$$

$$S_v(\tilde{l}) = \int_{u_{\tilde{l}}}^{1} \left[\frac{(1-\beta)l_1 + (\beta - u_{\tilde{l}})\underline{l} + (1-\beta)l_1 + (\beta - u_{\tilde{l}})\overline{l}}{1 - u_{\tilde{l}}}\right] \frac{1-\beta}{2(1-u_{\tilde{l}})} d\beta$$

$$= \int_{u_{\tilde{l}}}^{1} \left[\left(\underline{l} + \overline{l}\right) + \frac{\left(2l_1 - \underline{l} - \overline{l}\right)(1-\beta)}{1 - u_{\tilde{l}}}\right] \frac{1-\beta}{2(1-u_{\tilde{l}})} d\beta$$

$$= \frac{\left(\underline{l} + 4l_1 + \overline{l}\right)(1 - u_{\tilde{l}})}{12}$$

The ambiguity of the TIFN $\tilde{l} = \langle(\underline{l}, l_1, \overline{l}); w_{\tilde{l}}, u_{\tilde{l}}\rangle$ are denoted as $T_\mu(\tilde{l})$ and $T_v(\tilde{l})$, defined as

$$T_\mu(\tilde{l}) = \int_0^{w_{\tilde{l}}} \left[\frac{(w_{\tilde{l}} - \alpha)\overline{l} + \alpha l_1 - (w_{\tilde{l}} - \alpha)\underline{l} + \alpha l_1}{w_{\tilde{l}}}\right] \frac{\alpha}{2w_{\tilde{l}}}\ d\alpha$$

$$= \int_0^{w_{\tilde{l}}} \left[\left(\overline{l} - \underline{l}\right) - \frac{\left(\overline{l} - \underline{l}\right)\alpha}{w_{\tilde{l}}}\right] \frac{\alpha}{2w_{\tilde{l}}}\ d\alpha$$

$$= \frac{\left(\overline{l} - \underline{l}\right)w_{\tilde{l}}}{12}$$

$$T_v(\tilde{l}) = \int_{u_{\tilde{l}}}^{1} \left[\frac{(1-\beta)l_1 + (\beta - u_{\tilde{l}})\overline{l} - (1-\beta)l_1 + (\beta - u_{\tilde{l}})\underline{l}}{1 - u_{\tilde{l}}}\right] \frac{1-\beta}{2(1-u_{\tilde{l}})} d\beta$$

$$= \int_{u_{\tilde{l}}}^{1} \left[(\bar{l} - \underline{l}) - \frac{(\bar{l} - \underline{l})(1 - \beta)}{1 - u_{\tilde{l}}} \right] \frac{1 - \beta}{2(1 - u_{\tilde{l}})} d\beta$$

$$= \frac{(\bar{l} - \underline{l})(1 - u_{\tilde{l}})}{12}$$

Definition 2.3:

For the TIFN $\tilde{l} = \langle (\underline{l}, l_1, \bar{l}); w_{\tilde{l}}, u_{\tilde{l}} \rangle$ and the ranking function or defuzzify function is defined as $\mathcal{R} : \mathcal{F}(R) \to R$ where $\mathcal{F}(R)$ is the collection of all Triangular Intuitionistic fuzzy numbers defined on R, and the function maps each TIFN into the real line. And the ranking method also used to evaluate the relation between values and ambiguities of membership and non-membership functions of Triangular Intuitionistic fuzzy numbers defined as,

$$\mathcal{R}(\tilde{l}) = \frac{\mathbb{P}(\tilde{l}) + \mathbb{Q}(\tilde{l})}{2}$$

Where,

$$\mathbb{P}(\tilde{l}) = C_\mu(\tilde{l}) + C_v(\tilde{l}) = \frac{(\underline{l} + 4l_1 + \bar{l})(w_{\tilde{l}} + 1 - u_{\tilde{l}})}{12},$$

$$\mathbb{Q}(\tilde{l}) = D_\mu(\tilde{l}) + D_v(\tilde{l}) = \frac{(\bar{l} - \underline{l})(w_{\tilde{l}} + 1 - u_{\tilde{l}})}{12}$$

2.3 Equilibrium in Triopoly game :

The general structure of a triopoly game is defined by the coalition of set of players $N = \{1,2,3\}$, strategies or actions $s_i, i = 1,2,3,$, and the payoff function $p_i \in R, i = 1,2,3$, strategy space for each player $i, S_i, i = 1,2,3$,and strategy combinations between strategies (s_1, s_2, s_3). According to Nash a finite non- cooperative game has at least one equilibrium. In a n-person game each player choose from their finite strategy set from the strategy space[ ]. Each strategy in the player's strategy space counters with each strategies from other player's strategy space in the aim of attaining the highest payoff .

