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RESEARCH ARTICLE



The χ^2 Test as Compared to the T-Test and the Takiar Z Test for Small Samples below 30

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Abstract

In the present communication, the application of the χ^2 test is explored for small samples and the results are compared to the t-test and the Takiar Z test. The validity of the applicable Yates correction is also explored. The independent pairs of samples of size 12, 15, 18, 20, 24 and 30 are drawn from the Normal, Uniform and Exponential Population and compared for the possible significant differences or distribution differences. Thus, for each sample size, 500 mean comparisons are done by the t-test and the Takiar Z test and the distribution comparison are done by the Chi-square test. Proceeding, in a similar way, for the Uniform and Exponential Populations, the sample comparisons are made. Overall, the study pertains to 9000 samples comparisons.

The results surprisingly shows that the application of simple χ^2 test picks up correctly about 73.5% of the significant differences in the normal samples as against 64.3% picked up by the χ^2 test with the Yates correction. This clearly shows that the application of Yates correction is not beneficial. This is also observed to be true in the case of sample comparisons taken from the Uniform and Exponential distribution.

The comparison of the performance of the t-test and the Takiar Z test for small samples reveals that the Takiar Z test out-perform at each sample size and in case of each sample comparisons. Relatively, the validity is better in the case of Takiar Z test to the tune of 6-8%. Further, the performance of the Takiar Z test is found to be superior to χ^2 test and the t-test irrespective of the sample size and the type of population. Thus, for small samples Takiar Z test is recommended.

Key Words: χ^2 test, Small samples, Takiar Z test, t-test, Normal samples, Uniform samples, Exponential samples.

Introduction

The χ^2 test is one of the most popular non-parametric test. It is used mainly for categorical data and often deals with frequencies. The χ^2 test examines whether a series of frequencies obtained in selected categories are consistent with the numbers expected in those categories on some specific hypothesis. In literature, apart from finding the association between two categorical variables, typically we find the application of χ^2 test for testing the Goodness of fit and the Test of proportions. Two important conditions which are laid down for the χ^2 test to be valid is, the sample size should be more than 50 and no cell frequency to be below 5 in any cell. For 2X2 table, if any cell frequency is less than 5, the Yates correction is advocated (Gupta SC, Kapoor VK 2000, Gupta SC 2012). The application of Chi-square test for small samples and validity of the Yates correction is yet to be explored.

For small samples, the t-test is in use. It has been shown that for the small samples, the Takiar Z test is a better option than the t-test (Takiar R, 2024). The validity of the t-test for the small samples have been discussed in detail in my earlier papers (Takiar R-1, 2023, Takiar R-2, 2023). In the present communication, the application of the Chi-square is explored for small samples and the results are compared to the t-test and the Takiar Z test. The objectives of the present communication are therefore:

Objectives

- To explore the application of the χ^2 test for small samples below 30 and compare the results with the t-test and the Takiar Z test.
- To assess the validity of the Yates correction.

Materials and Methods

For $m \times n$ Contingency table, the χ^2 is defined as

$$\chi^2 = \sum \sum \frac{[O(I,J) - E(I,J)]^2}{E(I,J)} \quad \text{for all } I \text{ \& } J \text{ where } I=1,2,3,\dots,m \text{ and } J = 1,2,3,\dots,n.$$

Degrees of freedom = $(m-1)*(n-1)$ and

$O(I,J)$ is the Observed frequency and $E(I,J)$ is the Expected frequency.

For the validity of the χ^2 test, the following conditions are laid down.

- The sample observations should be independent
- $\sum \sum O(I, J) = \sum \sum E(I, J)$

- The total frequency should be greater than 50.
- No Cell frequency should be less than 5.
- If any cell frequency is less than 5, then it is pooled with preceding or succeeding frequency so that pooled frequency is more than 5.

Yates Correction

In a 2X2 contingency table, if any cell frequency is less than 5 then the Yates correction is advocated and 0.5 is added to that cell frequency and accordingly other cell frequencies are adjusted.

Assumptions used for the t-test and the Takiar Z test

- The samples are drawn independently from the Normal Population.
- The samples have been drawn from the Populations who have comparable variances.

Selection of Populations and their Parameters

For the study purposes, three types of Populations are generated and shown in Table 1 with respective Parameters. For the generation of populations, the function key, "Random Number Generation" provided in Excel is used appropriately. The pairs of populations generated for each population are chosen to be significantly different from each other.

