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Strictly I- Pseudo Regular I-spaces

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Abstract

In this paper Strictly I-Pseudo Regular I-Spaces and I-Completely regular I- spaces have been defined and a few important properties of these spaces have been justified. This is the 2nd paper of I-Pseudo Regular I-Spaces. I-Pseudo Normal is not defined for I-Spaces.

Keywords: I-Regular, I-Compact, I-Continuous, Strictly I-Pseudo Regular I-Spaces

Mathematics Subject Classification: 54D10, 54D15, 54A05, 54A40, 54G12, 54C08 . .

1. Introduction

A topological space X is said to be **Regular** if for each non- empty closed set F of X and any point $x \in X$, such that $x \notin F$ (i.e. x is the external point of F), there exist two disjoint open sets V and W such that $x \in V$ and $F \subseteq W$. In paper [3] a pseudo regular topological spaces has been defined by replacing a closed subset F by a compact subset K in the definition of a regular space. The concept of Pseudo regularity has been extended to I- spaces and U- spaces in previous paper [2]. These spaces were introduced and studied in [4], [5] and [6]. Here the concept of Strictly Pseudo regularity of [1] has been extended to I- spaces. A number of important theorems regarding these spaces have been established.

Here we have introduced Strictly pseudo regular I- spaces and studied their important properties. Many results have been proved about these I-spaces. We have also established characterizations of such I-spaces.

II. Preliminaries

We begin with some basic definitions and examples related to I – spaces.

I-Spaces

Definition 2.1[6]: Let X be anon- empty set. A collection I of subsets of X is called an I-structure on X if (i) $X, \Phi \in$ I (ii)$G_1, G_2, G_3, \cdots\cdots, G_n \in$ I implies $G_1 \cap G_2 \cap G_3 \cap \cdots\cdots \cap G_n \in$ I . Then (X, I) is called an I-space.

The members of I are called I-open set and the complement of I- open set is called I- closed set.

Example 2.1: Let X = {a, b, c, d}, **I** = {X, Φ, {a},{a, b}, {c},{a, c}, {b, c, d}}. Here (X, **I**) is a I- space but not a topological space and U- space.

Example 2.2: Let X = R, I = Finite unions of the sets in C , where C = $\{R, \Phi\} \cup \{[a, b] | a, b \in R\}$

Then I is an I- structure on R. **Thus (R, I) is an I- space**.

Definition 2.2 [6]: Let (X, **I**) be an I – space. An I- **open cover** of subset K is a collection {Gα} of I- open sets such that K⊆$\bigcup_{\alpha} G_{\alpha}$.

Definition 2.3 [6]: An I-space X is said to be **I- compact** if for every I-open cover of X has a finite sub-cover.

A subset K of a I- space X is said to be **I- compact** if every I-open cover of K has finite sub-cover.

Thus, if (X, **I**) be an I- space, and A ⊆X, then A is said to be **I- compact** if for each $\{I_\alpha | I_\alpha \in \mathbf{I}\}$ such that A $\subseteq \bigcup_\alpha I_\alpha, thereexitI_{\alpha_1}, I_{\alpha_2}, \cdots\cdots, I_{\alpha_n} suchthatA \subseteq I_{\alpha_1} \cup I_{\alpha_2} \cup \cdots\cdots \cup I_{\alpha_n}, forsomen \in N$.

Example 2.3: K is I- compact if K contains only intersections of the form [a, a] = {a}, and these must be finite in number. Thus K must be a finite set.

For, let K be a compact subset of R, for some n∈N and x∈R with x∉K.

We know that I = $\{R, \Phi\} \cup \{[a, b] | a, b \in R\}$. Since K is I- compact, $K = \{x_1, x_2, \cdots\cdots, x_n\}, wherex_i \neq x_j, \forall i, j, forsomepositive\ int\ e\ gern. IfK \subseteq R, K \supseteq [a, b], forsomea, b \in R, a < b$,then K is not compact, since [a, b] is not so.

Definition 2.4 [6]: If X, Y are I-spaces then a map f: X → Y is said to be **I-continuous** if for each I-open set H in Y, f $^{-1}$(H) is an I-open set in X .

