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THE MAD TEST FOR LARGE SAMPLE COMPARISONS

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ABSTRACT

In a recent study, a test based on the Mean Absolute Deviation (MAD) was developed and demonstrated superior performance compared to the t-test (Takiar R, 2024). This study aims to extend the application of the MAD test to the large samples. The test fundamentally utilizes the relationship between the Range and the Mean Absolute Deviation, proposing several critical scores associated with sample size to assess the comparability of two samples.

The MAD test recommends critical scores of 2, 3, and 4 for sample sizes of less than 30, between 30 and 50, and greater than 50, respectively. A score exceeding the suggested threshold for the specified sample size indicates significant differences in the distributions of the samples. Conversely, a score equal to or less than the threshold suggests comparable distributions among the samples.

For the present study, two pairs of normal populations, each consisting of 200 observations, were generated using an Excel function: (P1, P2) and (P3, P4) with Skewness and Kurtosis values closer to 0. From each population, 500 random samples of sizes 30, 50, 75, 100, 125, 150, and 175 were generated using a V-basic program and compared within each set of (P1, P2) and (P3, P4). All sample comparisons were conducted using both the Z-test and the MAD test, with a focus on comparing their significant results. On average, the

MAD test resulted in 86.7% of correct decisions, compared to 89.0% observed with the Z-test. Therefore, the MAD test can be claimed as performing comparably to the Z-test. Notably, the applications of the MAD test have shown to be consistently effective, regardless of the sample size being small (below 30) or large (above 30).

Keywords: Test of Significance, MAD test, Z-test, Performance

INTRODUCTION

In Research, one of the important problems often faced is to compare two or more series of data in a meaningful way so that some valid conclusion can be drawn. To know whether two random samples obtained are comparable or not, in statistics, often a test of significance like t-test or Z-test is advocated. A test of significance is a statistical procedure employed to analyze a set of data to test whether the conclusions drawn thereby supports a research hypothesis or not? The statistically significant difference between two sample means is often regarded as a proof of the existence of difference between them. Similarly, the non-significant difference between two means is regarded as a proof of comparability among them. It is understandable that whether you conduct the t-test or the Z-test for comparison of two sample means, the use of the Standard Deviation is inevitable. Besides the Standard Deviation there is another measure of dispersion known as the Mean Absolute Deviation (MAD). In literature, hardly there is any evidence that MAD is used as a test of significance.

In a recent study, it was shown that the Range and MAD are closely related to each other and the Range can be expressed in terms of MAD as Range = MAD[0.689*ln(n)+3.275]. The MAD was further shown to be useful for evaluating the significant differences between two sample distributions. The analysis revealed that the samples drawn from two different populations, on comparison, tend to differ by at least three uncommon values. The MAD test essentially rely on the number of uncommon values termed as the score rather on the specified level of probability of occurring the differences in the means. The MAD test was demonstrated to be performing better as compared to the t-test by picking up correctly 78% of the true significant differences as compared to 60% picked up by the t-test (Takiar R 2024). The present study aims to extend the application of MAD test to large samples.

OBJECTIVES

- To develop a methodology to extend the applications of MAD test to large samples for comparing two sample distributions or means.
- To compare the outcomes of MAD test with that of Z-test when two samples are known to be drawn from two different normal populations.
- To compare the outcomes of MAD test with that of Z-test when two samples are known to be drawn from the same normal population.

MATERIAL AND METHODS

MEAN ABSOLUTE DEVIATION FORMULA FOR DISCRETE DATA

The formula to calculate the Mean Absolute Deviation is given by

Mean Absolute Deviation = MAD = $\frac{[\Sigma|x_i - \bar{x}|]}{n}$ where

Σ represents the summation of values, x_i represents the ith value in the data set

$\bar{x}$ represents the Mean of the data set, n represents the number of data values

| | represents the absolute value, ignoring the sign of the deviation

MEAN ABSOLUTE DEVIATION FORMULA FOR CONTINUOUS DATA

$$\text{MAD} = \frac{\Sigma f_i |(x_i - \bar{x})|}{n} \quad \text{for i = 1,2,3, …… } n$$

For a given set of data, in Excel, a function key is available namely AVEDEV for calculation of the MAD values.

