Vol. 14. Issue.1. 2026 (Jan-March)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

*A Peer Reviewed International Research Journal*

http://www.bomsr.com
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

# Qualitative–Quantitative Approach for Coding Theorems on Comprehensive Fuzzy Useful R-Norm Verma Information Measure

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DOI:10.33329/bomsr.14.1.1

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**Article Info**
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Article Received: 26/12/2025
Article Accepted: 20/01/2026
Published online: 29/01/2026

**Abstract**

For the generalized fuzzy useful R-norm Verma information measure, where R and a serve as two flexible parameters, this communication aids in the development of a coding theorem. Additionally, we look at the codeword length in both scenarios and we find that one is more effective than the other.

**Key words:** Fuzzy set, Useful R-norm, Information measure, Code-word length, Decipherable code, Discrete probability distribution.

## Introduction

By incorporating a parameter 'R' (or 'a') to provide flexibility, particularly for "useful" information (considering event utility) and non-additive scenarios, the R-norm information measure is a generalised framework in information theory that extends standard measures like Shannon's entropy. It finds applications in fuzzy logic, coding theory, and decision-making by better handling uncertainty and imprecision, frequently through axiomatic definitions and relationships to other divergence measures.

Mathematical studies of the issues related to massage transmission, storage, and communication led to the development of information theory. It started with Shannon's [16] seminal work "The Mathematical Theory of Communication". Renyi [15], Arimoto [2], Sharma and Taneja [17], Verma [19, 20], Peerzada et al. [14], Deluca and Termini [13], and Kaufmann [12] all examined different generalisations of Shannon entropy. The human miracle and many real-world objectives are based on uncertainty and ambivalence. Contrast is found in the choices we make, the words we use, and the information we take in.

Bockee and Lubbe [5] R-norm information measure of a discrete probability distribution $P = (P_1, P_2 \dots, P_N), p_i \geq 0,\ i = 1,2 \dots, n$. Where $\sum_{i=1}^{n} p_i = 1$ and

$$R^* = \{R: R > 0, R \neq 1\}$$

given by
$$H_R(P) = \frac{R}{R-1}\left[1 - \left(\sum_{i=1}^{n} p_i^R\right)^{\frac{1}{R}}\right], \tag{1.1}$$

the R-norm information measure (1.1) is a real function $\Delta_n \rightarrow R^+$, defined on $\Delta_n$ where $n \geq 2$ and $R^+$ is the set of positive real numbers. This measure is different from Shannon's entropy [16], Renyi [15] and Havrda and Charvat [10] and Daroczy [7].

The most interesting property of this measure is that when $R \rightarrow 1$, R-norm information measure (1.1) approaches to Shannon's entropy and in case $R \rightarrow \infty$, $H_R(P) \rightarrow (1 - \max p_i)$, $i = 1,2 \dots, n$.

The measure (1.1) has been generalized by Hooda and Anant [2] as

$$H_R^{\beta}(P) = \frac{R}{R+\beta-2}\left[1 - \left(\sum_{i=1}^{n} p_i^{\frac{R}{2-\beta}}\right)^{\frac{2-\beta}{R}}\right], 0 < \beta \leq 1, R(> 0) \neq 1 \tag{1.2}$$

(1.1) has been called as the generalized R-norm entropy if degree $\beta$ which reduces to (1,2) when $\beta = 1$. In case $R = 1$, (1.2) reduces to

$$H_1^{\beta}(P) = \frac{1}{\beta-1}\left[1 - \left(\sum_{i=1}^{n} p_i^{\frac{1}{2-\beta}}\right)^{2-\beta}\right], 0 < \beta \leq 1. \tag{1.3}$$

Setting $r = \frac{1}{2-\beta}$ in (1.3), we get

$$H^r(P) = \frac{r}{r-1}\left[1 - \left(\sum_{i=1}^{n} p_i^r\right)^{\frac{1}{r}}\right], \frac{1}{2} < r \leq 1. \tag{1.4}$$

which is a measure mentioned by Arimoto [2] as an example of a generalized class of information measure.