In the non- cooperative game s_j are the strategy set of other players, and each player i have the knowledge about $s_j, j \in N$ and $p_j(s)$ is the payoff function in the strategy set s_j. So in the game each player i choose his best action according to j and $p_i(s/t_i)$ is the best outcome of the player i with respect to s , and $t_i(s)$ is the collection of best possible outcomes of $i \in N$. Because there may be more than one maximum payoff for s_i. So a strategy $\bar{s}$ is said to be a best response if $\bar{s} \in t_i(\bar{s})$ that is $\bar{s}$ is the equilibrium if $\bar{s} \in s_i$ and $p_i(\bar{s}) = \max_{s_i} p_i(\bar{s}; s_i)$, $i \in N$.

3.Model for solving triopoly game with TIFN payoffs

In the non-cooperative triopoly game it is assumed that the players are well aware of their own strategies as well as the other's and the players are considered rational. In this method a survey is conducted on the users to find the payoff matrix of each player. There are three suppliers involved in the market share of same product. Here $\tilde{L}, \tilde{F}$ and $\widetilde{H}$ are the three competitors in the market. The following notations are used in the construction of the model

$\tilde{l}_{ijk}$, ($i, j, k = 1,2,3$) is the estimated payoff of player $\tilde{L}$ if $\tilde{L}$ chooses i, $\tilde{F}$ chooses j and $\widetilde{H}$ chooses k. Where i, j, k are the strategies available to each player,

$\tilde{f}_{ijk}$, $(i,j,k = 1,2,3)$ is the estimated payoff of $\tilde{F}$ with the strategy combinations of $\tilde{L}$ and $\widetilde{H}$ with $\tilde{F}$,

$\tilde{h}_{ijk}$, $(i,j,k = 1,2,3)$ is the estimated payoff of $\widetilde{H}$ with the strategy combinations of $\tilde{L}$ and $\tilde{F}$ with $\widetilde{H}$,

$\tilde{L}_o, \tilde{F}_o$ and $\widetilde{H}_o$ are the number of users currently using the service before beginning the survey,

$\tilde{l}_o, \tilde{f}_o$ and $\tilde{h}_o$ are the number of users surveyed for each suppliers $\tilde{L}, \tilde{F}$ and $\widetilde{H}$,

$\tilde{L}\tilde{L}_{ijk}$, $(i,j,k = 1,2,3)$ is the number of users of $\tilde{L}$ chooses to remain as the customers of $\tilde{L}$ after the survey,

$\tilde{F}\tilde{F}_{ijk}, \widetilde{H}\widetilde{H}_{ijk}$, $(i,j,k = 1,2,3)$ are the number of users of $\tilde{F}$ and $\widetilde{H}$ chooses to retain their choice as being the customers of $\tilde{F}$ and $\widetilde{H}$ after the survey,

$\tilde{L}\tilde{F}_{ijk}$, $(i,j,k = 1,2,3)$ is the surveyed users of $\tilde{L}$ chooses to become the customers of $\tilde{F}$ after the survey,

Similarly , $\tilde{L}\widetilde{H}_{ijk}$, $\tilde{F}\tilde{L}_{ijk}$, $\tilde{F}\widetilde{H}_{ijk}$, $\widetilde{H}\tilde{L}_{ijk}$, *and* $\widetilde{H}\tilde{F}_{ijk}$ are the number of the users changing their choices to other service providers after the survey ,

$\underline{\tilde{L}}_{ijk}$, $\underline{\tilde{F}}_{ijk}$ and $\underline{\widetilde{H}}_{ijk}$ are the numbers of surveyed users chooses to discontinue the service regarding the companies choices.

