Vol.3.Issue.1.2015

http://www.bomsr.com

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

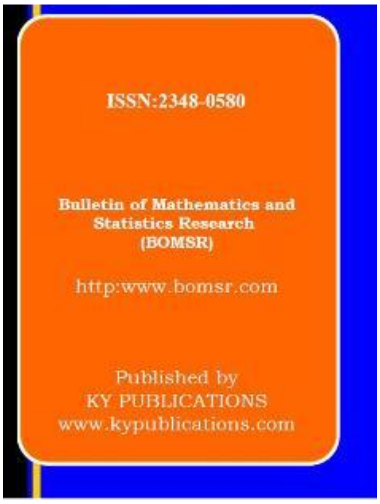
ISSN:2348-0580

RESEARCH ARTICLE

INTEGRAL SOLUTIONS OF THE BINARY QUADRATIC EQUATION $x^2-3xy+y^2+5x=0$

S.VIDHYALAKSHMI[1], M.A.GOPALAN[2], T.R.USHARANI[*3]

[1,2,3]Department of Mathematics,Shrimati Indira Gandhi College,Trichy.



ABSTRACT

The binary quadratic equation $x^2-3xy+y^2+5x=0$ represents a hyperbola. In this paper we obtain a sequence of its integral solutions and present a few interesting relations among them.

Keywords :Binary quadratic equation, Integral solutions.

MSC subject classification: 11D09.

Article Info:
Article received :16/12/2014
Revised on:28/12/2014
Accepted on:02/01/2015

INTRODUCTION

The binary quadratic Diophantine equations (both homogeneous and non homogeneous) are rich in variety [1-6]In [7-16] the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solutions. These results have motivated us to search for infinitely many non-zero integral solutions of an another interesting binary quadratic equation given by $x^2-3xy+y^2+5x=0$ The recurrence relations satisfied by the solutions x and y are given. Also a few interesting properties among the solutions are exhibited.

METHOD OF ANALYSIS:

The Diophantine equation under consideration is

$$x^2-3xy+y^2+5x=0 \tag{1}$$

It is to be noted that (1) represents a hyperbola.

Pattern1:

By shifting the origin to the centre (2,3), (1) reduces to

$$X^2 - 3XY + Y^2 + 5 = 0 \tag{2}$$

where $x = X - 2, y = Y - 3$ (3)

Again setting

$$X = M + N, Y = M - N \tag{4}$$

in (2) it simplifies to the equation

$$M^2 = 5N^2 + 5 \tag{5}$$

Now, consider the Pellian equation

$$M^2 = 5N^2 + 1 \tag{6}$$

whose general solution $(\tilde{M}_n, \tilde{N}_n)$ is given by

$$\tilde{M}_n = \frac{f}{2} \text{ and } \tilde{N}_n = \frac{g}{2\sqrt{5}}$$

in which

$$f = \left[\left(9+4\sqrt{5}\right)^{n+1} + \left(9+4\sqrt{5}\right)^{n+1}\right],$$

$$g = \left[\left(9+4\sqrt{5}\right)^{n+1} - \left(9+4\sqrt{5}\right)^{n+1}\right] \quad n = -1,0,1,2,3$$

Applying Brahmagupta lemma between the solutions of (x_0, y_0) an $(\tilde{M}_n, \tilde{N}_n)$, the general solutions of (5) is found to be

$$\begin{aligned} M_{n+1} &= M_0\tilde{M}_n + \sqrt{D}N_0\tilde{N}_n = \frac{5f}{2} + \sqrt{5}g \\ N_{n+1} &= N_0\tilde{M}_n + M_0\tilde{N}_n = \frac{5f}{2} + \frac{g\sqrt{5}}{2} \end{aligned} \tag{7}$$

$$n = -1,0,1,2,3$$

Taking advantage of (3), (4) and (7), the sequence of integral solutions of (1) can be written as

$$\left.\begin{aligned} x_{n+1} &= \frac{7f}{2} + \frac{3}{2}\sqrt{5}g + 2 \\ y_{n+1} &= \frac{3f}{2} + \frac{g\sqrt{5}}{2} + 3 \end{aligned}\right\} \tag{8}$$

$$n = -1,0,1,2,3$$

A few numerical examples are given below:

n	x_{n+1}	y_{n+1}
-1	9	6
0	225	50
1	2209	846
2	39605	15130
3	710649	271446
4	12752045	4870850

A few interesting properties satisfied by the solutions(8) are given below:

1.The values of x are odd and y are even and both values are positive.