As an illustration of a broad measure of distribution, Boekee and Lubbe [5] recognised and examined the R-norm information measure put forth by Arimoto [2]. Hooda and Sharma [8] suggested modifications to the fuzzy information rule in line with the R standard measures suggested by Hooda and Ram [9], which are comparable to the measurements of Boekee and Lubbe [5]. Additionally, expanded on the findings of measurements to gauge R-regulatory data. The measurements provided are measured by Kumar and Choudhary [18] using the following model for a random experiment S,

$$S_N = [E; P; U]$$

Where $E = (E_1, E_2, \dots, E_N)$ is a finite system of events happening with respective probabilities $P = (P_1, P_2 \dots, P_N), p_i \geq 0$ and $\sum p_i = 1$ and credited with utilities $U = (u_1, u_2, \dots, u_N), u_i \geq 0, i = 1,2, \dots, N$. Denote the model by $E$, where

$$E = \begin{pmatrix} E_1 & E_2 \dots E_N \\ p_1 & p_2 \dots p_N \\ u_1 & u_2 \dots u_N \end{pmatrix}. \tag{1.5}$$

We call (1.5) a Utility Information Scheme (UIS) proposed a measure of information called 'useful information' for this scheme, given by

$$H(U;P) = -\sum u_i p_i \ln p_i,$$

where $H(U;P)$ reduces to Shannon's [9] entropy when the utility aspect of the scheme is ignored i.e. when $u_i = 1$for each $i$. Throughout the paper, $\Sigma$ will stand for $i = 1,2 \dots, N$.

Information is mostly used to eliminate ambiguity and uncertainty. The amount of probability uncertainty that is overlooked during the experiment is actually used to restrict the data that is presented; this measure of uncertainty is referred to as a measure of information up to the degree of uncertainty. It is a gauge of ambiguity and uncertainty.

Zadeh [22] created the theory of fuzzy sets (FS), which is a condensed version of conventional set theory for representing ambiguous and indeterminate occurrences. This idea is a useful tool for understanding how the human system behaves, when judgements, perceptions, and emotions are crucial. Ambiguity in a restricted set of concepts is defined as the degree of ambiguity that reveals the extent of the ambiguity or issue within us, determining whether or not an element is part of the set. The idea of exponential resonance was expanded to include fuzzy phenomena by Bhandari and Pal [4]. Ambiguous metrics due to information uncertainty were examined by Kapur [11].

Lotfi A. Zadeh's fuzzy set theory [22] has been widely applied in numerous scientific and technological fields. Fuzzy measures have already been applied to computer science, engineering, fuzzy traffic control, fuzzy aviation control, medicine, and decision making, among other fields. For example, the publications Aczel [3], Kapur [11], Verma [18, 21], Renyi [15], and Bockee and Van Der Lubbe [5] provide the lower bound for the mean code-word length of a uniquely decipherable code in terms of Shannon's [16] entropy. Kapur [11] has demonstrated connections between coding and probability entropy. Therefore, in instances when probabilistic measures of entropy are ineffective, the concept of fuzziness might be investigated rather than using probability.

Given a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, a fuzzy subset of it is defined as follows:

$$A = \{(x_i, \mu_A(x_i)) : x_i \in X, \mu_A(x_i) \in [0,1]\}$$

where $\mu_A(x_i) : X \to [0,1]$ is a membership function that provides the degree to which element $x_i$ belongs to the set $A$. It is defined as follows:

$$\mu_A(x_i) = \begin{cases} 0 \; if \; x_i \notin A \; and \; there \; is \; no \; ambiguity, \\ 1 \; if \; x_i \in A \; and \; there \; is \; no \; ambiguity, \\ 0.5 \; if \; x_i \in A \; or \; x_i \notin A \; and \; there \; is \; no \; ambiguity. \end{cases}$$

Some fuzzy set concepts that we will require for our discussion, according to Zadeh []

