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RESEARCH ARTICLE

MODIFIED DIFFERENCE-CUM DUAL TO EXPONENTIAL TYPE ESTIMATOR OF THE POPULATION MEAN UNDER NON-RESPONSE

Mukesh Kumar[1*], Pooja Yadav[2]
[1]Associate Professor, Department of Statistics, University of Lucknow, Lucknow-226007, India
*Email:mukesh.stat@gmail.com;
[2]Research Scholar, Department of Statistics, University of Lucknow,Lucknow-226007, India
Email:yadavpooja3831@gmail.com
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ABSTRACT

Survey sampling is almost essential for the study of large populations. The estimators of population parameters such as population total, mean, variance, etc., are proposed to obtain the best estimate of say, GDP, income, commodities, etc. on various occasions. The auxiliary information is frequently used to get the best estimate. Further, in most of the sample surveys, many targeted and required responses are generally not received due to various reasons. Statistically, it is termed a non-response problem and without proper handling of this issue, might produce incorrect estimates and results. Therefore, a technique proposed by Hansen and Hurwitz (1946) is used to deal with the problem of non-response. In this technique, the targeted units considered into two groups one as a response group and the other as a non-response group. Further, a sub-sample from the non-response group was contacted, again to obtain the response. Both the sample responses and subsample responses were collectively used for estimation, which produced quite satisfactory results.

In this research, we have proposed a generalized estimator of population mean for the variable under study with the use of auxiliary variables in the presence of non-response. The bias and mean squared error (MSE) expressions for the proposed estimator have been obtained considering the first order of approximation. The MSE of the suggested estimator has been compared with the MSE of similar existing estimators theoretically and empirically. For, empirical findings, percent relative efficiency (PRE), has been reported and higher PRE means a more efficient estimator. The proposed estimator is shown to be generalized and performs better in terms of efficiency than most of the existing estimators and hence suitable for many real-life applications.

Keywords: Auxiliary information, Bias, MSE, PRE, Exponential estimator, non-response error.

Introduction

Sample surveys remain quite important for many years because it is appropriate for resource constraints that come with conducting census surveys for large populations. There are two types of errors in surveys, the first is known as sampling error and the other as non-sampling error. The non-sampling errors can occur for several reasons like mistakes in the questionnaire, respondent information, and/or data processing and collecting, etc. Non-sampling errors such as non-response have a greater impact on the suitability and performance of the estimator than do sampling errors. Non-response can occur in sampling as well census studies and have an important role in surveys. In the non-response technique, the targeted units are divided into two groups one as a response group and the other as a non-response group. Further, a sub-sample from the non-response group was contacted again generally in person to obtain the responses. Both responses from the sample and subsample are used for estimation purposes which provides satisfactory results.

In sample surveys, auxiliary information is utilized for both the selection and estimation phases to increase the estimator's effectiveness. At the estimation stage, the use of auxiliary information started with the work of Cochran (1940). Cochran suggested ratio method of estimation, for the estimation of population mean and/or variance, etc. whenever the correlation between study variate say, (y) and auxiliary variate say (x) is highly positive. Along the same line, whenever the correlation is highly negative, the product method of estimation suggested by Robson (1957) and revisited by Murthy (1964) has been considered.

Srivenkataramana (1980) first proposed dual to ratio estimator and in another work, Bandyopadhyaya (1980) proposed dual to product estimator. The technique of dual and dual class of estimators is appropriate and performs better at many occasions. Bahl and Tuteja (1991) proposed exponential ratio as well as exponential product type estimators for population mean estimation under simple random sampling. These estimators are in line of classical ratio and product estimators.

Hansen and Hurwitz (1946) technique of dealing non-response shall be combined with classical ratio estimator and related estimators for estimating the population mean under non-response. Rao (1986) introduced ratio estimator of population mean in case of non-response and later, Khare and Srivastava (1993) proposed a product estimator of population mean in presence of non-response.

