Vol.3.Issue.2.2015

http://www.bomsr.com

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

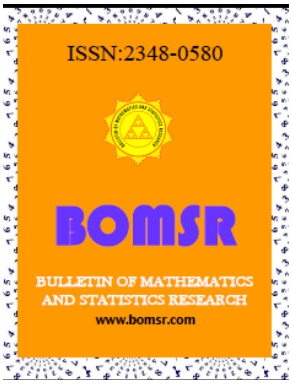
ISSN:2348-0580

RESEARCH ARTICLE

ON CONTRA δ-PRECONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

SANJOY KUMARBISWAS[1*], NASIMA AKTHER[2]

[1,2]Department of Mathematics, University of Rajshahi, Rajshahi, Bangladesh



ABSTRACT

In this paper,we introduce the notion of contra δ-precontinuous functions in bitopological spaces. Further we obtain a characterization and preservation theorems for contra δ-precontinuous functions in bitopological spaces.

Keywords: Contra precontinuous functions, contra-continuous functions, RC- continuous functions, perfectly continuous functions, bitopologicalspaces,contra δ-precontinuous functions.

1. INTRODUCTION

The notion of contra-continuous functions (Donchev 1996[1]), perfectly continuous functions (Noiri 1984a[9]), contra precontinuous functions(Jafari and Noiri 2002[6]) or RC- continuous functions due to (Donchev and Noiri 1999[2]) plays a significant role in general topology. In this paper, we introduce and study the notion of weak form of strong continuity, RC-continuity, perfectly continuity, contra- precontinuity and contra continuity in bitopological spaces. Also investigated the relationships between graphs and contra δ-precontinuous functions in bitopological spaces, which is a generalization of [16].

2.PRELIMINARIES

In this paper, the spaces (X,T_1,T_2) and (X,T) denote respectively the bitopological space and topological space.

Let (X,T_1,T_2) be a bitopological space and let A be a subset of X, then the closure and interior of A with respect to $T_i$are denoted by iCl(A) and iInt(A) respectively , for i = 1,2 .

Definition 2.1:A subset Aof a bitopological space (X,T_1,T_2) is said to be

(i) (i,j)- regular open [13] if A = iInt(jCl(A)) where $i\neq j$, i,j = 1,2.

(ii) (i,j)-regular closed [14] if A = iCl(jInt(A)) where $i\neq j$, i,j = 1,2.

(iii) (i,j)- preopen [15] if $A\subset$iInt(jCl(A)) where $i\neq j$, i,j = 1,2.

(iv) (i,j)- semi-open [14] if $A\subset$jCl(iInt(A)) where $i\neq j$, i,j = 1,2.

Remark 2.1: From above definition 2.1, we have (i)$\Rightarrow$(iii) and (ii)$\Rightarrow$(iv) but converse are not true. For these we have shown the following example.

Example 2.1: Let X = {a,b,c,d} with topologies T_1={X, ϕ ,{a},{b,c}}, T_2={X, ϕ ,{b},{c,d}} and A = {c,d}be a subset of X. Then jCl(A) = {a,c,d}and iInt(jCl(A)) = {a}. Therefore iInt(jCl(A)) $\not\subset$ A. Hence (iii)does not imply(i).

Again, let A = {a,b}be a subset of X. Then jInt(A) = {b}and iCl(jInt(A)) = {b,c,d}. Therefore iCl(jInt(A)) $\not\subset$ A. Hence (iv)does not imply(ii).

Definition 2.2: A subset Aof a bitopological space (X,T_1,T_2) is said to be

(i) The union of all(i,j)- regular open sets of X contained in A is called (i,j)- δ - interior of a subset Aof X and is denoted by (i,j)- δ -(Int(A))(Velicko 1968[12]).

(ii) A is called (i,j)- δ -open if A = (i,j)- δ -(Int(A)) (Velicko 1968[12]).

(iii) The complement of a (i,j)- δ -open set is called (i,j)- δ -closed . Equivalently, A is (i,j)- δ -closed iff A = (i,j)- δ -(Cl(A)) where (i,j)- δ -(Cl(A)) = {x$\in$X: A$\cap$U$\neq\varphi$, U is (i,j)- δ -open, x$\in$U}

(iv) A subset A of X is said to be (i,j)- δ -preopen if A $\subset$iInt((i,j) δ -Cl(A)). The family of all (i,j)- δ -preopen sets of X containing a point $x \in$X is denoted by (i,j)- δ PO(X,x)(M. et al.1982, R and M 1993[9]).

(v) The complement of a (i,j)- δ -preopen set is called (i,j)- δ -preclosed(El-Deeb et al. 1983[4]) .

(vi) The intersection of all(i,j)- δ -preclosed sets of X containing A is called the (i,j)- δ -preclosure of A and is denoted by (i,j)- δ -p(Cl(A)).