**Containment** If $A \subset B \Leftrightarrow \mu_A(x_i) \le \mu_B(x_i) \; \forall x_i \in X$

**Equality** If $A = B \Leftrightarrow \mu_A(x_i) = \mu_B(x_i) \; \forall x_i \in X$

**Complement** If $A^c$ is complement of $A \Leftrightarrow \mu_{A^c}(x_i) = 1 - \mu_A(x_i) \; \forall x_i \in X$

**Union** If $A \cup B$ is union of $A$ and $B \Leftrightarrow \mu_{A\cup B}(x_i) = Max\,\{\mu_A(x_i), \mu_B(x_i)\}\ \forall x_i \in X$

**Intersection** If $A \cap B$ is union of $A$ and $B \Leftrightarrow \mu_{A\cap B}(x_i) = Min\,\{\mu_A(x_i), \mu_B(x_i)\}\ \forall x_i \in X$

**Product** If $AB$ is product of $A$ and $B \Leftrightarrow \mu_{AB}(x_i) = \mu_A(x_i), \mu_B(x_i)\ \forall x_i \in X$

**Sum** If $A + B$ is sum of $A$ and $B \Leftrightarrow \mu_{A+B}(x_i) = \mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i)\mu_B(x_i)\ \forall x_i \in X$

## 2. New Results

**Claim 2.1** The generalized word length $L_R(A{:}\,U)$ meets the inequality if the code word lengths $l_1, l_2, \ldots, l_n$ satisfy $\sum_{i=1}^{n} u_i log_D\left(\frac{(1+a\mu_A(x_i))(\mu_A(x_i))^{-1}}{(1+a)^{\mu_A(x_i)}}\right).D^{-l_{i_1}\left(\frac{R-1}{R}\right)} \leq 1$ for any integers $>1$ $L_R(A{:}\,U) = V_R(A{:}\,U)$, $a > 0, R(\neq 1) > 0$. Where equality, for Verma $i.e.$ hybrid Burg measure, is valid if

$$-l_{i_1} = \frac{R}{1-R} log_D\left(\prod_{i=1}^{n} u_i\left(\left(\frac{1+a\mu_A(x_i)}{\mu_B(x_i)+a\mu_A(x_i)}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)+a\mu_{A^c}(x_i)}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$
$$\left.\frac{(1+a)^{\mu_A(x_i)}.\mu_A(x_i)}{1+a\mu_A(x_i)}.\frac{(1+a)^{\mu_{A^c}(x_i)}.\mu_{A^c}(x_i)}{1+a\mu_{A^c}(x_i)}\right)$$

**Proof:** Since $L_R(A{:}\,U) = V_R(A{:}\,U)$

$$i.e. \sum_{i=1}^{n} u_i log_D\left(\left(\frac{(1+a\mu_A(x_i))(\mu_A(x_i))^{-1}}{(1+a)^{\mu_A(x_i)}}\right).D^{-l_{i_1}\left(\frac{R-1}{R}\right)}\right) =$$
$$\sum_{i=1}^{n} u_i log_D\left(\left(\frac{1+a\mu_A(x_i)}{\mu_B(x_i)+a\mu_A(x_i)}\right)^{\mu_B(x_i)} + \left(\frac{1+a\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)+a\mu_{A^c}(x_i)}\right)^{\mu_{B^c}(x_i)}\right)$$

$$i.e.\ log_D D^{-l_{i_1}\frac{R-1}{R}} = \sum_{i=1}^{n} u_i log_D\left(\frac{(1+a\mu_A(x_i))^{\mu_B(x_i)}}{(\mu_B(x_i)+a\mu_A(x_i))^{\mu_B(x_i)}}.\frac{(1+a\mu_{A^c}(x_i))^{\mu_{B^c}(x_i)}}{(\mu_{B^c}(x_i)+a\mu_{A^c}(x_i))^{\mu_{B^c}(x_i)}}.\right.$$
$$\left.\frac{(1+a)^{\mu_A(x_i)}}{(1+a\mu_A(x_i))(\mu_A(x_i))^{-1}}.\frac{(1+a)^{\mu_{A^c}(x_i)}}{(1+a\mu_{A^c}(x_i))(\mu_{A^c}(x_i))^{-1}}\right)$$