2. $x_{2n+1} \equiv 0 \pmod 5$

3. $x_{n+1} + y_{n+1} \equiv 0 \pmod 5$

4.Each of the following is a nasty number:

(a) $6\left(3y_{2n+2} - x_{2n+2} - 5\right)$

(b) $30[5\left(3y_{n+1} - x_{n+1} - 7\right)^2 - \left(3x_{n+1} - 7y_{n+1} + 15\right)^2]$

5. $3y_{3n+3} - x_{3n+3} + 9y_{2n+1} - 3x_{n+1} - 28$ is a cubical integer.

6. $\left(3x_{2n+2} - 7y_{2n+2} + 15\right)^2 + 20\left(3y_{2n+2} - x_{2n+2} - 5\right)$ is 5 times a biquadratic integer.

7. $3y_{4n+4} - x_{4n+4} + 4(3y_{2n+2} - x_{2n+2}) - 29$ is a biquadratic integer.

Remarkable observations:

I: By considering suitable linear transformations between the solutions of (1), one may get integer solutions for the Parabola and Hyperbola

(a) It is to be noted that the Parabola

$$X^2 = Y$$

is satisfied for the following set of values of X and Y

$$X = \left(3y_{2n+2} - x_{2n+2} - 5\right)$$

$$Y = 3y_{4n+4} - x_{4n+4} + 4(3y_{2n+2} - x_{2n+2}) - 29$$

(b) The Parabola

$$Y^2 = 5X - 20$$

is satisfied for the following set of values of X and Y

$$X = \left(3y_{2n+2} - x_{2n+2} - 5\right)$$

$$Y = 3x_{n+1} - 7y_{n+1} + 15$$

(c) The Parabola

$$Y^2 = X + 4$$

is satisfied for the following set of values of X and Y

$$X = 3y_{2n+2} - x_{2n+2} - 9$$

$$Y = 3x_{n+1} - 7y_{n+1} - 7$$

II The Hyperbola

$$5X^2 - Y^2 = 20$$

is satisfied for the following set of values of X and Y

$$X = 3y_{n+1} - x_{n+1} - 7$$

$$Y = 3x_{n+1} - 7y_{n+1} + 15$$

Pattern2:

Treating (1) as a quadratic in x, the distinct non-zero integral solutions of (1) are given by

$$x_{n+1} = \frac{1}{2}[3y_{n+1} - \alpha_{n+1} - 5]$$

where

$$\alpha_{n+1}=\frac{5f}{2}+\frac{3}{2}\sqrt{5}g$$

$$y_{n+1}=\frac{3f}{2}+\frac{g\sqrt{5}}{2}+3$$

A few numerical examples are given below:

n	x_{n+1}	y_{n+1}
-1	4	6
0	20	50
1	324	846
2	5780	15130

Pattern3:

Treating (1) as a quadratic in y and solving the distinct non-zero integral solutions of (1) are obtained as

$$y_{n+1}=\frac{1}{2}[3x_{n+1}+\alpha_{n+1}]$$

where

$$\alpha_{n+1}=\frac{15f}{2}+\frac{35g}{2\sqrt{5}}$$

$$x_{n+1}=\frac{7f}{2}+\frac{15g}{2\sqrt{5}}+2$$

A few numerical examples are given below:

n	x_{n+1}	y_{n+1}
-1	9	21
0	125	325
1	2209	5781
2	39605	103685

The above values of x_n and y_n in the above patterns satisfy respectively the following recurrence relations.

$$x_{n+3}-18x_{n+2}+x_{n+1}+32=0$$

$$y_{n+3}-18y_{n+2}+y_{n+1}+48=0'$$

$$n=-1,0,1,2....$$

CONCLUSION

In this paper, we have made an attempt to obtain a complete set of non-trival distinct solutions for the non-homonegeneous binary quadratic equation .To conclude, one may search for other choices of solutions to the considered binary equation and further, quadratic equations with multi-variables.