SELECTION OF POPULATION AND GENERATION OF SAMPLES

For the study purposes, two pairs of normal populations are generated namely (P1, P2) and (P3, P4) each consisting of 200 samples. The populations exhibited their skewness and Kurtosis values closer to zero, confirming their normality. Within each pair, the populations are chosen to be significantly different from each other with respect to their means. The details of the populations are shown in Table 1.

SCHEME OF COMPARISONS

From each population, 500 random samples of size 30, 50, 75,100, 125, 150 and 175 are generated using a V-basic program.

The Scheme of Sample comparisons by the population and sample size is shown in Table 2 and Table 3.

Table 1: Comparison of Sample Means by Z-test

Parameter	Sample's Population		Sample's Population	
	P1	P2	P3	P4
Mean	36.2	40.8	38.2	43.8
SD	8.37	11.67	9.05	12.55
Skewness	0.002	-0.065	-0.021	0.015
Kurtosis	0.013	0.014	-0.025	-0.04
n	200	200	200	200
Z - value	4.9		5.11	
P-value	< 0.001		< 0.001	

Table 2: Number of Comparisons by type of Sample’s Population and Sample size

Sample size	Pair of Sample Populations	Number of Mean Comparisons	Sample size	Pair of Sample Populations	Number of Mean Comparisons
30	P1 with P2	500	125	P1 with P2	500
	P1 with P1	500		P1 with P1	500
50	P1 with P2	500	150	P1 with P2	500
	P1 with P1	500		P1 with P1	500
75	P1 with P2	500	175	P1 with P2	500
	P1 with P1	500		P1 with P1	500
100	P1 with P2	500			
	P1 with P1	500			

Table 3 Number of Comparisons by type of Population and Sample size

Sample size Each	Pair of Sample Populations	Number of Mean Comparisons	Sample size Each	Pair of Sample Populations	Number of Mean Comparisons
30	P3 with P4	500	125	P3 with P4	500
	P3 with P3	500		P3 with P3	500
50	P3 with P4	500	150	P3 with P4	500
	P3 with P3	500		P3 with P3	500
75	P3 with P4	500	175	P3 with P4	500
	P3 with P3	500		P3 with P3	500
100	P3 with P4	500			
	P3 with P3	500			

COMPARISONS OF SAMPLES BY T-TEST AND MAD TEST

All the sample comparisons are made by the Z-test and the MAD test, and their outcomes are compared. For more details of MAD test methodology, the readers may refer to my earlier publication (Takiar R, 2024).

DEFINITION OF SIGNIFICANT AND NON-SIGNIFICANT RESULTS

SIGNIFICANT RESULT: The test results showing the outcome as significant when samples of P1 with that of P2 or samples of P3 with that of P4 are compared**.**

NON-SIGNIFICANT RESULT: The test results showing the outcome as non-significant when two independent samples drawn from P1 or P3, are compared.

CORRECT RESULT: When the test picks up correctly, either the significant or non-significant results.

RESULTS

The results of correct significance by different scores when pooled for sample comparisons, drawn from two pairs of populations namely (P1,P2) and (P3,P4), are shown in Table 4.

The change in score will affect the percentage of obtaining the correct results. Ideally, the score which corresponds to the maximum percentage of correct results is taken as the desired cut-off score for the MAD test. Accordingly, for the sample size of 30, the MAD test correctly picks up 70.6% of the Non-significant and 79.3% of the significant results. Overall, 75% corrects results are obtained when you choose 2 as the score. For the sample size of 50 and 75, the score of 3 is found to be the most desirable cut-off and accordingly, 81.3% -87.7% correct results are obtained. For the sample size of 100 and above, the score of 4 is found to be the most desirable score. The percentage of correct results obtained in these cases ranged from 91.2% to 93.8% which is quite satisfactory. However, it is important to know how the MAD test compares in performance in relation to Z-test in correctly picking up the significant or non-significant differences.