In this work, we suggested modified difference cum dual to exponential type estimator for population mean through utilizing the auxiliary information in the presence of non-response.

Notations

Let N be indicating the population size and n as the sample size. The total population can be divided in N_1 and N_2 group, such that $N_1+N_2 = N$ where N_1 belong to the group who respond and N_2 belong to the group who do not respondents. In sample of n units we take n_1 those who have responded and n_2 take who have not responded. From non-responded group of n_2, a sub-sample of size h (where, $h = \frac{n_2}{k}$, k>1) is taken and contacted again in person for obtaining responses. Here $\frac{N_1}{N} = \frac{n_1}{n}$ and $\frac{N_2}{N} = \frac{n_2}{n}$ considered. Further, we define the following.

$W_1 = \frac{N_1}{N}$, $W_2 = \frac{N_2}{N}$, $\lambda = \frac{1}{n} - \frac{1}{N}$, $g = \frac{n}{N-n}$ $\alpha = \frac{W_2(k-1)}{n}$, $w_1 = \frac{n_1}{n}$, $w_2 = \frac{n_2}{n}$,

$\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, $\overline{Y}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} Y_i$, $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$, $\overline{X}_2 = \frac{1}{N_2}\sum_{i=1}^{N_2} X_i$, $C_y = \frac{S_y}{\overline{Y}}$

$C_{y(2)} = \frac{S_{y(2)}}{\overline{Y}}$ $C_x = \frac{S_x}{\overline{X}}$ $C_{x(2)} = \frac{S_{x(2)}}{\overline{X}}$ $S_y^2 = \frac{\sum_{i=1}^{N}(y_i-\overline{Y})^2}{N-1}$ $S_x^2 = \frac{\sum_{i=1}^{N}(x_i-\overline{X})^2}{N-1}$,

$S_{y(2)}^2 = \frac{\sum_{i=1}^{N_2}(y_i-\overline{Y}_2)^2}{N_2-1}$, $S_{x(2)}^2 = \frac{\sum_{i=1}^{N_2}(x_i-\overline{X}_2)^2}{N_2-1}$, $C_{yx} = \rho_{yx}C_yC_x$, $C_{yx(2)} = \rho_{yx(2)}C_{y(2)}C_{x(2)}$

Some Existing Estimators

Cochran introduced the ratio estimator,

$$\bar{y}_R = \bar{y}\frac{\overline{X}}{\bar{x}} \tag{1}$$

The corresponding MSE given as

$$MSE(\bar{y}_R) = \overline{Y}^2\lambda(C_y^2 + C_x^2 - 2C_{xy}) \tag{2}$$

Robson given the product estimator, and latter Murthy revisited it

$$\bar{y}_P = \bar{y}\frac{\bar{x}}{\overline{X}} \tag{3}$$

The corresponding MSE given as

$$MSE(\bar{y}_P) = \overline{Y}^2\lambda(C_y^2 + C_x^2 + 2C_{xy}) \tag{4}$$

Watson suggested the regression estimator,

$$\bar{y}_{lr} = \bar{y} + b(\overline{X} - \bar{x}) \tag{5}$$

The corresponding MSE given as

$$MSE(\bar{y}_{lr}) = \overline{Y}^2\lambda C_y^2(1 - \rho^2) \tag{6}$$

Where b represents the regression coefficient of y on x in respect of the sample.

Srivenkataramana (1980) proposed a transformation $\bar{x}^\beta = \frac{N\overline{X}-n\bar{x}}{N-n}$ for auxiliary variable x and suggested dual to classical ratio estimator for estimating the population mean $\overline{Y}$, followed by Bandyopadhyaya (1980) suggested dual to product estimator as

$$\overline{Y}_R^\beta = \bar{y}\left(\frac{\bar{x}^\beta}{\overline{X}}\right) \tag{7}$$