Table 1: Description of Populations, Type and Selected Parameter values

Population	Type of Population	Size (N)	Mean	SD	Z value	P - Value
Normal	A	200	55.5	16.05	8.03	< 0.001
	B	200	44.2	11.73		
Uniform	C	250	72.7	43.84	3.57	< 0.001
	D	250	60.4	32.35		
Exponential	E	200	31.9	17.76	4.03	< 0.001
	F	200	25.0	16.46		

Selection of Samples

The Scheme of sample selection by the type of population and the sample size is shown in Table 2. As shown in the Table 2, for each population, 500 random samples are generated for each sample size of 12, 15, 18, 20, 24 and 30. The random samples of given size are generated using a basic program developed by me.

Scheme of comparisons of the sample means and the sample distributions

The samples drawn from the Normal Population A and B are compared for the possible significant differences in means and the distribution differences by the χ^2 test.

Table 2: Scheme of Sample Selection according to Population, Type, Sample size and Number of Samples drawn

Population	Type	Sample Size						Total
		12	15	18	20	24	30	
Normal	A	500	500	500	500	500	500	3000
	B	500	500	500	500	500	500	3000
Uniform	C	500	500	500	500	500	500	3000
	D	500	500	500	500	500	500	3000
Exponential	E	500	500	500	500	500	500	3000
	F	500	500	500	500	500	500	3000

Thus, for each sample size, 500 mean comparisons are done by the t-test and the Takiar Z test and the distribution comparison are done by the Chi-square test.

For the application of the chi-square test, the observations from the Population A (Sample A) are divided according to median value in 2 categories. Based on the median value of Sample A, the Sample values from the population B are also divided into two categories. Thus, we get a 2X2 contingency table as shown in Table 3.

Table 3: 2X2 Contingency Table

Sample	Below Median of Sample A	Above Median of Sample A	Total
Sample A	a	b	a+b
Sample B	c	d	c+d
Total	a+c	b+d	a+b+c+d

This scheme allow use to make 500 sample comparisons for each sample size. Proceeding, in a similar way, for the Uniform and Exponential Populations, the sample comparisons are made.

Assessment of Yates Correction on Significance

For 2X2 Contingency table, It is often claimed that whenever the cell frequency is less than 5 in any cell, the Yates correction to be applied as a continuity correction. Therefore, to assess the impact of continuity correction on true significance, the χ^2 was calculated in two ways, first by the application of simple χ^2 and the second, by the application of χ^2 with the feasible Yates correction. The results obtained thereby are compared to assessing the validity

of the Yates correction. The objective is to see if there is really a gain in the validity by using the Yates correction.

Validity of the test

The Validity is defined as capacity of a test to identify correctly the significant differences in the distributions when the pairs of samples known to have been drawn from two different populations are compared. In formula, it is given as:

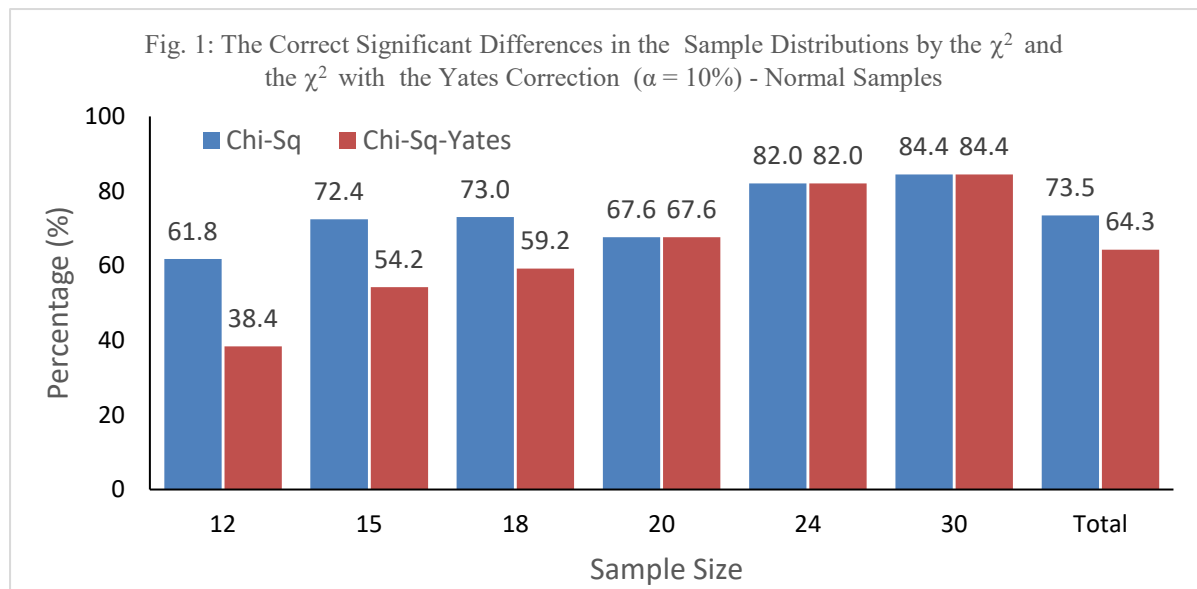
$$\text{Percentage Validity} = \frac{\text{Number of Correct Significant differences observed}}{\text{The total number of paired sample compared}} \times 100$$