$$i.e. -l_{i_1} = \left(\sum_{i=1}^{n} u_i log_D\left(\left(\frac{1+a\mu_A(x_i)}{\mu_B(x_i)+a\mu_A(x_i)}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)+a\mu_{A^c}(x_i)}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$
$$\left.\frac{(1+a)^{\mu_A(x_i)}}{(1+a\mu_A(x_i))(\mu_A(x_i))^{-1}}.\frac{(1+a)^{\mu_{A^c}(x_i)}}{(1+a\mu_{A^c}(x_i))(\mu_{A^c}(x_i))^{-1}}\right)^{\frac{R}{R-1}}$$

$$i.e. \quad = log_D\left(\prod_{i=1}^{n} u_i\left(\left(\frac{1+a\mu_A(x_i)}{\mu_B(x_i)+a\mu_A(x_i)}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)+a\mu_{A^c}(x_i)}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$
$$\left.\left.\frac{(1+a)^{\mu_A(x_i)}}{(1+a\mu_A(x_i))(\mu_A(x_i))^{-1}}.\frac{(1+a)^{\mu_{A^c}(x_i)}}{(1+a\mu_{A^c}(x_i))(\mu_{A^c}(x_i))^{-1}}\right)\right)^{\frac{R}{R-1}}$$

$$= \frac{R}{R-1} log_D\left(\prod_{i=1}^{n} u_i\left(\left(\frac{1+a\mu_A(x_i)}{\mu_B(x_i)+a\mu_A(x_i)}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)+a\mu_{A^c}(x_i)}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$
$$\left.\frac{(1+a)^{\mu_A(x_i)}.\mu_A(x_i)}{1+a\mu_A(x_i)}.\frac{(1+a)^{\mu_{A^c}(x_i)}.\mu_{A^c}(x_i)}{1+a\mu_{A^c}(x_i)}\right)$$

Hence the result.

**Claim 2.2** The generalized word length $L_R(A:U)$ meets the inequality if the code-word lengths $l_1, l_2, \dots, l_n$ satisfy $\sum_{i=1}^{n} u_i \log_D\left(\frac{(1+a\mu_A(x_i))\mu_A(x_i)^{-\mu_A(x_i)}}{(1+a)^{\mu_A(x_i)}}.\frac{(1+a\mu_{A^c}(x_i))\mu_{A^c}(x_i)^{-\mu_{A^c}(x_i)}}{(1+a)^{\mu_{A^c}(x_i)}}\right).D^{-l_{i_2}\left(\frac{R-1}{R}\right)} \le 1$ for any integers $> 1$ $L_R(A:U) = V_R(A:U)$, $a > 0, R(\ne 1) > 0$. Where equality, for modified Verma $i.e.$ hybrid Shannon measure, is valid if

$$-l_{i_2} = \frac{R}{R-1}\log_D\left(\prod_{i=1}^{n} u_i\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$

$$\left.\left.\left(\frac{(1+a)\mu_A(x_i)}{\mu_B(x_i)}\right)^{\mu_A(x_i)}.\left(\frac{(1+a)\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)^{\mu_{A^c}(x_i)}.\left(1+a\mu_A(x_i)\right)^{-1}.\left(1+a\mu_{A^c}(x_i)\right)^{-1}\right)\right).$$