The MSE expression is given as

$$\mathrm{MSE}\left(\overline{Y}_R^{\beta}\right) = \overline{Y}^2\lambda(C_y^2 + g^2C_x^2 - 2gC_{xy}) \tag{8}$$

$$\overline{Y}_P^{\beta} = \bar{y}\left(\frac{\overline{X}}{\bar{x}^{\beta}}\right) \tag{9}$$

The MSE expression is given as

$$\mathrm{MSE}\left(\overline{Y}_P^{\beta}\right) = \overline{Y}^2\lambda(C_y^2 + g^2C_x^2 + 2gC_{xy}) \quad \text{where} \quad g = \frac{n}{N-n} \tag{10}$$

Bahl and Tuteja (1991) proposed exponential ratio type estimator and also suggested exponential product type estimator for estimating the population mean $\overline{Y}$ in case of simple random sampling. The functional form and corresponding MSE expression up to first order of approximation are given as,

$$\bar{y}_{BTR} = \bar{y}\exp\left(\frac{\overline{X}-\bar{x}}{\overline{X}+\bar{x}}\right) \tag{11}$$

The MSE expression is given as

$$\mathrm{MSE}(\bar{y}_{BTR}) = \overline{Y}^2\left[\lambda\left(C_y^2 + \frac{C_x^2}{4} - C_{yx}\right)\right] \tag{12}$$

$$\bar{y}_{BTP} = \bar{y}\exp\left(\frac{\bar{x}-\overline{X}}{\bar{x}+\overline{X}}\right) \tag{13}$$

The MSE expression is given as

$$\mathrm{MSE}(\bar{y}_{BTP}) = \overline{Y}^2\left[\lambda\left(C_y^2 + \frac{C_x^2}{4} + C_{yx}\right)\right] \tag{14}$$

Hansen and Hurwitz (1946), suggested an unbiased estimator of $\overline{Y}$ in case of non-response

$$\bar{y}^* = w_1\bar{y}_1 + w_2\bar{y}_2 \tag{15}$$

where $\bar{y}_1$ is the mean of the sample responses and $\bar{y}_2$ is the mean of sub-sample responses

The variance is given as,

$$V(\bar{y}^*) = \overline{Y}^2(\lambda C_y^2 + \alpha C_{y(2)}^2) \tag{16}$$

Later, Rao (1986) suggested ratio estimator, followed by product and regression estimator while there is presence of non-response on study variable as well as on the auxiliary variable under the population $\overline{X}$ known. The corresponding form and MSE provide below

$$\bar{y}_R^* = \bar{y}^*\left(\frac{\overline{X}}{\bar{x}^*}\right) \tag{17}$$

The MSE up to the first order of approximation given as,

$$\mathrm{MSE}(\bar{y}_R^*) = \overline{Y}^2\left[\lambda\left(C_y^2 + C_x^2 - 2C_{yx}\right) + \alpha\left(C_{y(2)}^2 + C_{x(2)}^2 - 2C_{yx(2)}\right)\right] \tag{18}$$

$$\bar{y}_P^* = \bar{y}^*\left(\frac{\bar{x}^*}{\overline{X}}\right) \tag{19}$$

The MSE considering first order of approximation presented as,

$$\mathrm{MSE}(\bar{y}_P^*) = \overline{Y}^2\left[\lambda\left(C_y^2 + C_x^2 + 2C_{yx}\right) + \alpha\left(C_{y(2)}^2 + C_{x(2)}^2 + 2C_{yx(2)}\right)\right] \tag{20}$$

$$\bar{y}_{lr}^* = \bar{y}^* + b_{yx}(\overline{X} - \bar{x}^*) \tag{21}$$

The MSE takes the form,

$$MSE(\bar{y}_{lr}^{*}) = \bar{Y}^2\left[\lambda\left(1-\rho_{yx}^2\right)C_y^2 + \alpha\left(1-\rho_{yx(2)}^2\right)C_{y(2)}^2\right] \qquad (22)$$