Significance and α level

In a series of papers, it was shown that for the small sample size, the validity of the t-test, Z-EV test and the Mann-Whitney test are lower at 5% α level as compared to those of 10% α level. With the rise of α level to 10%, a substantial absolute gain of more than 10% in the Validity of all the tests was observed and it was recommended that for small samples, it is better to use 10% α level (Takiar R 2021, Takiar R-1 2023, Takiar R-2 2023, Takiar R, 2024). Accordingly, in the present communication for all significant differences, the α level is chosen to be 10%.

Results

The comparison of sample distributions, pertaining to the Normal samples, obtained by the χ^2 test and the χ^2 test with the Yates correction, resulting in the correct significant differences, for the sample size varying from 12 to 30 are shown in Fig. 1.

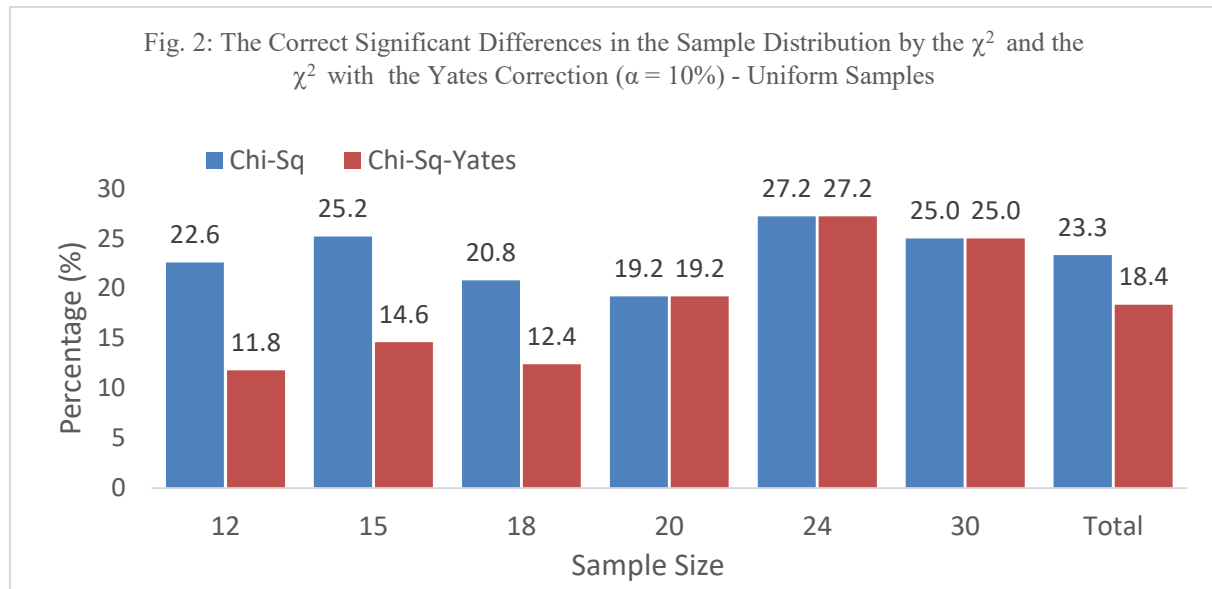


The results exhibited in Fig. 1; surprisingly shows that for each sample size the application of Yates correction, as anticipated, do not improve in picking up the correct percentage of significant differences in the samples compared. The results when pooled for all the samples, shows that the application of simple χ^2 test picks up correctly about 73.5% of the significant differences in the samples as against 64.3% picked up by the χ^2 test with the feasible Yates correction. This clearly shows that the application of Yates correction is not

resulting in improving of the validity and on the contrary registering a fall in it thereby suggesting that its application is not necessary.