The users might have a hesitation concerning the choice of using a particular service provider and they could change their decision due to lack of information or having second thoughts. Using the Intuitionistic Fuzzy Set theory is a better way to deal with the situation. Here $w_{\tilde{l}}$, $u_{\tilde{l}}$, $w_{\tilde{f}}, u_{\tilde{f}}, w_{\tilde{h}}$ and $u_{\tilde{h}}$ are the degree of acceptance and hesitation for each player for the strategy combination of i, j and k. The estimated payoff is calculated by using the following equations,

$$\tilde{l}_{ijk} = \langle\left(\frac{\tilde{L}\tilde{L}_{ijk}}{\tilde{l}_o}.\tilde{L}_o + \frac{\tilde{F}\tilde{L}_{ijk}}{\tilde{f}_o}.\tilde{F}_o + \frac{\widetilde{H}\tilde{L}_{ijk}}{\tilde{h}_o}.\widetilde{H}_o - \frac{\underline{\tilde{L}}_{ijk}}{\tilde{l}_o}.\tilde{L}_o\right); w_{\tilde{l}}, u_{\tilde{l}}\rangle \quad (1)$$

$$\tilde{f}_{ijk} = \langle\left(\frac{\tilde{L}\tilde{F}_{ijk}}{\tilde{l}_o}.\tilde{L}_o + \frac{\tilde{F}\tilde{F}_{ijk}}{\tilde{f}_o}.\tilde{F}_o + \frac{\widetilde{H}\tilde{F}_{ijk}}{\tilde{h}_o}.\widetilde{H}_o - \frac{\underline{\tilde{F}}_{ijk}}{\tilde{f}_o}.\tilde{F}_o\right); w_{\tilde{f}}, u_{\tilde{f}}\rangle \quad (2)$$

$$\tilde{h}_{ijk} = \langle\left(\frac{\tilde{L}\widetilde{H}_{ijk}}{\tilde{l}_o}.\tilde{L}_o + \frac{\tilde{F}\widetilde{H}_{ijk}}{\tilde{f}_o}.\tilde{F}_o + \frac{\widetilde{H}\widetilde{H}_{ijk}}{\tilde{h}_o}.\widetilde{H}_o - \frac{\underline{\widetilde{H}}_{ijk}}{\tilde{h}_o}.\widetilde{H}_o\right); w_{\tilde{h}}, u_{\tilde{h}}\rangle \quad (3)$$

4.Numerical Example

The model explained above is applied on the market share of internet service providers[11] where three companies $\tilde{L}, \tilde{F}$ and $\widetilde{H}$ are supplying internet service for the customers. Among the three $\tilde{L}$ is the oldest one in the market, so the customers using their service are higher than the others.

The companies approaches a situation about price change. Three competitors are trying to attract the customers as maximum as possible with an aim of increasing the profit. Here each companies have strategies which are same for all the three regarding the price change.

The companies are the players in the triopoly structure, each of them working independently and have the knowledge about other players strategies which are

i. Retaining the current price (CP)

ii. Decreasing the price (LP)

iii. Increasing the price (HP)

In the market the number of customers using $\tilde{L}$ are 1000, customers using $\tilde{F}$ are 600 and using $\tilde{H}$ are 400.

The survey is conducted through phone calling and Email in which the companies inform about their options in price change to the customers and the customers decides according to that. Here the customers also have three choices, they may continue their existing service providers ,or changing to other service providers, or discontinue the service.

The number strategy combinations for each player for $i, j, k = 1,2,3$ is $n(S_i) = 3^3 = 27$. The results of the survey with respect to the company's choices are given in the tables 1,2 and 3.