In the same line considering the presence of non-response over study and auxiliary variables, the corresponding Bahl and Tuteja (1991) exponential estimator take the following form,

$$\bar{y}^{*}{}_{BTR} = \bar{y}^{*}\exp\left[\frac{\bar{X}-\bar{x}^{*}}{\bar{X}+\bar{x}^{*}}\right] \qquad (23)$$

$$MSE(\bar{y}_{BTR}^{*}) = \bar{Y}^2\left[\lambda\left(C_y^2 + \frac{C_x^2}{4} - \rho_{yx}C_yC_x\right) + \alpha\left(C_{y(2)}^2 + \frac{C_{x(2)}^2}{4} - \rho_{yx(2)}C_{y(2)}C_{x(2)}\right)\right] \qquad (24)$$

Kadilar and Cingi (2004) proposed estimator in the presence of non-response on study and auxiliary variable take the following form,

$$\bar{y}_{KC}^{*} = \bar{y}^{*} + b_{yx}(\bar{X}-\bar{x}^{*})\left(\frac{\bar{X}}{\bar{x}^{*}}\right) \qquad (25)$$

$$MSE(\bar{y}_{KC}^{*}) = \bar{Y}^2\left[\lambda\left\{C_y^2 + C_x^2\left(1-\rho_{yx}^2\right)\right\} + \alpha\left\{C_{y(2)}^2 + C_{x(2)}^2\left(1-\rho_{yx(2)}^2\right)\right\}\right] \qquad (26)$$

Proposed Estimator

In search of improved estimation, we have taken motivation from the work by Sharma and Kumar (2020) and Rafia Jan et al. (2024), we proposed a modified difference-cum dual to exponential type estimator in generalized form for the estimation of population mean under non-response with the transformation of variable.

Let a and b be known constant or function of known auxiliary variable X. The corresponding transformation of variable is given as

$$Z = aX + b$$

Therefore $\bar{Z} = a\bar{X} + b$ and $\bar{z} = a\bar{x} + b$

Where $\bar{Z}$ and $\bar{z}$ are the corresponding population and sample mean under transformation

The proposed estimator t written as,

$$t = \{w_1\bar{y}^{*} + w_2(\bar{X}-\bar{x}^{*})\}\exp\left[\frac{\delta(\bar{z}^{*\beta}-\bar{Z})}{\bar{z}^{*\beta}+\bar{Z}}\right] \qquad (27)$$

Where $\bar{z}^{*\beta} = \frac{\bar{Z}N-\bar{z}^{*}n}{N-n}$ dual transformation under non-response, δ are arbitrary chosen constant and $g = \frac{n}{N-n}$.

Let us define sampling error for both study and auxiliary variable,

$e_0^{*} = \frac{\bar{y}^{*}}{\bar{Y}} - 1$ and $e_1^{*} = \frac{\bar{x}^{*}}{\bar{X}} - 1$ such that,

$\bar{y}^{*} = \bar{Y}(1+e_0^{*})$ $\bar{x}^{*} = \bar{X}(1+e_1^{*})$

$E(e_0^{*}) = E(e_1^{*}) = 0$

$E(e_0^{*2}) = (\lambda C_y^2 + \alpha C_{y(2)}^2)$, $E(e_1^{*2}) = (\lambda C_x^2 + \alpha C_{x(2)}^2)$

$E(e_0^{*}e_1^{*}) = (\lambda\rho_{yx}C_xC_y + \alpha\rho_{yx(2)}C_{y(2)}C_{x(2)})$

Where $\lambda = \frac{1}{n} - \frac{1}{N}$ is constant, C_y and C_x are the coefficient of variation for the study variable Y and auxiliary variable X respectively. ρ representing correlation coefficient between variables.