The χ^2 test is a non-parametric test and therefore if you apply it to other than normal samples, the results should be better than applying a parametric test to the samples.

The comparison of sample distributions, pertaining to the Uniform samples, obtained by the χ^2 test and χ^2 test with the Yates correction, resulting in the correct significant differences, for the samples of size varying from 12 to 30 are shown in Fig. 2.

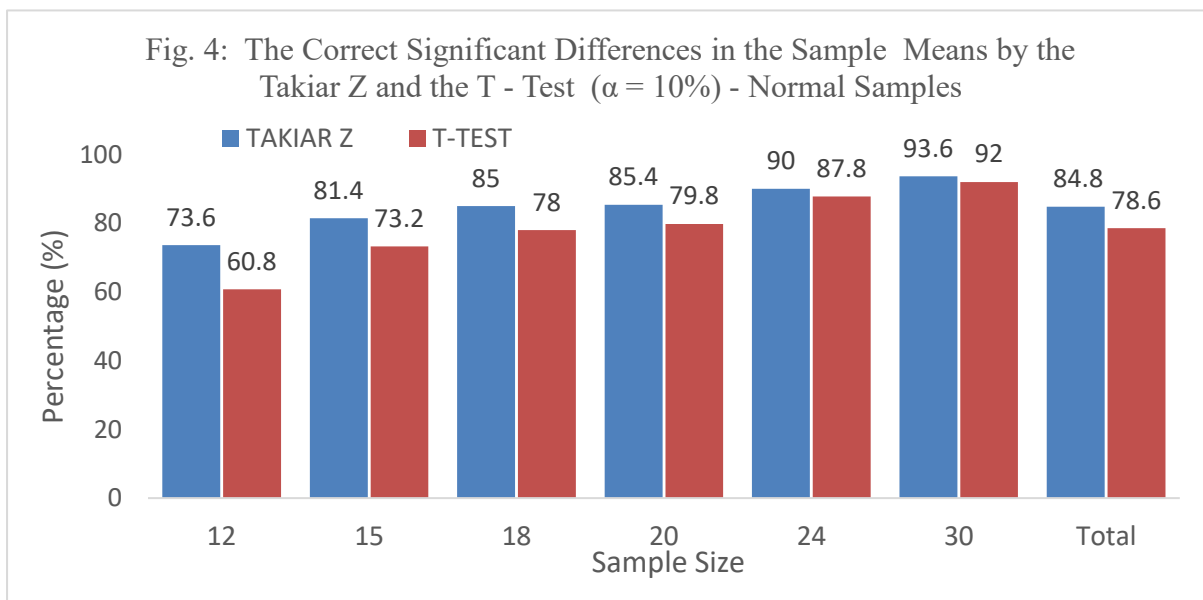
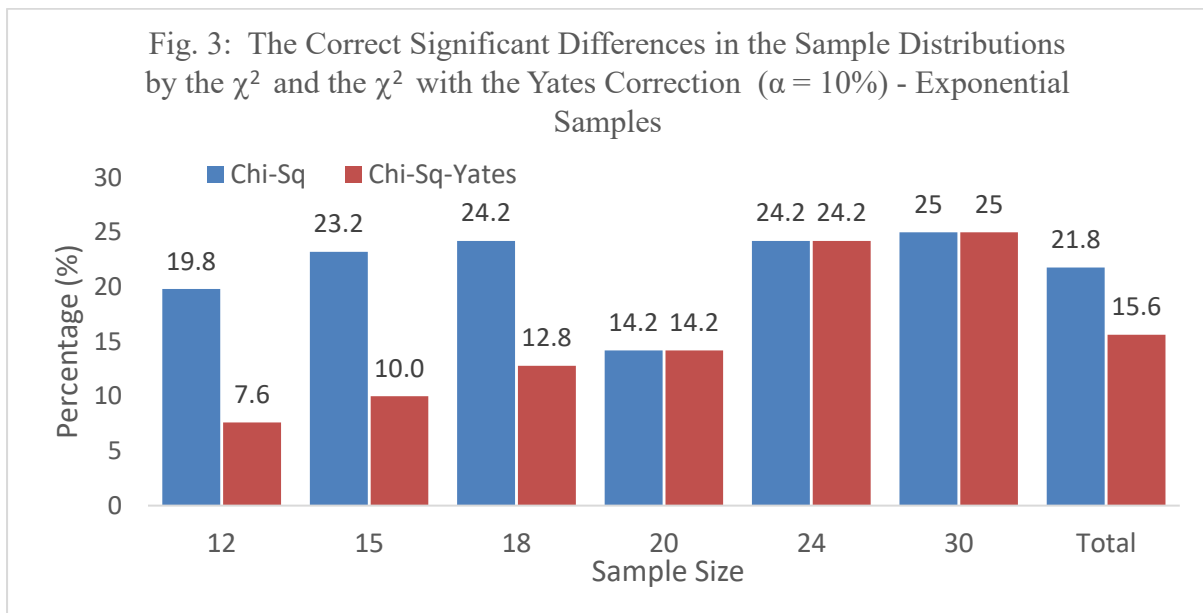