(vii) The union of all(i,j)- δ -preopen sets of X contained in A is called the (i,j)- δ -preinterior of A and is denoted by (i,j)- δ -p(Int(A))(Raychoudhuri and Mukherjee 1993[9]).

(viii) Asubset U of X is said to be (i,j)- δ -pre neighbourhood (Raychoudhuri and Mukherjee 1993[9]) of a point $x \in$X if $\exists$ a (i,j)- δ -preopen set V such that $x \in$ V$\subset$U .

(ix) The family of all (i,j)- δ -open (resp. (i,j)- δ -preopen, semi-open, (i,j)- δ -preclosed , (i,j)-closed) sets of X containing a point $x \in$X is denoted by (i,j)- δ O(X,x) (resp. (i,j)- δ PO(X,x), (i,j)-SO(X,x) , (i,j)- δ PC(X,x),(i,j)-C(X,x)).

Definition 2.3:A function f: (X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) is said to be

(i) (i,j)-perfectly continuous ([2], Noiri 1984 a, N and P. 2007[8]) if f^{-1}(V) is T_i-clopen in X for each σ_i- open set V of Y, for i= 1,2.

(ii) (i,j)-contra-continuous (Dontchev 1996[1]) if f^{-1}(V) is T_i-closed in X for each σ_i- open set V of Y, for i, = 1,2.

(iii) (i,j)-RC- continuous (Dontchev and Noiri 1999[2]) if f^{-1}(V) is (i,j)- regular closed in X for each σ_i- open set V of Y, for i$\neq$ j, i,j = 1,2.

(iv) (i,j)-contra-precontinuous (Jafari and Noiri 2002[6]) if f^{-1}(V) is (i,j)- pre- closed in X for each σ_i- open set V of Y, for i$\neq$ j, i,j = 1,2.

(v) (i,j)-strongly- continuous (Levine 1960[7]) if f(iCl(jInt(A))) $\subset$ f(A) for every subset A of X .

3.Contra δ -precontinuous functions in bitopological spaces

Definition 3.1:A function f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) is said to be (i,j)-contra-δ -precontinuous at a point $x \in$X if for each σ_i- closed set V in Y with f(x) $\in$ V , $\exists$ a (i,j)- δ -preopen set U in X such that

$x \in$ U and f(U) $\subset$ V and f is called (i,j)-contra-δ -precontinuous if it has this property at each point of X .

Theorem 3.1: The following are equivalent for a function f: (X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) :

(i) f is(i,j)-contra-δ -precontinuous ;

(ii) the inverse image of a σ_{i}- closed set, i = 1,2 of Y is (i,j)- δ -preopen ;

(iii) the inverse image of a σ_{i}- open set, i = 1,2 of Y is (i,j)- δ -preclosed ;

Proof:(i) $\Rightarrow$(ii) . Let V be a σ_{i}- closed set, i = 1,2 in Y with $x \in f^{-1}$(V) . Since f(x) $\in$ V and f is (i,j)-contra-δ -precontinuous , $\exists$ a (i,j)- δ -preopen set U in X containing x such that f(U) $\subset$ V . It follows that $x \in$U$\subset f^{-1}$(V) . Hence f^{-1}(V) is (i,j)- δ -preopen.

(ii) $\Rightarrow$(iii) . Let U be a σ_{i}- open set, i = 1,2 of Y . Since Y\U is σ_{i}- closed , then by (ii) it follows that f^{-1}(Y\U) = X\ f^{-1}(U) is (i,j)- δ -preopen. Therefore f^{-1}(U) is (i,j)- δ -preclosed in X .

(iii) $\Rightarrow$(i) . Let $x \in$X and V be a σ_{i}- closed set, i = 1,2 in Y with f(x) $\in$ V . By (iii) , we have f^{-1}(Y\V) = X\ f^{-1}(V) is (i,j)- δ -preclosed and so f^{-1}(V) is (i,j)- δ -preopen . Let U = f^{-1}(V). We obtain that $x \in$U and f(U) $\subset$ V . This shows that f is(i,j)-contra-δ -precontinuous.

Remark 3.1: The following diagram holds:

(i,j)-strongly- continuous

$\Downarrow$

(i,j)-perfectly continuous

$\Downarrow$

(i,j)-RC- continuous

$\Downarrow$

(i,j)-contra-continuous

$\Downarrow$

(i,j)-contra-precontinuous

$\Downarrow$

(i,j)-contra-δ -precontinuous

None of these implications are reversible. For these we have shown the following examples.

Example 3.1: Let, X = {a,b,c,d} and T_1 = {X,ϕ,{a},{b,c}},T_2={X,ϕ,{b},{c,d}}. Let, f:(X,T_1,T_2)$\rightarrow$(X,T_1,T_2) be the identity function. Then f is (i,j)-perfectly continuous but not (i,j)-strongly- continuous. For, let A = {a.b} be a subset of X and f(A) = A, then f(iCl(jInt(A)))$\not\subset$ f(A.

