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ON THE TERNARY CUBIC DIOPHANTINE EQUATION $X^2+Y^2-XY=12^{2n}Z^3$

M.A.GOPALAN[1], S.DIVYA[2*]

[1]Professor, Department of Mathematics , Shrimati Indira Gandhi College,Trichy, Tamilnadu , India.
[2]M. Phil student, Department of Mathematics, Shrimati Indira Gandhi College, Trichy, Tamilnadu , India.

*** S.DIVYA**
Author for Correspondence

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ABSTRACT

The ternary cubic Diophantine equation is analyzed for its infinitely many non-zero distinct integral solutions. A few interesting properties among the solutions are presented.

Keywords: Ternary cubic, integral solutions.

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INTRODUCTION

Integral solutions for the cubic homogeneous or non-homogeneous Diophantine equations are an interesting concept, as it can be seen from [1, 2, 3]. In [4-17] a few special cases of ternary cubic Diophantine equations are studied. In this communication, we present the integral solutions of yet another ternary cubic equation $x^2+y^2-xy=12^{2n}z^3$. A few interesting relations between the solutions are obtained.

NOTATIONS:

$T_{m,n}$: Polygonal number

P_n^m : Pyramidal number

Pr_n : Pronic number

METHOD OF ANALYSIS:

The cubic equation under consideration is

$$x^2 + y^2 - xy = 12^{2n} z^3 \qquad (1)$$

Assuming $x = u + v$, $y = u - v$, $u \neq v$ (2)

in (1), it is written as

$$u^2 + 3v^2 = 12^{2n} z^3 \qquad (3)$$

Here, we present two different choices of solutions of (3) and hence, obtain two different patterns of solutions to (1).

Assume $z = z(a,b) = a^2 + 3b^2$, $a,b \neq 0$ (4)

PATTERN: 1

Write 12 as

$$12 = (3 + i\sqrt{3})(3 - i\sqrt{3}) \qquad (5)$$

Using (4) and (5) in (3) and employing the method of factorization, define

$$(u + i\sqrt{3}v) = (3 + i\sqrt{3})^{2n} (a + i\sqrt{3}b)^3 \qquad (6)$$

Where in we write

$$(3 + i\sqrt{3})^{2n} = (\alpha + i\beta) \qquad (7)$$

On comparing real and imaginary parts on both sides of (6), we get

$$u = \alpha(a^3 - 9ab^2) + \beta(9b^3 - 9a^2b)$$

$$v = \alpha(3a^2b - 3b^3) + \beta(a^3 - 9ab^2)$$

Substituting the values of u and v in (2), we get

$$x = x(a,b) = (a^3 - 9ab^2)(\alpha + \beta) + (3a^2b - 3b^3)(\alpha - 3\beta)$$

$$y = y(a,b) = (a^3 - 9ab^2)(\alpha - \beta) + (3b^3 - 3a^2b)(\alpha + 3\beta)$$

For simplicity, taking n=1 in (7), the corresponding integer solutions to (1) are found to be

$$x = x(a,b) = 12a^3 - 108ab^2 - 36a^2b + 12b^3$$

$$y = y(a,b) = 72b^3 - 72a^2b$$

along with (4).

Properties:

- $x(a,1) - y(a,1) - 72P_a^3 \equiv 0 (mod\, 6)$
- $x(a,1) - 72P_a^3 + 144T_{3,a} \equiv 0 (mod\, 12)$
- $y(1,b) = 432P_{b-1}^3$
- $y(a,-1) - 72T_{4,a} \equiv 0 (mod\, 72)$
- $z(a,1) - y(a,1) - 73T_{4,a} \equiv 0 (mod\, 3)$

PATTERN: 2

Here, instead of (7), assume

$$(3 + i\sqrt{3}) = r(\cos\theta + i\sin\theta) \qquad (8)$$

Following the procedure as in pattern: 1, employing the method of factorization, define

$$(u + i\sqrt{3}v) = 12^n \left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)(a + i\sqrt{3}b)^3 \qquad (9)$$

On comparing the real and imaginary parts, we get

$$u = 12^n \cos\frac{n\pi}{3}(a^3 - 9ab^2) + 3\sqrt{3}(12^n \sin\frac{n\pi}{3})(b^3 - a^2b)$$

$$v = 12^n \sin\frac{n\pi}{3}(\frac{\sqrt{3}a^3}{3} - 3\sqrt{3}ab^2) + (12^n \cos\frac{n\pi}{3})(3a^2b - 3b^3)$$

Substituting the values of u and v in (2), we get

$$x = x(a,b) = 12^n \cos\frac{n\pi}{3}[a^3 - 9ab^2 - 3b^3 + 3a^2b] + 12^n \sin\frac{n\pi}{3}[3\sqrt{3}b^3 - 3\sqrt{3}a^2b + \frac{\sqrt{3}a^3}{3} - 3\sqrt{3}ab^2]$$

$$y = y(a,b) = 12^n \cos\frac{n\pi}{3}[a^3 - 9ab^2 + 3b^3 - 3a^2b] + 12^n \sin\frac{n\pi}{3}[3\sqrt{3}b^3 - 3\sqrt{3}a^2b - \frac{\sqrt{3}a^3}{3} + 3\sqrt{3}ab^2]$$

