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RESEARCH ARTICLE

INTEGER POINTS ON THE HYPERBOLA $x^2 - 5xy + y^2 + 5x = 0$

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ABSTRACT

The binary quadratic equation $x^2 - 5xy + y^2 + 5x = 0$ representing hyperbola is considered. Different patterns of solutions are obtained. A few interesting recurrence relations satisfied by x and y are exhibited.

Keywords: binary quadratic, hyperbola, integer solutions.

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INTRODUCTION

The binary quadratic equation offers an unlimited field for research because of their variety [1-5]. In this context one may also refer [6-19]. This communication concerns with yet another interesting binary quadratic equation $x^2 - 5xy + y^2 + 5x = 0$ for determining its infinitely many non-zero integral solutions. Also a few interesting relations among the solutions are presented.

2. METHOD OF ANALYSIS:

The hyperbola under consideration is

$$x^2 - 5xy + y^2 + 5x = 0 \qquad (1)$$

Treating (1) as a quadratic in y and solving for y, we get

$$y=\frac{1}{2}[5x\pm\sqrt{21x^2-20x}] \quad (2)$$

Let $\alpha^2=21x^2-20x$ (3)

Substituting $x=\frac{X+10}{21}$ (4)

in (3), we have

$$X^2=21\alpha^2+100 \quad (5)$$

The smallest positive integer solution of (5) is

$\alpha_0=1$ and $X_0=11$

To find the other solution of (5), consider the pellian equation

$$X^2=21\alpha^2+1$$

whose general solution $\left(\overline{X_n},\overline{\alpha_n}\right)$ is given by

$$\overline{X_n}=\frac{1}{2}\left[\left(55+12\sqrt{21}\right)^{n+1}+\left(55-12\sqrt{21}\right)^{n+1}\right]$$

$$\overline{\alpha_n}=\frac{1}{2\sqrt{2}}\left[\left(55+12\sqrt{21}\right)^{n+1}-\left(55-12\sqrt{21}\right)^{n+1}\right]$$

Applying Brahmagupta Lemma between (X_0,α_0) and $\left(\overline{X_n},\overline{\alpha_n}\right)$, the general solutions to (3) are given by,

$$X_{n+1}=X_0\overline{X_n}+21\alpha_0\overline{\alpha_n} \quad (6)$$

$$\alpha_{n+1}=\alpha_0\overline{X_n}+X_0\overline{\alpha_n} \quad (7)$$

Substituting (6) in (4) and employing (2) by considering the positive sign, the corresponding integer solutions to (1) are given by

$$x_{n+1}=\frac{1}{42}\left[11f+\sqrt{21}g+20\right],\quad n=-1,1,3,5,......$$

$$y_{n+1}=\frac{1}{42}\left[38f+8\sqrt{21}g+50\right],\quad n=-1,1,3,5,......$$

where

$$f=\left(55+12\sqrt{21}\right)^{n+1}+\left(55-12\sqrt{21}\right)^{n+1}$$

$$g=\left(55+12\sqrt{21}\right)^{n+1}-\left(55-12\sqrt{21}\right)^{n+1}$$

PROPERTIES:

1) $\frac{1}{25}\left[168x_{2n+2}-21y_{2n+2}-60\right]$ is a perfect square.

2) $\frac{1}{25}\left[168x_{3n+3}-21y_{3n+3}+504x_{n+1}-63y_{n+1}-440\right]$ is a Cubical integer.

3) The following expression is a Bi-quadratic integer

$$\frac{1}{25}\left[168x_{4n+4}-21y_{4n+4}+4\left[28224x_{n+1}^2+441y_{n+1}^2-4620y_{n+1}-7056x_{n+1}y_{n+1}+3690x_{n+1}\right]+1209840\right]$$

4) $7854x_{n+1}-42x_{n+2}\equiv 120 \pmod{1200}$.

5) $863898x_{n+1}-42x_{n+3}\equiv 15380 \pmod{132000}$.

$6) 863898 x_{n+2} - 7854 x_{n+3} \equiv 840 (\text{mod} 1200).$

$7) 4704 y_{n+1} - 42 y_{n+2} \equiv 0 (\text{mod} 150).$

$8) 517398 y_{n+1} - 42 y_{n+3} \equiv 5400 (\text{mod} 16500).$

$9) 4139184 y_{n+2} - 37632 y_{n+3} \equiv 0 (\text{mod} 1200).$

Also, taking the negative sign in (2), the other set of solutions to (1) is given by

$$x_{n+1} = \frac{1}{42}\left(11f + \sqrt{21}g + 20\right), \quad n = -1,1,3,5,......$$

$$y_{n+1} = \frac{1}{42}\left[17f - 3\sqrt{21}g + 50\right], \quad n = -1,1,3,5,......$$

PROPERTIES:

- ❖ $273 y_{n+1} - 21 y_{n+2} \equiv 17 (\text{mod} 21)$.
- ❖ $30051 y_{n+2} - 273 y_{n+3} \equiv 50 (\text{mod} 200)$.
- ❖ $30051 y_{n+1} - 21 y_{n+3} \equiv 845 (\text{mod} 2327)$
- ❖ $\frac{1}{25}\left[63 x_{2n+2} + 21 y_{2n+2} - 5\right]$ is a perfect square.
- ❖ $\frac{1}{25}\left[63 x_{3n+3} + 21 y_{3n+3} + 189 x_{n+1} + 63 y_{n+1} - 220\right]$ is a Cubical integer
- ❖ The following expression is a Bi-quadratic integer

$$\frac{1}{25}\left[63 x_{4n+4} + 21 y_{4n+4}\right] + \frac{4}{25}\left[3969 x_{n+1}^2 + 441 y_{n+1}^2 + 2646 x_{n+1} y_{n+1} - 2310 y_{n+1} - 6930 x_{n+1}\right] - \frac{20}{25}$$

3. CONCLUSION: As the binary quadratic equations are rich in variety, one may consider other choices of hyperbolas and search for their non-trivial distinct integral solutions along with the corresponding properties.

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