(1). The customers asked what would they do if $\tilde{L}$ chooses to retain current price					
other player's choice		Response from customers			
$\tilde{F}$	$\tilde{H}$	Remains at $\tilde{L}$	Changes to $\tilde{F}$	Changes to $\tilde{H}$	Discontinue service
CP (1)	CP (1)	$\langle (395,397,400); 0.61,0.3 \rangle$	$\langle (51,53,55); 0.5,0.32 \rangle$	$\langle (36,38,40); 0.4,0.5 \rangle$	$\langle (6,7,9); 0.4,0.6 \rangle$
…	…	…	…	…	…
LP (2)	CP (1)	$\langle (406,407,410); 0.38,0.46 \rangle$	$\langle (66,68,69); 0.42,0.51 \rangle$	$\langle (11,13,16); 0.29,0.41 \rangle$	$\langle (2,3,5); 0.31,0.42 \rangle$
LP (2)	LP (2)	$\langle (385,387,388); 0.41,0.45 \rangle$	$\langle (55,56,58); 0.42,0.33 \rangle$	$\langle (45,47,49); 0.3,0.5 \rangle$	$\langle (3,5,6); 0.61,0.32 \rangle$
…	…	…	…	…	…
HP (3)	HP (3)	$\langle (400,402,406); 0.58,0.36 \rangle$	$\langle (38,40,42); 0.5,0.3 \rangle$	$\langle (35,39,41); 0.37,0.48 \rangle$	$\langle (7,9,11); 0.6,0.25 \rangle$
(2)The customers asked what would they do if $\tilde{L}$ chooses to decrease the price					
other player's choice		Response from customers			
$\tilde{F}$	$\tilde{H}$	Remains at $\tilde{L}$	Changes to $\tilde{F}$	Changes to $\tilde{H}$	Discontinue service
CP	CP	$\langle (423,426,430); 0.52,0.33 \rangle$	$\langle (33,35,38); 0.61,0.28 \rangle$	$\langle (20,23,25); 0.42,0.35 \rangle$	$\langle (4,6,7); 0.38,0.5 \rangle$

(1)	(2)				
…	…	…	…	…	…
LP (2)	CP (1)	$\langle (419,421,424); 0.53,0.26 \rangle$	$\langle (42,45,47); 0.49,0.32 \rangle$	$\langle (18,20,23); 0.55,0.25 \rangle$	$\langle (3,5,6); 0.52,0.37 \rangle$
LP (2)	LP (2)	$\langle (415,417,419); 0.42,0.38 \rangle$	$\langle (49,51,53); 0.62,0.28 \rangle$	$\langle (20,22,25); 0.32,0.45 \rangle$	$\langle (0,1,3); 0.5,0.3 \rangle$
…	…	…	…	…	…
HP (3)	HP (3)	$\langle (417,420,422); 0.5,0.3 \rangle$	$\langle (33,35,37); 0.35,0.42 \rangle$	$\langle (26,29,31); 0.38,0.42 \rangle$	$\langle (5,6,8); 0.6,0.25 \rangle$
(3)The customers asked what would they do if $\tilde{L}$ chooses to increase the price					
other player's choice		Response from customers			
$\tilde{F}$	$\tilde{H}$	Remains at $\tilde{L}$	Changes to $\tilde{F}$	Changes to $\tilde{H}$	Discontinue service
CP (1)	CP (1)	$\langle (338,340,344); 0.51,0.35 \rangle$	$\langle (71,73,76); 0.48,0.32 \rangle$	$\langle (67,70,72); 0.6,0.22 \rangle$	$\langle (5,7,8); 0.4,0.48 \rangle$
…	…	…	…	…	…
LP (2)	CP (1)	$\langle (358,360,362); 0.51,0.25 \rangle$	$\langle (78,80,81); 0.58,0.23 \rangle$	$\langle (40,43,45); 0.28,0.45 \rangle$	$\langle (8,10,12); 0.52,0.31 \rangle$
…	…	…	…	…	…
HP (3)	LP (2)	$\langle (329,332,334); 0.38,0.27 \rangle$	$\langle (74,77,79); 0.41,0.28 \rangle$	$\langle (73,75,78); 0.25,0.61 \rangle$	(5,7,9); 0.3,0.41
HP (3)	HP (3)	$\langle 300,304,308); 0.43,0.35 \rangle$	$\langle (74,77,80); 0.51,0.25 \rangle$	$\langle (69,72,74); 0.42,0.33 \rangle$	$\langle (12,15,18); 0.4,0.3 \rangle$

