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RESEARCH ARTICLE

INTEGRAL SOLUTIONS OF TERNARY QUADRATIC DIOPHANTINE EQUATIONS $7X^2+2Y^2=135Z^2$

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ABSTRACT

The ternary quadratic homogeneous equation representing a cone given by $7X^2+2Y^2=135Z^2$ is analyzed for its non-zero distinct integer points on it. Six different patterns of integer solutions satisfying the cone under consideration are given. A few interesting relations between the solutions and special number patterns are presented. Given an integral solution on the considered cone, three triples of integers generated from the given solution are exhibited.

Keywords: Ternary quadratic ,integral solutions

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Notations:

P_n^m : Pyramid number of rank n with size m

$T_{m,n}$: Polygonal number of rank n with size m

INTRODUCTON

The ternary quadratic Diophantine equations offer an unlimited field for research due to their variety [1,20]. For an extensive review of various problems. One may refer [2,19]. This communication concerns with yet another interesting ternary quadratic equation representing $7X^2+2Y^2=135Z^2$ a cone for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions and special numbers are presented

2. METHOD OF ANALYSIS: The ternary quadratic equation to be solved to be given by,

$$7x^2 + 2y^2 = 135z^2 \quad (1)$$

It is seen that (1) is satisfied by (391, -1478, 289) (117,-412, 57) and (133,-146, 59) However, we have other choices of solutions which are presented below.

Introducing the linear transformations,

$$x = X + 2T \text{ , } y = X - 7T \quad (2)$$

in (1), it is written as,

$$x^2 + 14t^2 = 15z^2 \quad (3)$$

2.1 choice: 1

Let

$$z = a^2 + 14b^2 \text{ , } a, b \neq 0 \quad (4)$$

Write 15 as,

$$15 = (1 + i\sqrt{14})(1 - i\sqrt{14}) \quad (5)$$

Substituting (4) and (5) in (3) and employing the method of factorization, define

$$(x + i\sqrt{14t}) = (1 + i\sqrt{14})(a + i\sqrt{14}b)^2$$

Equating real and imaginary parts, we get

$$X = X(a,b) = a^2 - 14b^2 - 28ab \quad (6)$$

$$T = T(a,b) = a^2 - 14b^2 + 2ab$$

Using (6) in (2) we have,

$$\left.\begin{aligned} x &= 3a^2 - 42b^2 - 24ab \\ y &= -6a^2 + 84b^2 - 42ab \end{aligned}\right\} \quad (7)$$

Thus (7) and (4) represent the non-zero distinct integral solutions of (1).

Properties:

- $x(,1) - T_{8,a} \equiv -20(Mod 22)$
- $x(a,1) + z(a,1) - T_{10,a} \equiv -7(\text{mod} 21)$
- $z(1,b) - T_{30,b} \equiv 1(Mod 13)$
- $2\{x(a, a(a+1)) + 168T_{3,a}^2 + 48p_a^5\}$ is a Nasty number

2.2 Choice:2

Equation (3) can be written as,

$$x^2 = 15z^2 - 14t^2 \quad (8)$$

Take $z = \alpha + 14\beta$, $t = \alpha + 15\beta$ (9)

Substituting (9) in (8) it becomes,

$$\alpha^2 = 210\beta^2 + x^2 \quad (10)$$

Which is satisfied by,

$$\beta = 2pq \text{ , } x = 210p^2 - q^2 \text{ , } \alpha = 210p^2 + q^2 \quad (11)$$

Thus, from (11), (9) and (2) represents the corresponding integral solutions of (1) are,

$$\left.\begin{aligned} x &= 630p^2 + q^2 + 60pq \\ y &= -1260p^2 - 8q^2 - 210pq \\ z &= 210p^2 + q^2 + 28pq \end{aligned}\right\} \quad (12)$$

Properties:

- $z(p,1)-T_{202,p}-T_{222,p}\equiv 1(\text{mod}236)$
- $x(p,1)-T_{642,p}-T_{622,p}\equiv 1(\text{mod}688)$
- $z(p,2)-T_{422,p}\equiv 4(\text{mod}56)$
- $T_{-14,q}-y(1,q)\equiv 165(Mod219)$

2.3 Choice:3

Equation (2) is rewritten in the form of ratio as,

$$\frac{X+Z}{Z-T}=\frac{14(Z+T)}{X-Z}=\frac{P}{Q},Q\neq 0 \qquad (13)$$

which is equivalent to the following two equations.

$$QX+Z(Q-P)+PT=0$$
$$PX-Z(14Q+P)-14QT=0$$

Employing the method of cross multiplication, we get

$$\left.\begin{aligned} X&=P^2-14Q^2+28PQ \\ T&=P^2-14Q^2-2PQ \end{aligned}\right\} \qquad (14)$$
$$Z=P^2+14Q^2 \qquad (15)$$

Using the values of X and T in (2), we have

$$\left.\begin{aligned} x&=3P^2-42Q^2+24PQ \\ y&=-6P^2+84Q^2+42PQ \end{aligned}\right\} \qquad (16)$$

Thus (16), (15) represent the non-zero integral solutions of (1).