The Proposed generalized class of estimator can be rewritten as

$$t = \{w_1\overline{Y}(1+e_0^*) - w_2\overline{X}e_1^*\}\exp\left[\frac{\delta\theta e_1^*}{1-\theta e_1^*}\right] \quad (28)$$

Where $\theta = \frac{ga\overline{X}}{2(a\overline{X}+b)}$

$t = \{w_1\overline{Y}(1+e_0^*) - w_2\overline{X}e_1^*\}[1 - \delta\theta e_1^* - \delta\theta^2 e_1^{*2} + \frac{\delta^2\theta^2 e_1^{*2}}{2}]$

$$t - \overline{Y} = (w_1 - 1)\overline{Y} + w_1\overline{Y}\left[e_0^* - \delta\theta e_1^* + \frac{\delta(\delta-2)\theta^2 e_1^{*2}}{2} - \delta\theta e_0^* e_1^*\right] - w_2\overline{X}[e_1^* - \delta\theta e_1^{*2}] \quad (29)$$

Taking expectation on both side and taking the terms of maximum degree two, we get bias,

$$\mathrm{Bias}(t) = (w_1 - 1)\overline{Y} + w_1\overline{Y}\left[\frac{\delta(\delta-2)\theta^2}{2}\left\{\lambda C_x^2 + \alpha C_{x(2)}^2\right\} - \theta\delta\left\{\lambda\rho_{yx}C_yC_x + \alpha\rho_{yx(2)}C_{y(2)}C_{x(2)}\right\}\right] + w_2\overline{X}\delta\theta\left[\lambda C_x^2 + \alpha C_{x(2)}^2\right] \quad (30)$$

Or

$$\mathrm{Bias}(t) = (w_1 - 1)\overline{Y} + w_1\overline{Y}\left[\frac{\delta(\delta-2)\theta^2}{2}\alpha_2 - \theta\alpha_3\right] + w_2\overline{X}\delta\theta\alpha_2 \quad (31)$$

Squaring $(t - \overline{Y})$ both sides and taking expectation, we get MSE as

$$\mathrm{MSE}(t) = (w_1 - 1)^2\overline{Y}^2 + w_1^2\overline{Y}^2[\alpha_1 + \delta^2\theta^2\alpha_2 - 2\theta\delta\alpha_3] + w_2^2\overline{X}^2\alpha_2 + 2w_1(w_1 - 1)\ \overline{Y}^2\left[\frac{\delta(\delta-2)\theta^2}{2}\alpha_2 - \delta\theta\ \alpha_3\right] - 2w_1w_2\overline{XY}[\alpha_3 - \delta\theta\alpha_2] + 2(w_1 - 1)w_2\overline{YX}\delta\theta\alpha_2 \quad (32)$$

Where $\alpha_1 = \lambda Cy^2 + \alpha C_{y(2)}^2$ $\quad \alpha_2 = \lambda C_x^2 + \alpha C_{x(2)}^2$

$\alpha_3 = \lambda\rho_{yx}C_yC_x + \alpha\rho_{yx(2)}C_{y(2)}C_{x(2)}$

Or

$$\mathrm{MSE}(t) = w1^2R_1 + w_2^2R_2 + w_1w_2R_3 - w_1R_4 - w_2R_5 + R_6 \quad (33)$$

Differentiating with respect to w_1 and w_2, and simplifying the optimum values represented as,

$w_1^* = \frac{2R_2R_4 - R_3R_5}{(4R_1R_2 - R_3^2)}$ $\qquad w_2^* = \frac{2R_1R_5 - R_4R_3}{(4R_1R_2 - R_3^2)}$

Where,

$R_1 = \overline{Y}^2[1 + \alpha_1 + \delta^2\theta^2\alpha_2 - 4\theta\delta\alpha_3 + \delta(\delta-2)\theta^2\alpha_2]$, $\quad R_2 = \overline{X}^2\alpha_2$

$R_3 = 4\overline{YX}\delta\theta\alpha_2 - 2\overline{XY}\alpha_3$, $\quad R_4 = \overline{Y}^2[2 + \delta(\delta-2)\theta^2\alpha_2 - 2\delta\theta\alpha_3]$