For simplicity, putting n=3 in the above equations, the corresponding integer solutions to (1) are given by

$$x = x(a,b) = -1728a^3 + 15552ab^2 + 5184b^3 - 5184a^2b$$

$$y = y(a,b) = -1728a^3 + 15552ab^2 - 5184b^3 + 5184a^2b$$

along with (4)

Properties:

- $3\{y(a,1) - x(a,1)\}$ is a nasty number.
- $x(-a,1) - 3456P_a^5 + 13824T_{3,a} \equiv 0 (mod\, 12)$
- $y(-1,b) - 15552P_b^4 + 23328Pr_b \equiv 0 (mod\, 2)$
- $x(-a,1) - 1728P_a^3 + 10368Pr_a \equiv 0 (mod\, 12)$
- $z(2,b) - 3T_{4,b} \equiv 0 (mod\, 4)$

CONCLUSION: To conclude, we may search for other patterns of solutions to (1) along with their properties.

REFERENCES

[1] Dickson. L. E., History of the Theory of Numbers, Vol. 2, Diophantine Analysis, New York, Dover, (2005) .

[2] Mordell. L. J., Diophantine Equations, Academic Press, New York, 1969.

[3] Carmichael. R. D., The Theory of Numbers and Diophantine Analysis, New York, Dover, (1959) .

[4] Gopalan. M. A., Anbuselvi. R., Integral solutions of Ternary Cubic Diophantine Equations $x^2 + y^2 + 4N = zxy$, Pure and Applied Mathematics Sciences, Vol. LXVII (No. 1-2): 107-111, March(2008) .

[5] Gopalan. M.A., Manju Somnath and Vanitha, N., On ternary Cubic Diophantine Equations $x^2 + y^2 = 2z^3$, Advances in Theoretical and Applied Mathematics, Vol.1 (No. 3) Pg. 227-231 (2006) .

[6] Gopalan. M.A., Manju Somnath and Vanitha, N., On ternary Cubic Diophantine Equations $x^2 - y^2 = z^3$, ActaCiencia Indica, Vol.XXXIIIM (NO.3) Pg. 705-707 (2007) .

[7] Gopalan. M.A., and Pandichelvi. V., Integral solutions of Ternary Cubic Equation $x^2 - xy + y^2 = (k^2 - 2k + 4)z^3$, Pacific Asian Journal of Mathematics, Vol.2, (No. 1-2),January-December (2008) Pg. 91-96.

[8] Gopalan. M.A., and Kalinga Rani. J., Integral solutions of $x^2 - ay^2 = (a-1)z^3$ ($a-1$ and a are square free), Impact J. Sci. Tech., Vol.2(4) Pg.201-204.

[9] Gopalan. M.A., Devibala. S., and Manju Somnath., Integral solutions of $x^3 + x + y^3 + y = 4z(z-2)(z+2)$, Impact J. Sci. Tech., Vol.2(2) Pg. 65-69 .

[10] Gopalan. M.A., Manju Somnath and Vanitha. N., Ternary Cubic Diophantine Equation $2^{2a-1}(x^2+y^2)=z^3$, ActaCiencia Indica, Vol.XXXIVM, (NO.3) Pg. 113-137 (2008) .

[11] Gopalan. M.A., and Kalinga Rani. J., Integral solutions of $x^3+y^3+8k(x+y)=(2k+1)z^3$ Bulletin of Pure and Applied Sciences, Vol. 29E(No.1), 95-99(2010) .

[12] Gopalan. M.A., and Janaki. G., Integral solutions of $x^2-y^2+xy=(m^2-5n^2)z^3$, Antarctica J. math., 7(1), 63-67 (2010) .

[13] Gopalan. M.A., and Shanmuganandham. P., On the Equation $x^2-y^2+xy=(n^2+4n-1)z^3$, Bulletin of pure and Applied sciences, Vol. 29E, Issue 2(2010) Pg. 231-235 .

[14] Gopalan. M.A., and Vijayalakshmi A., Integral solutions of ternary cubic Equation $x^2+y^2-xy+2(x+y+z)=z^3$, Antarctica Journal of Mathematics, Vol.7, (No.4), (2010), 455-460.

[15] Gopalan. M.A., and Kalinga Rani. J., Integral solutions of Ternary Cubic Equation $x^3+y^3+4z^2=3xy(x+y)$, Antarctica Journal of Mathematics, 7(3), (2010), 311-315.

[16] Gopalan. M.A., and Pandichelvi. V., On the Ternary Cubic Equation $y^2+gz^2=(k^2+g)x^3$, Impact J. Sci. Tech., Vol.4 No.4,(2010), 117-123.

[17] Gopalan. M.A., and Geetha. K., on the ternary cubic Diophantine equation $x^2+y^2-xy=z^3$, Bessel J.Math., 3(2),(2013), 119-123.