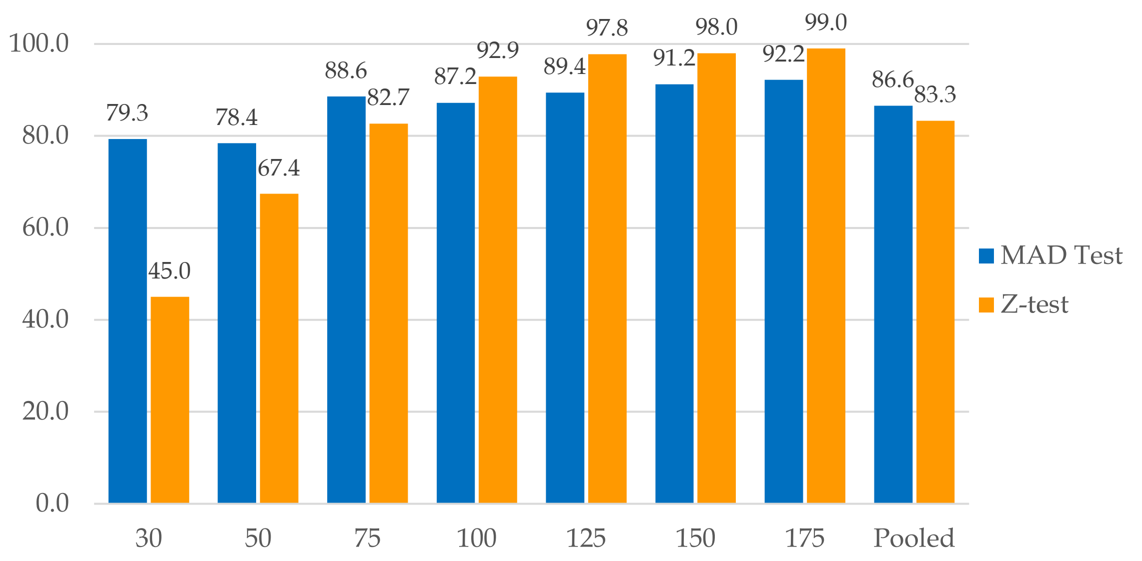
COMPARISON OF CORRECT DECISION OBTAINED BY THE Z TEST AND THE MAD TEST

The comparison of percentage of correct significant results obtained by the MAD test and the Z test is shown in Fig. 1. For the sample size of 30-50, the MAD test performs far better than the Z-test. After the sample size of 75, the Z-test performs better as compared to the MAD test. On an average, Z test correctly picks up 83.3% of the significant differences as compared to 86.6 % observed in the case of MAD test. Overall, the MAD test performs better as compared to the Z-test.

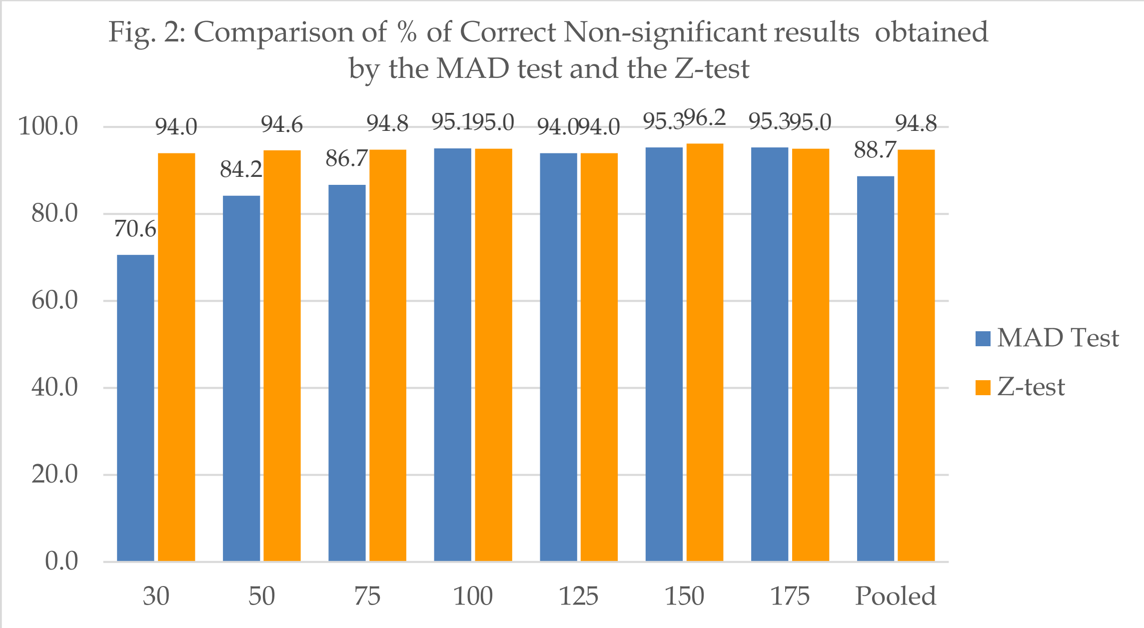
Table 4: The Percentage of True Non-significant and Significant Comparisons by the MAD test