**Proof:** Since $V_R(A:U) = L_R(A:U)$

$$i.e. \sum_{i=1}^{n} u_i \log_D\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}\mu_B(x_i)^{-\mu_A(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}\mu_{B^c}(x_i)^{-\mu_{A^c}(x_i)}\right) =$$

$$\sum_{i=1}^{n} u_i \log_D\left(\left(\frac{(1+a\mu_A(x_i))\mu_A(x_i)^{-\mu_A(x_i)}}{(1+a)^{\mu_A(x_i)}}.\frac{(1+a\mu_{A^c}(x_i))\mu_{A^c}(x_i)^{-\mu_{A^c}(x_i)}}{(1+a)^{\mu_{A^c}(x_i)}}\right).D^{-l_{i_2}\left(\frac{R-1}{R}\right)}\right)$$

$$i.e. \log_D D^{-l_{i_2}\left(\frac{R-1}{R}\right)} = \sum_{i=1}^{n} u_i \log_D\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\right.$$

$$\left.\left(\frac{(1+a)^{\mu_A(x_i)}\mu_B(x_i)^{-\mu_A(x_i)}}{(1+a\mu_A(x_i))\mu_A(x_i)^{-\mu_A(x_i)}}.\frac{(1+a)^{\mu_{A^c}(x_i)}\mu_{B^c}(x_i)^{-\mu_{A^c}(x_i)}}{(1+a\mu_{A^c}(x_i))\mu_{A^c}(x_i)^{-\mu_{A^c}(x_i)}}\right)\right)$$

$$= \sum_{i=1}^{n} u_i \log_D\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\frac{(1+a)^{\mu_A(x_i)}}{(1+a\mu_A(x_i))}.\frac{(1+a)^{\mu_{A^c}(x_i)}}{(1+a\mu_{A^c}(x_i))}.\right.$$

$$\left.\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^{\mu_A(x_i)}.\left(\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)^{\mu_{A^c}(x_i)}\right)$$

$$i.e. \log_D D^{-l_{i_2}\left(\frac{R-1}{R}\right)} = \sum_{i=1}^{n} u_i \log_D\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\right.$$

$$\left.\left(\frac{(1+a)\mu_A(x_i)}{\mu_B(x_i)}\right)^{\mu_A(x_i)}.\left(\frac{(1+a)\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)^{\mu_{A^c}(x_i)}.\left(1+a\mu_A(x_i)\right)^{-1}.\left(1+a\mu_{A^c}(x_i)\right)^{-1}\right)$$

$$i.e. \qquad = \left(\sum_{i=1}^{n} u_i \log_D\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$

$$\left(\frac{(1+a)\mu_A(x_i)}{\mu_B(x_i)}\right)^{\mu_A(x_i)}.\left(\frac{(1+a)\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)^{\mu_{A^c}(x_i)}.\left(1+a\mu_A(x_i)\right)^{-1}.\left(1+a\mu_{A^c}(x_i)\right)^{-1}\Bigg)\Bigg)^{\frac{R}{R-1}}$$

$$i.e. -l_{i_2} = \left(\sum_{i=1}^{n} u_i log_D\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$

$$\left(\frac{(1+a)\mu_A(x_i)}{\mu_B(x_i)}\right)^{\mu_A(x_i)}.\left(\frac{(1+a)\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)^{\mu_{A^c}(x_i)}.\left(1+a\mu_A(x_i)\right)^{-1}.\left(1+a\mu_{A^c}(x_i)\right)^{-1}\Bigg)\Bigg)^{\frac{R}{R-1}}$$

$$i.e. \qquad = \frac{R}{R-1} log_D\left(\prod_{i=1}^{n} u_i\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$

$$\left(\frac{(1+a)\mu_A(x_i)}{\mu_B(x_i)}\right)^{\mu_A(x_i)}.\left(\frac{(1+a)\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)^{\mu_{A^c}(x_i)}.\left(1+a\mu_A(x_i)\right)^{-1}.\left(1+a\mu_{A^c}(x_i)\right)^{-1}\Bigg)\Bigg)$$

$$i.e. \; l_{i_2} = \frac{R}{1-R} log_D\left(\prod_{i=1}^{n} u_i\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_{B^c}(x_i)}.\right.\right.$$

$$\left(\frac{(1+a)\mu_A(x_i)}{\mu_B(x_i)}\right)^{\mu_A(x_i)}.\left(\frac{(1+a)\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)^{\mu_{A^c}(x_i)}.\left(1+a\mu_A(x_i)\right)^{-1}.\left(1+a\mu_{A^c}(x_i)\right)^{-1}\Bigg)\Bigg)$$

Hence the result.