Properties:

- $y(p,1)-T_{-10,p}\equiv 14(\text{mod}35)$
- $x(p,1)-T_{8,p}\equiv -16(\text{mod}26)$
- $2x(p,p+1)+y(p,p+1)-180T_{422,p}=0$
- $6\{z(p,p(p+1))-28T_{3,p}^2\}$ is a Nasty number.

Note:

Equation (2) is also written in the form of ratio in three more ways as below,

$$1.\ \frac{X+Z}{14(Z-T)}=\frac{Z+T}{X-Z}=\frac{P}{Q},Q\neq 0 \qquad (17)$$

$$2.\ \frac{X+Z}{2(Z-T)}=\frac{7(Z+T)}{X-Z}=\frac{P}{Q},Q\neq 0 \qquad (18)$$

$$3.\ \frac{X+Z}{7(Z-T)}=\frac{2(Z+T)}{X-Z}=\frac{P}{Q},Q\neq 0 \qquad (19)$$

Following the procedure presented above in pattern 3, are may set 3 more different choices of integer solutions to (1)

Considering (16), the corresponding integer solutions of (1) are

$$x=42p^2-3q^2+24pq$$
$$y=-84p^2+6q^2+42pq$$

$z = 14p^2 + q^2$

1. Properties:

- $2x(a^2, a+1) + y(a^2, a+1) - 180p_a^5 = 0$
- $x(p,1) - T_{42,p} - T_{46,p} \equiv -3 \pmod{64}$
- $x(1,q) - T_{-4,q} \equiv 2 \pmod{20}$
- $z(p,p) - T_{32,p} \equiv \pmod{14}$

Considering (17), the corresponding integer solutions of (1) are

$x = 6p^2 - 21q^2 + 24pq$

$y = -12p^2 + 42q^2 + 42pq$

$z = 2p^2 + 7q^2$

2. Properties:

- $2x(a(a+1), a+2) + y(a(a+1), a+2) - 540p_a^3 = 0$
- $x(p,2) - T_{14,p} \equiv -31 \pmod{53}$
- $y(2,q) - T_{86,q} \equiv -48 \pmod{125}$
- $x(p, p(p+1)) + 84T_{3,p}^2 - 48p_a^5 = 6p^2$ is a Nasty number.

Considering (18), the corresponding integer solutions of (1) are

$x = 21p^2 - 6q^2 + 24pq$

$y = 12p^2 + 12q^2 + 42pq$

$z = 7p^2 + 2q^2$

3. Properties:

- $x(p,1) - T_{22,p} - T_{24,p} \equiv -1 \pmod{5}$
- $z(p,p) - T_{20,p} \equiv 0 \pmod{8}$
- $3\{z(p(p+1), p) - 28T_{3,p}^2\}$ is a Nasty number.
- $2\{y(p, p(p+1)) - 48T_{3,p}^2 - 84p_a^5\}$ is a Nasty number.

3. REMARKABLE OBSERVATION:

Let (x_0, y_0, z_0) be the positive initial solution of (1).Then each of the following three triples of integers based on x_0, y_0, z_0 also satisfy (1).

Triple:1

$$x_n = \frac{1}{2}(\alpha^n + \beta^n)x_0 + \frac{135}{12\sqrt{105}}(\alpha^n - \beta^n)z_0$$

$$y_n = 4^n y_0$$

$$z_n = \frac{7}{12\sqrt{105}}(\alpha^n - \beta^n)x_0 + \frac{1}{2}(\alpha^n + \beta^n)z_0$$

in which

$\alpha = 31 + 6\sqrt{105}$, $\beta = 31 - 6\sqrt{105}$

Triple : 2

$$x_n = 7^n x_0$$

$$y_n = \frac{1}{2}(\alpha^n + \beta^n) y_0 + \frac{11}{\sqrt{30}}(\alpha^n - \beta^n) z_0$$

$$z_n = \frac{7}{6\sqrt{30}}(\alpha^n - \beta^n) y_0 + \frac{1}{2}(\alpha^n - \beta^n) z_0$$

in which

$$\alpha = 263 + 96\sqrt{30}\ , \quad \beta = 263 - 96\sqrt{30}$$

Triple : 3

$$x_n = \frac{1}{18}\{(4\alpha^n + 14\beta^n) x_0 + (-4(\alpha^n + \beta^n)) y_0\}$$

$$y_n = \frac{1}{18}\{(-14(\alpha^n + \beta^n)) x_0 + (14\alpha^n + 4\beta^n) Y_0\}$$

$$z_n = 9^n z_0$$

in which

$$\alpha = +9\ , \quad \beta = -9$$

4. CONCLUSION: In this dissertation, the ternary quadratic Diophantine equations reffering a cone is analysed for is non-zero distinct integral Points. A few interesting properties between the solutions and special numbers are presented. To conclude, one may search for other patterns of solutions and their corresponding properties for the cone under consideration

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