$R_5 = 2\overline{YX}\delta\theta\alpha_2$, $\quad R_6 = \overline{Y}^2$

On using these optimum values of w_1^* and w_2^* in the equation of MSE, the minimum Mean Square Error (MSE) of the suggested estimator takes the following form.

$$\mathrm{Min\ MSE}(t) = \frac{R_3R_4R_5 - R_2R_4^2 - R_1R_5^2}{(4R_1R_2 - R_3^2)^2} + R_6 \quad (34)$$

Table 1: Some members of the proposed generalized class of estimator

S. No	Estimators	Value of Constant			
		w_1	w_2	δ	θ
1.	$t_0 = \bar{y}^*$	1	0	0	1
2.	$t_1 = \bar{y}^* \exp\left[\frac{\bar{x}^{*\beta} - \bar{X}}{\bar{x}^{*\beta} + \bar{X}}\right]$ dual exponential ratio type estimator under non response	1	0	1	1
3.	$t_2 = \bar{y}^* \exp\left[\frac{\bar{X} - \bar{x}^{*\beta}}{\bar{X} + \bar{x}^{*\beta}}\right]$ dual exponential product type estimator under non response	1	0	-1	1
4.	$t_3 = \bar{y}^* \exp\left[\frac{\bar{z}^{*\beta} - \bar{Z}}{\bar{z}^{*\beta} + \bar{Z}}\right]$	1	0	1	θ
5.	$t_4 = \bar{y}^* \exp\left[\frac{\bar{Z} - \bar{z}^{*\beta}}{\bar{Z} + \bar{z}^{*\beta}}\right]$	1	0	-1	θ
6.	$t_5 = [w_1\bar{y}^* + w_2(\bar{X} - \bar{x}^*)] \exp\left[\frac{\bar{z}^{*\beta} - \bar{Z}}{\bar{z}^{*\beta} + \bar{Z}}\right]$ Sharma and kumar (2020)	w_1	w_2	1	θ
7.	$t_6 = [w_1\bar{y}^* + w_2(\bar{X} - \bar{x}^*)] \exp\left[\frac{\bar{Z} - \bar{z}^{*\beta}}{\bar{Z} + \bar{z}^{*\beta}}\right]$	w_1	w_2	-1	θ

*θ depends on the values of a and b.

Theoretical Efficiency Comparison

Under this section, we compared the MSE of the proposed class of estimator with the variance/MSE of existing estimators.

I. Proposed estimator t will be more efficient than $\bar{y}^*$ if

$$Var(\bar{y}^*) - \min MSE(t) \geq 0$$

$$\bar{Y}^2\alpha_1 - \left[\frac{R_3R_4R_5 - R_2R_4^2 - R_1R_5^2}{(4R_1R_2 - R_3^2)^2} + R_6\right] \geq 0 \tag{35}$$

II. Proposed estimator t will be more efficient than $\bar{y}_R{}^*$ if

$$MSE(\bar{y}_R^*) - \min MSE(t) \geq 0 \text{ if}$$

$$\bar{Y}^2(\alpha_1 + \alpha_2 - 2\alpha_3) - \left[\frac{R_3R_4R_5 - R_2R_4^2 - R_1R_5^2}{(4R_1R_2 - R_3^2)^2} + R_6\right] \geq 0 \tag{36}$$

III. Proposed estimator t will be more efficient than $\bar{y}_{BT1}^*$ if

$$MSE(\bar{y}_{BTR}^*) - \min MSE(t) \geq 0 \text{ if}$$

$$\bar{Y}^2\left(\alpha_1 + \frac{\alpha_2}{4} - \alpha_3\right) - \left[\frac{R_3R_4R_5 - R_2R_4^2 - R_1R_5^2}{(4R_1R_2 - R_3^2)^2} + R_6\right] \geq 0 \tag{37}$$

IV. Proposed estimator t will be more efficient than $\bar{y}_{BT2}^*$ if

$$MSE(\bar{y}_{BTP}^*) - \min MSE(t) \geq 0 \text{ if}$$

$$\bar{Y}^2(\alpha_1 + \frac{\alpha_2}{4} + \alpha_3) - [\frac{R_3R_4R_5 - R_2R_4^2 - R_1R_5^2}{(4R_1R_2 - R_3^2)^2} + R_6] \geq 0 \quad (38)$$

The proposed estimator observed to be more efficient with above mentioned conditions.