The results exhibited in Fig. 2; again, shows that for each sample size the application of feasible Yates correction, as anticipated, do not improve in picking up the correct percentage significant differences in the sample distributions. The results when pooled for all the samples, shows that the application of simple χ^2 test picks up correctly about 23.3% of the significant differences in the samples as against 18.4% picked up by the χ^2 test with the Yates correction. This clearly shows that the application of Yates correction is not at all beneficial even if you apply it to non-normal samples. Further, the results show that the validity is below 30% if you apply the χ^2 test to Uniform samples.

The comparison of sample distributions, pertaining to the Exponential samples, obtained by the χ^2 test and χ^2 test with the Yates correction, resulting in the correct significant differences, for the samples of size varying from 12 to 30 are shown in Fig. 3.

The results shows that for each sample size the application of Yates correction do not improve in picking up the correct percentage significant differences in the samples compared. The results when pooled for all the samples, show that the application of simple χ^2 test picks up correctly about 21.8% of the significant differences in the samples as against 15.6% picked up by the χ^2 test with the Yates correction. This exhibit the failure of Yates correction in improving the validity. Further, the results show that the validity is below 25% if you choose the sample comparisons drawn from the Exponential distribution.

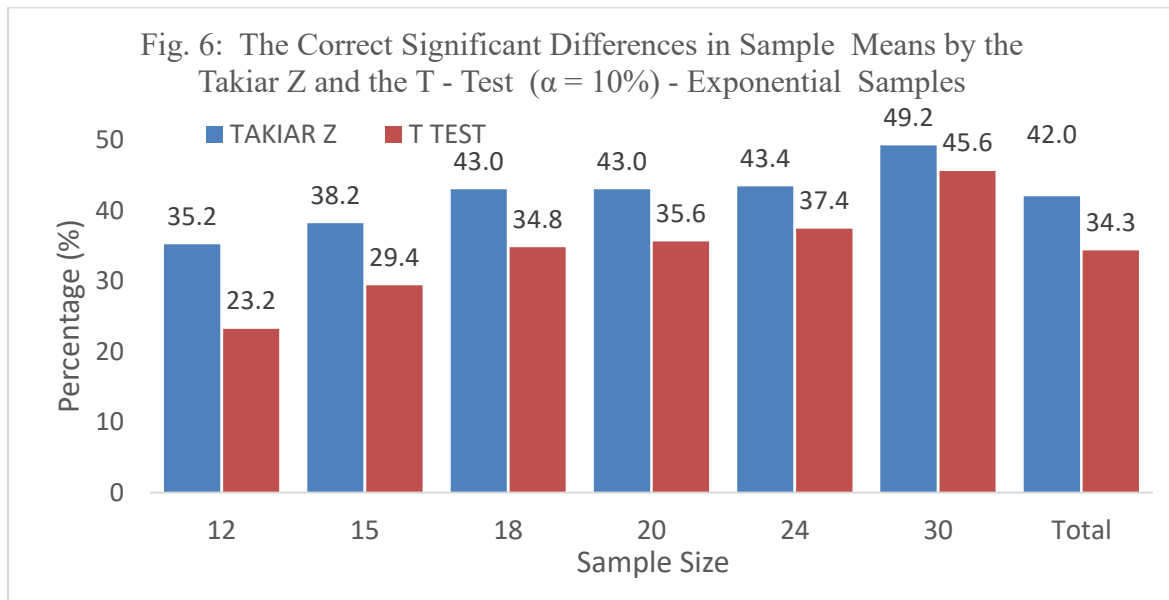
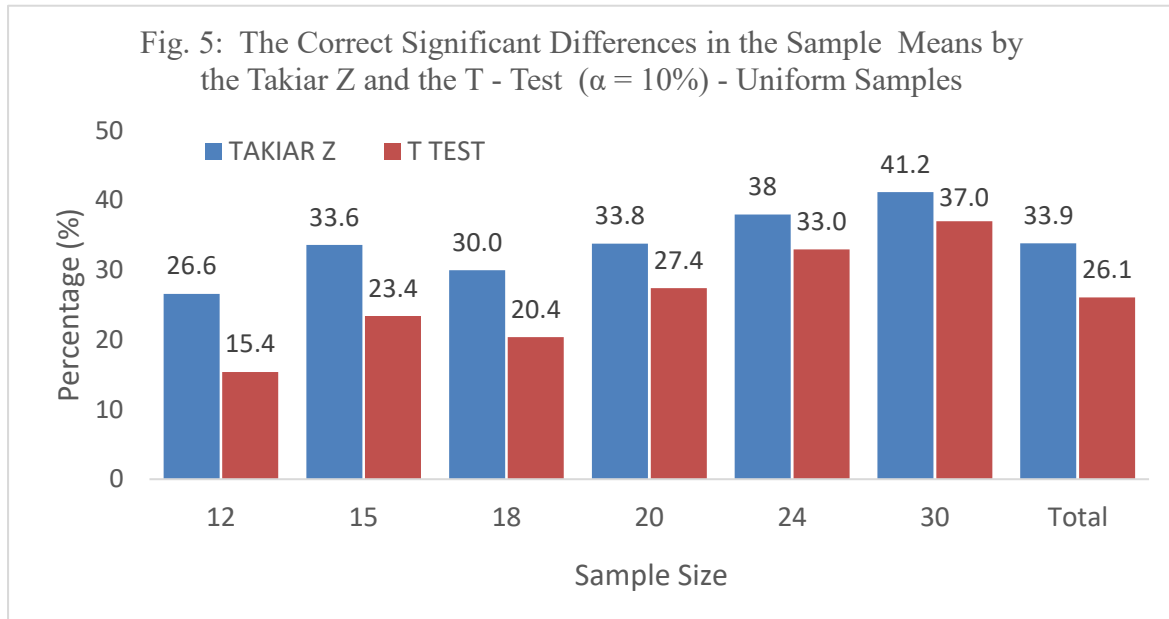
The results presented in Fig.4 for the Normal samples show that the performance of the Takiar Z test is better than the t-test consistently irrespective of the sample size selected.



Even at the sample size 12, the Takiar Z test picks up correctly 73.6% of the mean differences as against 60.8% by the t-test. For all the samples pooled from 12-30, the Takiar Z test picks up correctly 84.8% of the mean differences, as against 78.6% picked by the t-test (78.6%).

The results presented in Fig.5 for Uniform samples show that the performance of the Takiar Z test is better than the t-test consistently irrespective of the sample size selected. However, overall, for the Uniform samples, both the test show poor picking up of the true sample mean differences (below 35%).

The results presented in Fig.6 for Exponential samples again confirms that the performance of the Takiar Z test is better than the t-test. Even at the sample size of 12, the Takiar Z test picks up correctly 35.2% of the mean differences as against 23.2% by the t-test. For all the samples pooled from 12-30, the Takiar Z test picks up correctly 42.0% of the mean differences, more than seen for the t-test (34.3%).