Table 1: survey results for $\tilde{L}$

(1) The customers asked what would they do if $\tilde{F}$ chooses to retain the current price					
other player's choice		Response from customers			
$\tilde{L}$	$\tilde{H}$	Remains at $\tilde{F}$	Changes to $\tilde{L}$	Changes to $\tilde{H}$	Discontinue service
CP (1)	CP (1)	$\langle (269,272,274); 0.46,0.27 \rangle$	$\langle (35,37,41); 0.51,0.32 \rangle$	$\langle (24,27,29); 0.39,0.45 \rangle$	$\langle (3,5,6); 0.6,0.2 \rangle$
…	…	…	…	…	…
LP (2)	LP (2)	$\langle (253,255,259); 0.37,0.49 \rangle$	$\langle (43,45,49); 0.4,0.52 \rangle$	$\langle (32,35,37); 0.41,0.35 \rangle$	$\langle (1,3,5); 0.6,0.21 \rangle$
LP (2)	HP (3)	$\langle (261,264,267); 0.42,0.37 \rangle$	$\langle (50,53,56); 0.51,0.38 \rangle$	$\langle (14,17,19); 0.61,0.27 \rangle$	$\langle (4,6,8); 0.41,0.34 \rangle$
…	…	…	…	….	…
HP (3)	HP (3)	$\langle (269,273,275); 0.51,0.3 \rangle$	$\langle (34,36,40); 0.58,0.32 \rangle$	$\langle (24,27,30); 0.5,0.28 \rangle$	$\langle (2,3,5); 0.41,0.32 \rangle$

(2) The customers asked what would they do if $\tilde{F}$ chooses to decrease the price					
other player's choice		Response from customers			
$\tilde{L}$	$\tilde{H}$	Remains at $\tilde{F}$	Changes to $\tilde{L}$	Changes to $\tilde{H}$	Discontinue service
CP (1)	CP (1)	$\langle (371,319,321); 0.52,0.32 \rangle$	$\langle (15,17,18); 0.41,0.36 \rangle$	$\langle (5,7,8); 0.5,0.25 \rangle$	$\langle (0,2,3); 0.6,0.2 \rangle$
…	…	…	…	…	…
LP (2)	HP (3)	$\langle (324,327,330); 0.5,0.32 \rangle$	$\langle (7,9,11); 0.45,0.52 \rangle$	$\langle (5,7,9); 0.27,0.38 \rangle$	$\langle - - \rangle$
HP (3)	CP (2)	$\langle (328,332,334); 0.41,0.34 \rangle$	$\langle (5,8,10); 0.32,0.42 \rangle$	$\langle (1,2,4); 0.43,0.49 \rangle$	$\langle (0,1,2); 0.5,0.32 \rangle$
…	…	…	…	…	…
HP (3)	HP (3)	$\langle (300,302,306); 0.42,0.38 \rangle$	$\langle (16,21,23); 0.33,0.52 \rangle$	$\langle (14,16,21); 0.51,0.32 \rangle$	$\langle (1,2,4); 0.51,0.26 \rangle$

(3) The customers asked what would they do if $\tilde{F}$ chooses to increase the price					
other player's choice		Response from customers			
$\tilde{L}$	$\tilde{H}$	Remains at $\tilde{F}$	Changes to $\tilde{L}$	Changes to $\tilde{H}$	Discontinue service
CP (1)	CP (1)	$\langle (244,246,247); 0.52,0.24 \rangle$	$\langle (44,46,50); 0.6,0.22 \rangle$	$\langle (43,45,46); 0.5,0.32 \rangle$	$\langle (4,5,7); 0.5,0.28 \rangle$
CP (1)	LP (2)	$\langle (227,230,232); 0.5,0.3 \rangle$	$\langle (52,55,58); 0.3,0.54 \rangle$	$\langle (48,50,52); 0.6,0.3 \rangle$	$\langle (6,8,11); 0.3,0.53 \rangle$
…	…	…	…	…	…
HP (3)	CP (1)	$\langle (215,217,221); 0.51,0.32 \rangle$	$\langle (63,65,69); 0.34,0.42 \rangle$	$\langle (48,52,54); 0.51,0.33 \rangle$	$\langle (0,2,5); 0.6,0.19 \rangle$