Empirical Study

For above mentioned estimators, the percent relative efficiencies (PREs) are calculated from real populations. The formula used for PRE calculations is defined as $PRE(.) = \frac{V(\bar{y})}{MSE(.)} * 100$. The populations considered are given below and corresponding PREs values reported in the following tables.

1. Khare and Sinha (2004)
2. Statici and Kadilar (2011)
3. Khare and Srivastava (1995)

In population 1

y = weight in kg of children,

x = chest circumferences in cm.

In population 2

y= number of successful students,

x=number of teachers.

In population 3

y= Cultivated area (in acres),

x = population of village.

Table 2: Real data sets for the empirical study

Population I [Khare and Sinha (2004)]		**Population II [Staici and Kadilar (2011)]**		**Population III [Khare and Srivastava (1995)]**	
N=95	N_1=71	N_1=261	N_1= 196	N_1=700	W_2=0.20
n=35	N_2=24	n = 90	N_2 = 65	n=35	ρ_{xy} =0.778
$\bar{X}$=55.86	ρ_{xy} =0.85	$\bar{X}$ = 306.43	ρ_{xy} = 0.9705	$\bar{X}$=1755.53	$C_{y(2)}$=0.4087
$\bar{Y}$=19.5	$C_{y(2)}$=0.12075	$\bar{Y}$ = 222.57	$C_{y(2)}$=1.22	$\bar{Y}$=981.29	$C_{x(2)}$=0.5739
C_x=0.05860	$C_{x(2)}$=0.05402	C_x =1.7595	$C_{x(2)}$=1.23	C_x=0.8009	$\rho_{xy(2)}$= 0.445
C_y=0.15613	$C_{xy(2)}$=0.00395	C_y = 1.8654	$\rho_{xy(2)}$=0.9733	C_y=0.6254	
C_{xy} =0.00776	$\rho_{xy(2)}$=0.729				

Table 3: PREs obtained through empirical study

	PRE								
	Population I			**Population II**			**Population III**		
	k = 2	**k=3**	**k=4**	**k = 2**	**k=3**	**k=4**	**k=2**	**k=3**	**k=4**
t_y	100	100	100	100	100	100	100	100	100
t_R	195.5	193.3	191.7	1716.5	1730.4	1741.3	124.3	108.6	98.9
t_{ER}	121.1	121.0	121.0	173.6	174.37	175.0	208.3	188.9	176.2
t_{EP}	81.6	81.4	81.3	77.03	76.98	76.9	147.9	164.4	179.80
t_{prop}	**306.8**	**281.8**	**267.4**	**1726.4**	**1734.6**	**1743.6**	**210.9**	**189.3**	**176.2**

Conclusions

In this research, we proposed a generalized difference-cum-exponential type estimator of a population mean applying transformation on the auxiliary variable, in simple random sampling whenever non-response is present. The Bias and MSE of the proposed estimator obtained and the MSE of the suggested estimator compared with the MSE of existing estimators such as ratio estimator, exponential ratio estimator and exponential product estimators theoretically and empirically. Some existing estimators can be seen as the members of this generalized estimator. The real data set considered and PRE reported at different non-response rates. Here, we observed that the minimum MSE for the proposed estimator is smaller as compared to MSE of other existing estimators, and accordingly PRE reported is higher for the proposed estimator. Concludingly, the proposed estimator is more generalized and efficient under wider range of populations. Therefore, by suggesting this estimator, here is to contribute a new generalized and efficient estimator for the estimation of population mean under non-response in survey sampling. This proposed estimator might be used for various estimation purposes and real-life applications.

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