**Claim 2.3** The generalized code-word length meets the inequality

$$\frac{1}{R-1} log_D \prod_{i=1}^{n}\left(\left(\frac{1+a\mu_A(x_i)}{(1+a)^{\mu_A(x_i)}.\mu_A(x_i)^{\mu_A(x_i)}}\right).\left(\frac{1+a\mu_{A^c}(x_i)}{(1+a)^{\mu_{A^c}(x_i)}.\mu_{A^c}(x_i)^{\mu_{A^c}(x_i)}}\right).D^{-l_i}\right) \geq$$

$$\frac{1}{R-1} log_D \prod_{i=1}^{n}\left(\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\frac{1}{\mu_B(x_i)^{\mu_A(x_i)}}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_B(x_i)}.\frac{1}{\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}}\right)\right),$$

$$a > 0, R(\neq 1) > 0$$

if the code-word lengths $l_1, l_2, \dots, l_n$ satisfy the requirement

$$\left(\frac{1+a\mu_A(x_i)}{(1+a)\mu_A(x_i)}\mu_A(x_i) + \frac{1+a\mu_{A^c}(x_i)}{(1+a)\mu_{A^c}(x_i)}\mu_{A^c}(x_i)\right)D^{-\frac{l_i}{R-1}} \geq$$

$$\frac{1+a\mu_A(x_i)}{\left(1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}\right).\mu_B(x_i)^{\mu_A(x_i)}} + \frac{1+a\mu_{A^c}(x_i)}{\left(1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right).\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}},$$

$$a > 0, R(\neq 1) > 0$$

for the uniquely decipherable codes $D > 1$.

**Proof:** By Holder's inequality we have

$$\sum_{i=1}^{n} x_i y_i \geq \left(\sum_{i=1}^{n} x_i^{\gamma}\right)^{\frac{1}{\gamma}} \left(\sum_{i=1}^{n} y_i^{\delta}\right)^{\frac{1}{\delta}}$$

For all $x_i, y_i > 0, i = 1,2, \dots, n$ and $\frac{1}{\gamma} + \frac{1}{\delta} 1, \gamma < 1(\neq 0), \delta < 0$ or $\delta < 1(\neq 0), \gamma < 0$. We see the inequality holds iff there exists a positive constant $\mu$ such that $x_i^{\gamma} = \mu y_i^{\delta}$.

Making the substitutions

$$x_i = \left(\left(\frac{1+a\mu_A(x_i)}{(1+a)\mu_A(x_i)^{\mu_A(x_i)}}\right)\left(\frac{1+a\mu_{A^c}(x_i)}{(1+a)\mu_{A^c}(x_i)^{\mu_{A^c}(x_i)}}\right)\right)^{\frac{1}{R-1}} D^{\frac{-l_i}{R-1}} \quad \text{and}$$

$$y_i = \left(\left(\frac{1+a\mu_A(x_i)}{\left(1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)\mu_B(x_i)^{\mu_A(x_i)}}\right)\left(\frac{1+a\mu_{A^c}(x_i)}{\left(1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}}\right)\right)^{\frac{1}{1-R}}$$

Now, $\left(\sum_{i=1}^{n}\left(\left(\left(\frac{1+a\mu_A(x_i)}{(1+a)\mu_A(x_i)^{\mu_A(x_i)}}\right)\left(\frac{1+a\mu_{A^c}(x_i)}{(1+a)\mu_{A^c}(x_i)^{\mu_{A^c}(x_i)}}\right)\right)^{\frac{1}{R-1}}\right)^{R-1} . \left(D^{\frac{l_i}{R-1}}\right)^{R-1}\right)^{\frac{1}{R-1}}.$