…	…	…	…	…	…
HP (3)	HP (3)	$\langle (208,210,212); 0.71,0.18 \rangle$	$\langle (63,65,68); 0.31,0.52 \rangle$	$\langle (52,55,58); 0.27,0.43 \rangle$	$\langle (9,10,13); 0.51,0.35 \rangle$

Table 2: survey results for $\widetilde{F}$

(1)The customers asked what would they do if $\widetilde{H}$ chooses to retain the current price					
other player's choice		**Response from customers**			
$\widetilde{L}$	$\widetilde{F}$	**Remains at $\widetilde{H}$**	**Changes to $\widetilde{L}$**	**Changes to $\widetilde{F}$**	**Discontinue service**
CP (1)	CP (1)	$\langle (153,157,160); 0.61, 0.32 \rangle$	$\langle (49,50,53); 0.42, 0.52 \rangle$	$\langle (27,29,31); 0.51, 0.32 \rangle$	$\langle (3,4,6); 0.5, 0.31 \rangle$
CP (1)	LP (2)	$\langle (150,154,156); 0.52, 0.33 \rangle$	$\langle (44,47,49); 0.31, 0.42 \rangle$	$\langle (34,37,40); 0.51, 0.23 \rangle$	$\langle (2,4,5); 0.61, 0.23 \rangle$
…	…	…	…	…	…
HP (3)	CP (1)	$\langle (140,143,145); 0.53, 0.36 \rangle$	$\langle (48,51,53); 0.4, 0.53 \rangle$	$\langle (37,40,44); 0.71, 0.2 \rangle$	$\langle (5,6,7); 0.32, 0.55 \rangle$
…	…	…	…	…	…
HP (3)	HP (3)	$\langle (190,195,198); 0.4, 0.6 \rangle$	$\langle (25,27,29); 0.37, 0.42 \rangle$	$\langle (14,17,20); 0.5, 0.3 \rangle$	$\langle (1,2,3); 0.5, 0.33 \rangle$
(2) The customers asked what would they do if $\widetilde{H}$ chooses to decrease the price					
other player's choice		**Response from customers**			
$\widetilde{L}$	$\widetilde{F}$	**Remains at $\widetilde{H}$**	**Changes to $\widetilde{L}$**	**Changes to $\widetilde{F}$**	**Discontinue service**
CP (1)	CP (1)	$\langle (195,198,202); 0.5, 0.25 \rangle$	$\langle (23,27,30); 0.4, 0.6 \rangle$	$\langle (13,16,18); 0.5, 0.32 \rangle$	$\langle - - \rangle$
CP (1)	LP (2)	$\langle (178,181,184); 0.54, 0.31 \rangle$	$\langle (30,33,36); 0.39, 0.53 \rangle$	$\langle (18,21,25); 0.51, 0.28 \rangle$	$\langle (1,3,4); 0.6, 0.25 \rangle$
…	…	…	…	…	…
LP (2)	HP (3)	$\langle (214,217,220); 0.51, 0.36 \rangle$	$\langle (18,20,21); 0.54, 0.41 \rangle$	$\langle (5,7,9); 0.69, 0.18 \rangle$	$\langle - - \rangle$
…	…	…	…	…	…
HP (3)	HP (3)	$\langle (181,185,187); 0.51, 0.23 \rangle$	$\langle (35,37,39); 0.53, 0.33 \rangle$	$\langle (18,20,21); 0.61, 0.24 \rangle$	$\langle (0,1,3); 0.7, 0.2 \rangle$
(3)The customers asked what would they do if $\widetilde{H}$ chooses to increase the price					