Example 2.4: Let X = {a, b, c, d}, I = { Φ , X, {a}, {b}, {d}, {a, b}, {a, d}, {b, c, d}}, Y = {p, q, r, s},

I = {Y, Φ, {p}, {q}, {s}, {p, q},{p, s},{q, r, s}}. Let f: X → Y be defined by f(a) = p, f(b) = q, f(c) = r, f(d) = s. Then f is I-continuous.

Definition 2.5 [4]: A I-space X is called **Hausdorff** if, for each x, y ∈ X, x ≠ y, there exists disjoint I-open sets G and H in X such that x∈ G, y∈ H.

Example 2.5. Let X = {a, b, c, d}, I ={{a},{d},{b},{c},{b, c},{b, d}, {a, d}, {a, c},{c, d},{a, b, c}, {b, c, d},{a, c, d}, X, φ }. Then (**X**,I) is a Hausdorff I- space.

Pseudo Regular I - Spaces

Definition 2.6 [2]: An I- space X is said to be I - **pseudo regular** if for every I- compact subset K of X and for every x∈ X, x∉ K, there exist two I-open sets $I_1, I_2 \in \mathbf{I}$ with $K \subseteq I_1, x \in I_2, I_1 \cap I_2 = \Phi$.

Example 2.6: (**Example of an I-pseudo regular space**)

Let $[x_1 - \alpha_1, x_1 + \alpha_1], [x_2 - \alpha_2, x_2 + \alpha_2], \cdots\cdots\cdots, [x_n - \alpha_n, x_n + \alpha_n]$ be n closed intervals with each $\alpha_l < \frac{1}{2}|x_i - x_j|$and $\alpha_l < |x - x_i|$for each I, j, l.

Let $\alpha = \frac{1}{2} min\{\alpha_1, \alpha_2, \cdots\cdots \alpha_n\}. Then [x - \alpha, x + \alpha] \cap ([x_1 - \alpha_1, x_1 + \alpha_1] \cup \cdots\cdots \cup [x_n - \alpha_n x_n + \alpha_n]) = \Phi$. Now $x \in [x - \alpha, x + \alpha], K \subseteq [x_1 - \alpha_1, x_1 + \alpha_1] \cup \cdots\cdots \cup [x_n - \alpha_n, x_n + \alpha_n]$. Then **(R, I) is pseudo regular**.

III. Strictly Pseudo Regular I - Spaces

Definition 3.1. A I- space X is said to be I- **completely regular** if for any I- closed subset K of X and x∈ X which does not belongs to K, there exists a I- continuous function f: X → [0, 1] such that f(x) = 0 and f(K) = 1. Here [0, 1] is considered as a subspace of the usual I- space R.

Example 3.1. The set of real numbers R is I- completely regular

Definition 3.2 [1]: A I – space X will be called **Strictly I - pseudo- regular** if for each I- compact set K and for every x∈X with x ∉ K, there exists a continuous function f : X → [0,1] such that f(x) = 0 and f(K) = 1.

Example 3.2: Let K be a I- compact subset of R and let x∈R such that x ∉ K. Since R is I – Hausdorff, K is closed and since R is I- completely regular, there exists a continuous function f: X → [0, 1] such that f(x) = 0 and f(K) = 1.Thus R is strictly I- pseudo regular.

Theorem 3.1: Every strictly I- pseudo regular compact space is I- completely regular.

Proof: Let X be I- compact and strictly I-pseudo- regular. Let K be a closed subset of X and let x∈X with x ∉ K. Since X is I- compact, K is I- compact. Again, since X is strictly I – pseudo regular, there exists a I- continuous function f: X → [0, 1] such that f(x) = 0 and f(K) = 1.Therefore X is I- completely regular.

Theorem 3.2: Every I- completely regular Hausdorrf space is strictly I- pseudo regular.

Proof: Let X be a I- completely regular Hausdorrf space. Let K be a I- compact subset of X and x∈X with x ∉ K. Since X is I- Hausdorrf, K is I- closed. Now, since X is I – completely regular, there exists a I- continuous function f: X → [0, 1] such that f(x) = 0 and f(K) = 1.Therefore X is strictly I- pseudo regular.

Theorem 3.3: A I-space X is strictly I- pseudo regular if for each x∈X and any I- compact set K not containing x, there exists an I-open set H of X such that $x \in H \subseteq \bar{H} \subseteq K^C$.