$$\left(\sum_{i=1}^{n}\left(\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)} . \frac{1}{\mu_B(x_i)^{\mu_A(x_i)}} . \left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_B(x_i)} . \frac{1}{\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}}\right)^{\frac{1}{1-R}}\right)^{1-R}\right)^{\frac{1}{1-R}}$$

$$\leq \left((1+a)\mu_A(x_i)^{\mu_A(x_i)} . (1+a)\mu_A(x_i)^{\mu_A(x_i)}\right)^{\frac{1}{1-R}}.$$

$$\left(\left(1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)\mu_B(x_i)^{\mu_A(x_i)} . \left(1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}\right)\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}\right)^{\frac{1}{1-R}} D^{\frac{-l_i}{R-1}}$$

i.e. $\left(\sum_{i=1}^{n}\left(\frac{1+a\mu_A(x_i)}{(1+a)^{\mu_A(x_i)}.\mu_A(x_i)^{\mu_A(x_i)}}\right) . \left(\frac{1+a\mu_{A^c}(x_i)}{(1+a)^{\mu_{A^c}(x_i)}.\mu_{A^c}(x_i)^{\mu_{A^c}(x_i)}}\right) . D^{-l_i}\right)^{\frac{1}{R-1}}.$

$$\left(\sum_{i=1}^{n}\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)} . \frac{1}{\mu_B(x_i)^{\mu_A(x_i)}} . \left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_B(x_i)} . \frac{1}{\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}}\right)\right)^{\frac{1}{1-R}} \leq$$

1

i.e. $\left(\sum_{i=1}^{n}\left(\frac{1+a\mu_A(x_i)}{(1+a)^{\mu_A(x_i)}.\mu_A(x_i)^{\mu_A(x_i)}}\right) . \left(\frac{1+a\mu_{A^c}(x_i)}{(1+a)^{\mu_{A^c}(x_i)}.\mu_{A^c}(x_i)^{\mu_{A^c}(x_i)}}\right) . D^{-l_i}\right)^{\frac{1}{R-1}} \geq$

$$\left(\sum_{i=1}^{n}\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)} . \frac{1}{\mu_B(x_i)^{\mu_A(x_i)}} . \left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_B(x_i)} . \frac{1}{\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}}\right)\right)^{\frac{1}{1-R}}$$

Taking logarithm with base $D$ throughout to the above inequality

$$\frac{1}{R-1}log_D\prod_{i=1}^{n}\left(\left(\frac{1+a\mu_A(x_i)}{(1+a)^{\mu_A(x_i)}.\mu_A(x_i)^{\mu_A(x_i)}}\right).\left(\frac{1+a\mu_{A^c}(x_i)}{(1+a)^{\mu_{A^c}(x_i)}.\mu_{A^c}(x_i)^{\mu_{A^c}(x_i)}}\right).D^{-l_i}\right)\geq$$

$$\frac{1}{R-1}log_D\prod_{i=1}^{n}\left(\left(\left(\frac{1+a\mu_A(x_i)}{1+a\frac{\mu_A(x_i)}{\mu_B(x_i)}}\right)^{\mu_B(x_i)}.\frac{1}{\mu_B(x_i)^{\mu_A(x_i)}}.\left(\frac{1+a\mu_{A^c}(x_i)}{1+a\frac{\mu_{A^c}(x_i)}{\mu_{B^c}(x_i)}}\right)^{\mu_B(x_i)}.\frac{1}{\mu_{B^c}(x_i)^{\mu_{A^c}(x_i)}}\right)\right)$$

$a>0, R(\neq 1)>0.$

Hence the result.

**Final Remarks**

When the probability distribution $P$ belongs to the R-norm vector space, the R-norm information measure is defined and described. The family of generalized information measures now includes this new member.

In this investigation, we have established and characterized a new measure called the R-norm information measure, taking into account that physical systems have both quantitative and qualitative characterizations. This metric can be utilized in source coding where source symbols provide utility in addition to frequency of occurrence, and it can be further generalized in numerous ways.

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**Biography**

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