Sample size	Number of Samples	Outcome	Score			
			2	3	4	5
30	1000	Non-significant	70.6	85.6	92.8	96.0
	1000	Significant	79.3	62.4	46.5	32.3
	2000	**Pooled**	**75.0**	**74.0**	**69.7**	**64.2**
50	1000	Non-significant	69.1	84.2	92.9	97.6
	1000	Significant	88.4	78.4	68.3	55.4
	2000	**Pooled**	**78.8**	**81.3**	**80.6**	**76.5**
75	1000	Non-significant	71.3	86.7	95.0	97.9
	1000	Significant	94.4	88.6	79.1	69.4
	2000	**Pooled**	**82.9**	**87.7**	**87.1**	**83.7**
100	1000	Non-significant	73.3	88.2	95.1	97.4
	1000	Significant	95.9	92.6	87.2	81.5
	2000	**Pooled**	**84.6**	**90.4**	**91.2**	**89.5**
125	1000	Non-significant	70.9	87.0	94.0	97.3
	1000	Significant	96.9	93.4	89.4	84.0
	2000	**Pooled**	**83.9**	**90.2**	**91.7**	**90.7**
150	1000	Non-significant	70.2	88.9	95.3	98.5
	1000	Significant	98.4	95.2	91.2	87.0
	2000	**Pooled**	**84.3**	**92.1**	**93.3**	**92.8**
175	1000	Non-significant	74.5	89.4	95.3	98.5
	1000	Significant	98.0	95.3	92.2	87.7
	2000	**Pooled**	**86.3**	**92.4**	**93.8**	**93.1**

Fig. 1: Comparison of % of Correct Significant results obtained by the MAD test and the Z-test

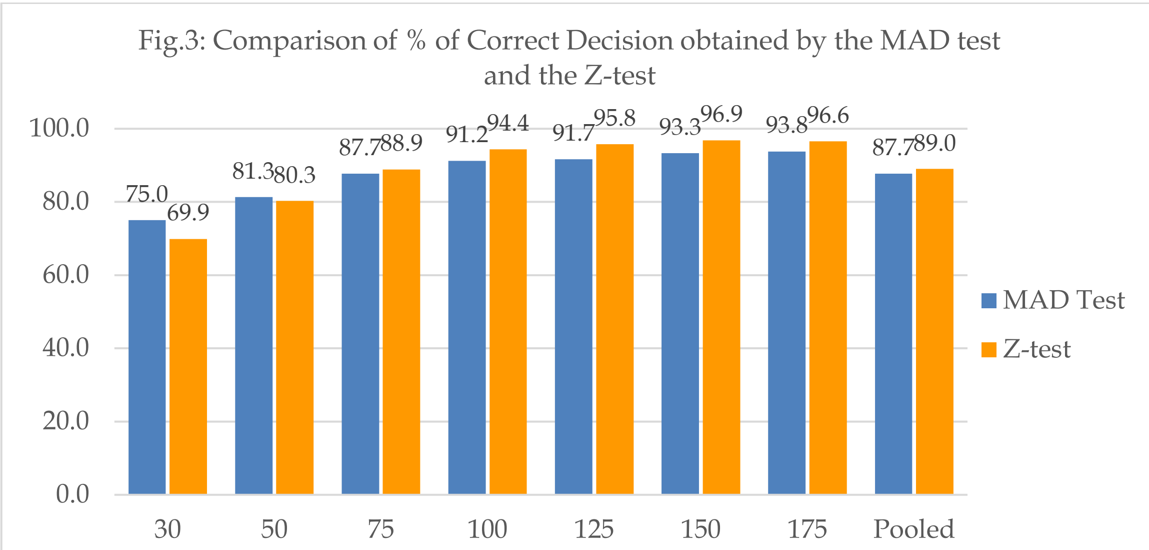


The comparison of % of correct non-significant results obtained by the MAD test and the Z test is shown in Fig. 2.