REFERENCES

[1]. Carmichael, R.D., The Theory of Numbers and Diophantine Analysis, Dover Publications, New York (1950).
[2]. Dickson. L. E., History of The Theory of Numbers, Vol.II, Chelsia Publicating Co, New York(1952)
[3]. Mordell,L,J.Diophantine Equations, Acadamic Press,London(1969).
[4]. Telang ,S.G., Number theory, Tata Mc Graw-Hill Publishing Company , NewDelhi(1996)

[5]. Nigel,P.Smart.,TheAlgorithm Resolutions of Diaphantine eqations,Cambridge University, Press, London(1999).

[6]. Banumathy.T.S.,A Modern Introduction to Ancient Indian Mathematics,Wiley Eastern Limited, London(1995)

[7]. Mollion,R.A, "All Solutions of the Diophatine Equations $X^2 - DY^2 = n$" Far EastJ,Math.Sci., Speical Volume,Part III,p.257-293(1998).

[8]. Gopalan.M.A., and Janaki.G., "Observations on $x^2 - y^2 + x + y + xy = 2$", Impact J.Sci.,Tech, Vol2(3)p.14, 3-148(2008).

[9]. Gopalan.M.A., and., Shanmuganadham,P.,and Vijayashankar,A., "On Binary Quadratic Equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$", Acta Ciencia Indica,Vol . XXXIVM. No.4,p.1803-1805(2008)

[10]. Gopalan,M.A.,Gokila,K.,andVidhyalakahmi,S., "On the Diophantine Equation $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$", ActaCienciaIndica, Vol.XXXIIIM No2, p. 567-570, (2007)

[11]. Gopalan,M.A., and Parvathy,G., "Integral Points On The Hyperbola $x^2 + 4xy + y^2 - 2x - 10y + 24 = 0$,"Antarctica J.Math,Vol 1(2),149-155, (2010).

[12]. Gopalan,M.A, Vidhyalakahmi,S, Sumathi.G and Lakshmi.K, ""Integral Pionts On The Hyperbola $x^2 + 6xy + y^2 + 40x + 8y + 40 = 0$" , Bessel J.Math. Vol 2(3), 159-164, Sep (2010).

[13]. Gopalan,M.A.,Vidhyalakahmi,S.,andDevibala,S.,On The Diophantine Equation $3x^2 + xy = 14$,Acta CienciaIndica,Vol.XXXIII M.No2,P.645-646(2007)

[14]. Gopalan,M.A, Vidhyalakahmi,S, Lakshmi.K and Sumathi.G, " Observation on $3x^2 + 10xy + 4y^2 - 4x + 2y - 7 = 0$" , Diophantus J.Maths.Vol.1(2),123-125,(2012)

[15]. Vidhyalakahmi,S, Gopalan,M.A and Lakshmi.K, " Observation On The Binary Quadratic Equation $3x^2 - 8xy + 3y^2 + 2x + 2y + 6 = 0$", Scholar Journal of Physics, Mathematics and Statistics, Vol.1(2), (Sep-Nov), 41-45, (2014).

[16]. Vidhyalakahmi. S, Gopalan,M.A and Lakshmi.K, "Integer Solution of the Binary Quadratic Equation $x^2 - 5xy + y^2 + 33x = 0$", International Journal of Innovative ScienceEngineering &Technology, Vol.1(6), 450-453, August(2014).