other player's choice		Response from customers			
$\tilde{L}$	$\tilde{F}$	**Remains at $\tilde{H}$**	**Changes to $\tilde{L}$**	**Changes to $\tilde{F}$**	**Discontinue service**
CP (1)	CP (1)	$\langle(155, 158, 163); 0.5, 0.32\rangle$	$\langle(45, 49, 51); 0.62, 0.27\rangle$	$\langle(24, 27, 30); 0.28, 0.52\rangle$	$\langle(3, 5, 6); 0.62, 0.32\rangle$
CP (1)	LP (2)	$\langle(127, 129, 132); 0.58, 0.34\rangle$	$\langle(54, 57, 60); 0.63, 0.25\rangle$	$\langle(44, 47, 50); 0.42, 0.38\rangle$	$\langle(5, 6, 8); 0.51, 0.33\rangle$
…	…	…	…	…	…
HP (3)	LP (2)	$\langle(139, 145, 149); 0.6, 0.32\rangle$	$\langle(46, 49, 51); 0.55, 0.34\rangle$	$\langle(38, 42, 44); 0.56, 0.35\rangle$	$\langle(3, 5, 7); 0.7, 0.21\rangle$
…	…	…	…	…	…
HP (3)	HP (3)	$\langle(120, 123, 125); 0.5, 0.3\rangle$	$\langle(62, 66, 70); 0.6, 0.33\rangle$	$\langle(39, 43, 45); 0.7, 0.12\rangle$	$\langle(5, 7, 10); 0.25, 0.62\rangle$

Table 3: survey results for $\tilde{H}$

Here $\tilde{L}_o = 1000$, $\tilde{F}_o = 600, \tilde{H}_o = 400$ and $\tilde{l}_o = 500, \tilde{f}_o = 350$, $\tilde{h}_o = 250$.

Using the equations (1),(2), (3) the payoffs are calculated for every combinations of strategies for each players . The calculated payoffs are given in the tables 4,5 and 6

Strategy combinations i, j, k	$\tilde{l}_{ijk}$
111	$\langle(910.4, 923.43, 943.09); 0.4, 0.6\rangle$
112	$\langle(922.4, 938.34, 956.69); 0.31, 0.55\rangle$
…	…
332	$\langle(830.17, 851.83, 869.89); 0.3, 0.42\rangle$
333	$\langle(771.2, 795.03, 820.57); 0.31, 0.52\rangle$

Table 4: Player $\tilde{L}'s$ Expected Payoffs

Strategy combinations i, j, k	$\tilde{f}_{ijk}$
111	$\langle(596.05, 610.12, 624.17); 0.46, 0.32\rangle$
112	$\langle(584.12, 597.77, 613.43); 0.41, 0.34\rangle$
…	…

332	$\langle(527.66, 549.17, 569.54); 0.25, 0.58\rangle$
333	$\langle(544.68, 564.66, 580); 0.51, 0.35\rangle$

Table 5: Player $\widetilde{F}'s$ Expected Payoffs

Strategy combinations i, j, k	$\widetilde{h}_{ijk}$
111	$\langle(348.34, 367.09, 380.91); 0.39, 0.5\rangle$
112	$\langle(359.71, 373.14, 388.69); 0.35, 0.52\rangle$
…	…
332	$\langle(449.77, 471.71, 492.46); 0.25, 0.61\rangle$
333	$\langle(403.14, 423.89, 439.43); 0.25, 0.62\rangle$

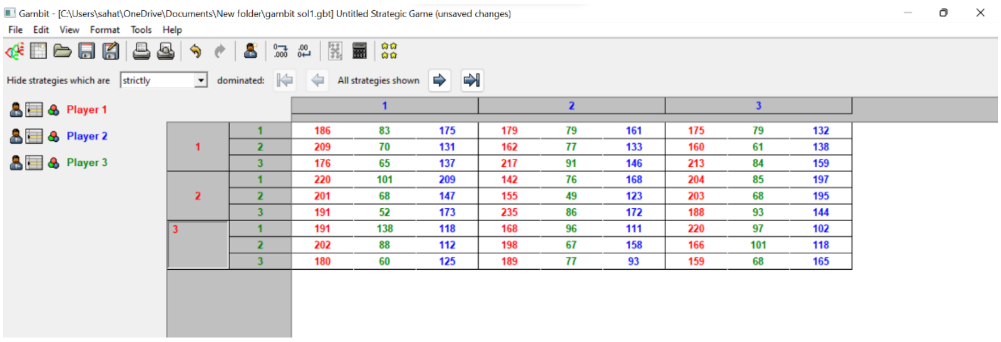
Table 6: Player $\widetilde{H}'s$ Expected Payoffs

The payoffs in the table in the form of TIFNs are converted into crisp payoff values using definition (2.3) . Calculated values are listed in the table 7,