For the sample size of 30-50, the Z - test performs far better as compared to the MAD test. After the sample size of 75, the performance of Z-test and MAD test appear to be comparable. On an average, Z test picks up 94.8% of correct non-significant differences as compared to 88.7% observed in the case of MAD test. Thus, Z-test performs better.

In view of the MAD test performing better in picking up the significant differences while the Z - test performs better in picking up the non-significant differences, it would be interesting to examine how do they compare when the results of the significant and non-significant are pooled? The comparison of pooled results of both the tests are shown in Fig. 3. Overall, the Z-test gave 89% of the correct results while the MAD test gave 87.7% of the correct results. On an average, both the test differ by 1.3% suggesting that they are almost comparable in their outcomes.



DISCUSSION

In my previous publication (Takiar R 2024), the development of MAD test of significance, based on the Mean Absolute Deviation, was shown. For the test, it was shown that a score of 2 is appropriate for evaluating the significant or non-significant differences among the pairs of samples when drawn from Normal populations. In the current study, the extension of application of MAD test to the large samples is explored.

It is concluded that based on the sample size, there is a need to change the level of score from 2 to 4 as shown in Table 5. A higher score than suggested in the table for a given sample size should be taken as the existence of significant differences in the distributions of the samples. Similarly, a score of equal to or less than suggested should be taken as the indicator of comparable distributions among the samples.

Table 5: Levels of score for Evaluating differences among the Pairs of Normal Samples

Sample size	< 50	50-99	≥ 100
Score	2	3	4

The MAD test almost give comparable results when seen against Z-test. It is observed that for sample size below 100, the MAD test performs better as compared to the Z-test. For small samples, below 30, it was already shown that the MAD test performs better as compared to the t-test, hitherto used for small samples. The results of the study therefore confirms that the MAD test is consistent in giving good results whether the sample size is small or large.

RECOMMENDATION

The MAD test is suitable for large as well as small samples for comparing the distributions among two samples.

SUMMARY OF OBSERVATIONS

- The extension of application of the MAD test to the large samples, is successfully evaluated.
- The study results are based on 14000 comparisons.
- The ability of the MAD test in picking up the correct significant or non-significant results are evaluated against the applications of Z-test, hitherto used as the Gold standard for mean comparisons.
- The MAD test correctly picks up the significant differences in 86.6% of the comparisons as against 83.3% seen in the case of Z-test.
- The MAD test correctly picks up the non-significant differences in 88.7% of the comparisons as against 94.8% seen in the case of Z-test.
- Overall, the MAD test picks up correctly in 87.7% of the comparisons as against in 89.0% of the comparisons seen in the case of Z -test.
- Levels of score for evaluating differences among the pairs of normal samples is shown in the following table.

Sample size	< 50	50-99	≥ 100
Score	2	3	4

- The MAD test applications are found to be good and consistent whether the sample size is small, below 30 or when the samples are large and above 30.

REFERENCE

[1]. Takiar R 2024: The MAD test for Small sample Comparisons – An Alternative to the t- test, *Bulletin of Mathematics and Statistics Research*, Vol. 12(4), Page 52-65.

Biography

Dr. Ramnath Takiar

I am a Postgraduate in Statistics from Osmania University, Hyderabad. I did my Ph.D. from Jai Narain Vyas University of Jodhpur, Jodhpur, while in service, as an external candidate. I worked as a research scientist (Statistician) for Indian Council of Medical Research from 1978 to 2013 and retired from the service as Scientist G (Director Grade Scientist). I am quite experienced in large scale data handling, data analysis and report writing. I have sixty-six research publications in national and International Journals related to various fields like Nutrition, Occupational Health, Fertility and Cancer epidemiology. During the tenure of my service, I attended three international conferences, namely in Goiana (Brazil-2006), Sydney (Australia-2008), and Yokohama (Japan-2010), and I presented a paper on each. I also attended the Summer School related to Cancer Epidemiology (Modul I and Module II) conducted by International Agency for Research in Cancer (IARC), Lyon, France from 19th to 30th June 2007. After my retirement, I joined my son at Ulaanbaatar, Mongolia. I worked in Ulaanbaatar as a professor and consultant from 2013 to 2018, and I was responsible for teaching and guiding Ph.D. students. I also taught Mathematics to undergraduates and Econometrics to MBA students. During my service there, I also functioned as the Executive Editor for the in-house Journal "International Journal of Management." I am still active in research and have published thirteen research papers during 2021-24.