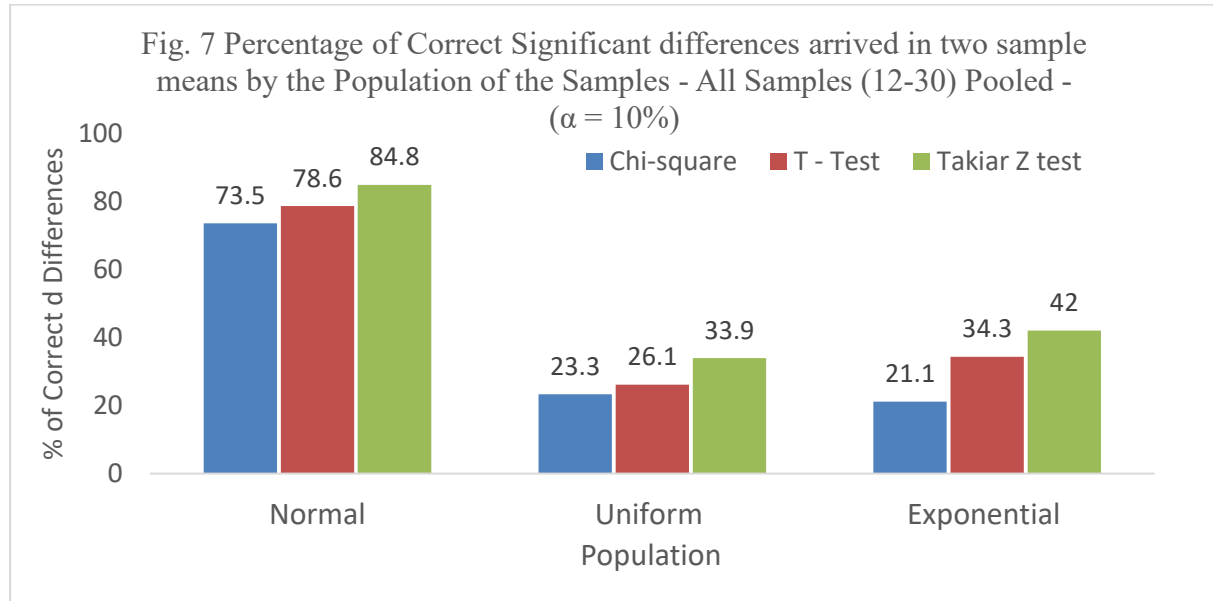


The comparison of performance of selected Significance test for samples drawn from Normal, Uniform and Exponential is shown in Fig. 7. Irrespective of types of samples, the performance of the Takiar Z test is better than the t-test and the χ^2 test. It is advocated that for non-normal samples, the application of non-parametric tests is best. But our results suggests that it is not true. In case of normal as well as non-normal samples, the performance of the Takiar Z test is adjudged to be the best.

Discussion

The present study explores the validity of using the application of Yates correction with the χ^2 test to three different types of sample distributions namely Normal, Uniform and Exponential. The results of the study surprisingly shows that for each sample size ranging from 12-30 and each distribution namely Normal, Uniform and Exponential, the application

of Yates correction, as anticipated, do not improve in picking up the correct percentage significant differences in the samples compared and exhibit a clear fall in the validity in the application of it.



The absolute fall in the validity is about 10% in the case of Normal samples and around 5% in the case of Uniform and Exponential samples. This further suggests that application of Yates correction is not necessary. For Normal samples, considering the validity of above 70% as acceptable, the χ^2 test can be applied when samples to be compared are with the size as low as 18. This is collaborated by considering the power of the χ^2 test as shown in Table 4.

Table 4: Correctly rejecting the Null hypothesis when the Alternative Hypothesis is true by the χ^2 test ($\alpha = 10\%$) - Normal Samples

Test of Significance	12	15	18	20	24	30
χ^2 test	90.2	89.8	92	92	92.4	91.4

The comparison of the performance of t-test and Takiar Z test for small samples reveals that the Takiar Z test out-perform at each sample size. Relatively, the validity is better in the case of Takiar Z test to the tune of 6-8%.

In case of non-normal samples or when the sample distributions are suspected to be following other than normal distribution, often the application of a non-parametric test is advocated against the use of a parametric test. The underlying assumption is that a non-parametric test will be performing better in such a situation. However, the current study results do not support this view. This collaborate with my earlier study where it was shown that in comparisons of sample means, drawn from Uniform or Exponential distribution, the validity of Mann-Whitney test was below 25% (Takiar R 2023-3). The Takiar Z test is shown to be performing better than the χ^2 test.

It is interesting to note that the performance of the non-parametric tests like Mann-Whitney or χ^2 test is better when samples are drawn from Normally distributed populations. In case of Uniform and Exponential population samples their performance also suffer. So, the expectation that a non-parametric test is better in case of non-normal samples is not necessarily true.