Proof: Let X be a strictly I pseudo regular space and Let K be I- compact in X. Let x $\notin$ K i.e., $x \in K^C$. Since X is strictly I- pseudo regular, there exists a I-continuous function f : X → [0, 1] such that f(x) = 0 and f(K) = 1. Let a, b∈ [0,1] and a < b. Then [0, a) and (b, 1] are two disjoint open sets of [0, 1]. Since f is I- continuous f $^{-1}$ ([0, a)) and f $^{-1}$ ((b, 1]) are two disjoint open sets of X and obviously $x \in f^{-1}([0, a))$ and $K \subseteq f^{-1}((b, 1])$. Let U = f $^{-1}$ ([0, a)) and V = f $^{-1}$ ((b, 1]). Then $x \in U, K \subseteq V and U \cap V = \varphi. Then \rightleftarrows U \subseteq V^C \subseteq K^C$.

$$So \bar{U} \subseteq \overline{V^C} = V^C \subseteq K^C. Writing U = H, we have x \in H \subseteq \bar{H} \subseteq K^C.$$

Theorem 3.4: Any subspace of a strictly I- pseudo regular space is strictly I pseudo – regular.

Proof: Let X be a strictly I- pseudo regular space and $Y \subseteq X$.. Let $y \in Y$ and K be a I - compact subset of Y such that $y \notin K$. Since$y \in Y$, so $y \in X$ and since K is I - compact in Y, so K is I - compact in X. Since X is strictly I- pseudo regular, there exists a continuous function f : X → [0, 1] such that f(y) = 0 and f(K) = 1. Therefore the restriction function $\bar{f}$ of f is a continuous function$\bar{f}$: Y → [0, 1] such that f(y) = 0 and f(K) = 1. Hence Y is strictly I- pseudo regular.

Corollary: Let X be a I- space and A, B are two strictly I- pseudo regular subspace of X. Then $A \cap B$ is strictly I- pseudo regular.

Proof: Since $A \cap B$ being a subspace of both A and B, $A \cap B$ is strictly I- pseudo regular by the above theorem.

Theorem 3.5: Every strictly I- pseudo regular space is I – Hausdorff.

Proof: Let X be a strictly I- pseudo regular space. Let $x, y \in X$. with $x \neq y$. Then {x} is a I – compact set and $y \notin \{x\}$. Since X is strictly I- pseudo regular, there exists a continuous function f : X → [0, 1] such that f(y) = 0 and f({x}) = 1. Let $a, b \in$ [0,1]. and a < b. Then [0, a) and (b, 1] are two disjoint open sets of [0, 1]. Since f is I - continuous, f $^{-1}$ ([0, a)) and f $^{-1}$ ((b, 1]) are two disjoint open sets of X and obviously $y \in f^{-1}([0, a))$ and $\{x\} \subseteq f^{-1}((b, 1])$ i.e., $x \in f^{-1}((b, 1])$. Therefore X is I - Hausdorff.

Theorem 3.6: Every strictly I- pseudo regular space is I- pseudo regular

Proof: Let X be a strictly I- pseudo regular space. Let K be a compact subset of X and $x \in X$. with $x \notin K$.. Since X is strictly I- pseudo regular, there exists a continuous function f : X → [0, 1] such that f(x) = 0 and f(K) = 1. Let $a, b \in$ [0,1]. and a < b. Then [0, a) and (b, 1] are two disjoint open sets of [0, 1]. Since f is I - continuous, f $^{-1}$ ([0, a)) and f $^{-1}$ ((b, 1]) are two disjoint open sets of X and obviously $x \in f^{-1}([0, a))$ and $K \subseteq f^{-1}((b, 1])$. Therefore X is I- pseudo regular.

Result and Discussion

It has been proved that Strictly I- pseudo regular compact space is I- completely regular and I- completely regular Hausdorrf space is strictly I- pseudo regular, A I-space X is strictly I- pseudo regular if for each x∈X and any I- compact set K not containing x, there exists an I- open set H of X such that $x \in H \subseteq \bar{H} \subseteq K^C$., Any subspace of a strictly I- pseudo regular space is strictly I pseudo – regular, strictly I- pseudo regular space is I – Hausdorff , strictly I- pseudo regular space is I- pseudo regular.

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