Strategies i, j, k	$\widetilde{L}$	$\widetilde{F}$	$\widetilde{H}$
111	185.99	175.22	82.7
112	179.45	161.3	78.5
…	…	…	…
332	188.73	93.19	76.58
333	158.7	165.23	67.58

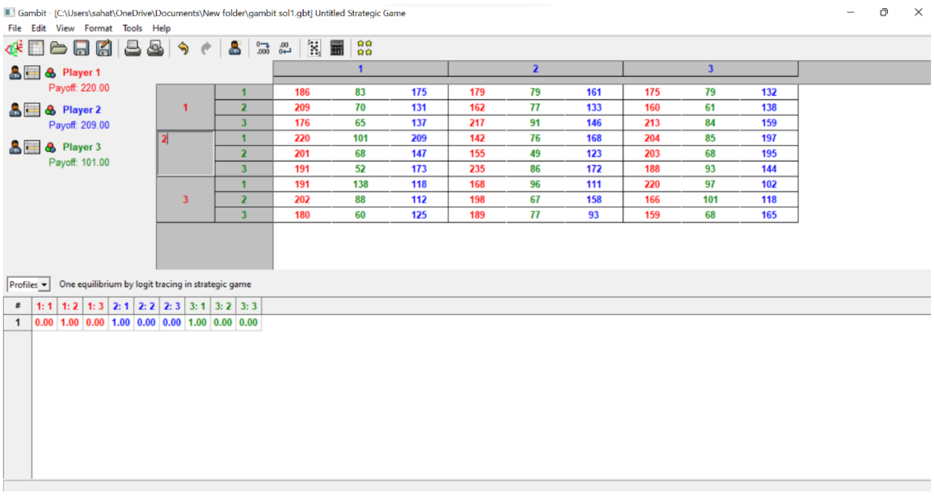
Table 7: Payoff values

The equilibrium solution is calculated using the GAMBIT 2022(version 16.0.2)program package .The estimated payoff values for every strategic combination are entered in the cells of the strategic game table given in figure 1. The standard algorithm process is applied to calculate the nash equilibrium solution shown in figure 2.



		1			2			3		
1	1	186	83	175	179	79	161	175	79	132
	2	209	70	131	162	77	133	160	61	138
	3	176	65	137	217	91	146	213	84	159
2	1	220	101	209	142	76	168	204	85	197
	2	201	68	147	155	49	123	203	68	195
	3	191	52	173	235	86	172	188	93	144
3	1	191	138	118	168	96	111	220	97	102
	2	202	88	112	198	67	158	166	101	118
	3	180	60	125	189	77	93	159	68	165

Figure 1. Payoff matrices



		1			2			3		
1	1	186	83	175	179	79	161	175	79	132
	2	209	70	131	162	77	133	160	61	138
	3	176	65	137	217	91	146	213	84	159
2	1	220	101	209	142	76	168	204	85	197
	2	201	68	147	155	49	123	203	68	195
	3	191	52	173	235	86	172	188	93	144
3	1	191	138	118	168	96	111	220	97	102
	2	202	88	112	198	67	158	166	101	118
	3	180	60	125	189	77	93	159	68	165

#	1: 1	1: 2	1: 3	2: 1	2: 2	2: 3	3: 1	3: 2	3: 3
1	0.00	1.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00

Figure 2. Equilibrium solution

From the figure 2 it is discovered that the equilibrium solution occurred at the strategy combination (2,1,1). The payoff values with respect to the strategies are 220 for $\tilde{L}$, 209 for $\tilde{F}$ and 101 for $\tilde{H}$. The game has only one possible equilibrium solution. So being the oldest in the market the best possible strategy for $\tilde{L}$ is to decrease their price to gain maximum profit and best strategic choices for $\tilde{F}$ and $\tilde{H}$ are to maintain their existing price.

Conclusion :

After analysing the constructed model for solving triopoly games with Triangular Intuitionistic Fuzzy payoffs we can deduce that the technique is more accurate in finding the equilibrium strategies than the previously used method. This method can be used directly when the survey is not necessary to form the payoff matrix. The players in the structure are non- cooperative and clearly knows their set of strategies and their opponents strategies. So this method is a benefit for the companies to choose the suitable pricing.

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