Based on the results of the study, in relation to the t-test for small samples whether they follow the normal distribution or otherwise, Takiar Z test is adjudged to be the best.

Summary of observations

- For the study purpose, all the sample comparisons are made for $\alpha = 10\%$ as this was shown to be resulting in improvement of the validity of the test.
- For the Normal samples, pooled for all the sample sizes from 12-30, the Correct Significant Differences in the Sample Distributions by the χ^2 test and the χ^2 with feasible Yates Correction are observed to be 73.5% and 64.3%, respectively.
- For the Uniform samples, , pooled for all the sample sizes, the Correct Significant Differences in the Sample Distributions by the χ^2 test and the χ^2 with Yates Correction are observed to be 23.3% and 18.4%, respectively.
- For the Exponential samples, pooled for all the sample sizes, the Correct Significant Differences in the Sample Distributions by the χ^2 test and the χ^2 with Yates Correction are observed to be 21.8% and 15.6%, respectively.
- Yates correction led to the fall in the validity of the χ^2 test.
- The application of a non-parametric test like χ^2 test to other than normally distributed samples, does not seem to be beneficial.
- For the Normal samples, pooled for all the sample sizes, the Correct Significant Differences in the Sample means by the Takiar Z test and the t-test are observed to be 84.8% and 78.6%, respectively. The Takiar Z test is scoring over the t-test.
- For the Uniform samples, pooled for all the sample sizes, the Correct Significant Differences in the Sample means by the Takiar Z test and the t-test are observed to be 33.9% and 26.1%, respectively. The Takiar Z test is scoring over the t-test.
- For the Exponential samples, pooled for all the sample sizes, the Correct Significant Differences in the Sample means by the Takiar Z test and the t-test are observed to be 33.9% and 26.1%, respectively. The Takiar Z test is scoring over the t-test.
- For the Normal samples, the validity of the χ^2 test, t-test and Takiar Z test is observed to be 73.5%, 78.6% and 84.8%, respectively. The Takiar Z test scoring over other tests.
- For the Uniform samples, the validity of the χ^2 test, t-test and Takiar Z test is observed to be 23.3%, 26.1% and 33.9%, respectively. The Takiar Z test scoring over other tests.
- For the Exponential samples, the validity of the χ^2 test, t-test and Takiar Z test is observed to be 21.1%, 34.3% and 42.0%, respectively. The Takiar Z test scoring over other tests.

- The theoretical belief that the application of a non-parametric test is better than the application of a parametric test to non-normal samples does not seem to be correct.

Conclusions

- The application of the simple χ^2 results in higher validity as compared to those of χ^2 with the Yates correction.
- The Yates correction is not necessary.
- The χ^2 can be applied to the normal samples with the size as low as 18 while it is claimed that the test should be applied to samples with size more than 50.
- The performance of the Takiar Z test is better than the t-test and χ^2 test. Hence, for small sample size, The Takiar Z can be used in place of t-test.

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Biography

Dr. Ramnath Takiar

I am a Post graduate in Statistics from Osmania University, Hyderabad. I did my Ph.D. from Jai Narain Vyas University of Jodhpur, Jodhpur, while in service, as an external candidate. I worked as a research scientist (Statistician) for Indian Council of Medical Research from 1978 to 2013 and retired from the service as Scientist G (Director Grade Scientist). I am quite experienced in large scale data handling, data analysis and report writing. I have 73 research publications, with 1250 citations to my credit, published in national and International Journals related to various fields like Nutrition, Occupational Health, Fertility and Cancer epidemiology. During the tenure of my service, I attended three International conferences

namely in Goiana (Brazil-2006), Sydney (Australia-2008) and Yokohoma (Japan-2010) and presented a paper in each. I also attended the Summer School related to Cancer Epidemiology (Modul I and Module II) conducted by International Agency for Research in Cancer (IARC), Lyon, France from 19th to 30th June 2007. After my retirement, I joined my son at Ulaanbaatar, Mongolia. I worked in Ulaanbaatar as a Professor and Consultant from 2013-2018 and was responsible for teaching and guiding the Ph.D. students. I also taught Mathematics to undergraduates and Econometrics to MBA students. During my service there, I also acted as the Executive Editor for the in-house Journal "International Journal of Management". I am also acting as a reviewer for few International Journals. I am still active in research and have published 15 research papers during 2021-25.