Example 3.2: Consider the topologies on X = {a,b,c} and Y = {p,q} respectively by

T_1 = {X,ϕ,{b},{a,c}},T_2={X,ϕ,{a},{b},{a,b},{b,c}} and σ_1 = {Y,ϕ,{p}}, σ_2 = {Y,ϕ,{q}}. Let, f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) be a map defined as f(a) =p, f(b) = q, f(c) = p. Then f is (i,j)-RC- continuous but not (i,j)-perfectly continuous , since f^{-1}(p) and f^{-1}(q) are clopen in T_1 but not in T_2.

Example 3.3: Consider the topologies on X = {a, b, c} and Y = {p, q, r} respectively by

T_1 = {X,ϕ,{c},{b, c}},T_2={X,ϕ,{a},{c},{a, c}} and σ_1 = {Y,ϕ,{p}}, σ_2 = {Y,ϕ,{p, q}}. Let, f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) be a map defined as f(a) =p, f(b) = q, f(c) = r. Then f is (i,j)-contra-continuous but not (i,j)-RC- continuous , since thenf^{-1}(p, q) is not regular closed in X.

Example 3.4: Consider the topologies on X = {a, b, c} and Y = {p, q, r} respectively by T_1 = {X, ϕ,{a, b},{b}}, T_2={X, ϕ,{a},{c},{a, c}} and σ_1 = {Y, ϕ,{p}}, σ_2 = {Y, ϕ,{r}}. Let, f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) be a map defined as f(a) =p, f(b) = q, f(c) = r. Then f is(i,j)-contra-precontinuous but not (i,j)-contra- continuous, since then f^{-1}(p) is not T_i closed in X.

Example 3.5: Let Rbe the set of all real numbers, P_2-be the countable extension topology on Ri.e,the topology with subbase$T_1 \cup T_2$, where T_1 is the Euclidian topology of Rand T_2 is the topology of countable complements of Rand σ_1 be the discrete topology of Rand P_1=σ_2= T_1 . Define a function f:(R,P_1,P_2)$\rightarrow$(R,σ_1,σ_2) as follows

$$f(x) = \begin{cases} 1\ if\ x\ is\ rational \\ 3\ f\ x\ is\ irrational \end{cases}$$

Then f is (i,j)-contra-δ-precontinuous but not (i,j)-contra-precontinuous since {1} is closed in (R,σ_1,σ_2) and f^{-1}({1}) = QwhereQis the set of rationals, is not (i,j)-preopen in (R,T_1,T_2).

Definition 3.2:A function f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) is said to be almost (i,j)-contra-precontinuous(Ekici 2004[3]) if f^{-1}(V) is (i,j)- preclosed in X for each (i,j)- regular open set V in Y.

Remark 3.2: Almost contra-precontinuity is a generalization of contra-precontinuity. Almost contra-precontinuity and contra-δ-precontinuity are independent. We have shown the following examples.

Example 3.6: If we take the function f such as in Example 3.3(i) then f is (i,j)-contra-δ-precontinuous but not almost (i,j)-contra-precontinuous.

Example 3.7: Let, X = {a,b,c,d,e}, T_1 = {X, ϕ,{b},{d},{b,d}}, T_2={X, ϕ,{a},{c},{a,c}} and Y = {a,b,c,d}, σ_1 = {Y, ϕ,{a}, {a,b},{a,c}}, σ_2 = {Y, ϕ,{b}, {b,c},{b,d}}. If we take a function f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) defined as f(a) =a, f(b) = b, f(c) = c, f(d) = d, f(e) = d. Then f is almost (i,j)-contra-precontinuous but not (i,j)-contra-δ-precontinuous.

For topological spaces, Noiri and Ekici stated that if A and B be subsets of a space (X,T) and if A $\in \delta$ PO(X) and B $\in \delta$ O(X), then A $\cap$ B$\in \delta$ PO(B)(Raychoudhuri and Mukherjee 1993[9]), then we can state and prove the following lemma.

Lemma 3.1: Let A and B be subsets of abitopological space(X,T_1,T_2). If A $\in$ (i,j)-δ PO(X) and B $\in$ (i,j)-δ O(X), then A $\cap$ B$\in$ (i,j)-δ PO(B).

Proof: We need to prove that A $\cap$ B$\subset$ iInt((i,j)-δ-Cl(A $\cap$ B)).

Let, x$\in$A $\cap$ B, then x$\in$iInt((i,j)-δ-Cl(A)) and x$\in$(i,j)-δ-Int(B), since A $\in$ (i,j)-δ PO(X) and B $\in$ (i,j)-δ O(X). This implies that$\exists$i-open set G such that, x$\in$G$\subset$ (i,j)-δ-Cl(A).

Also since x$\in$(i,j)-δ-Int(B), this implies that $\exists$(i,j)-δ-open set U such that x$\in$U$\subseteq$ B and hence U $\cap$ A$\neq \phi$. Therefore, $\forall$ (i,j)-δ-open set U containing x, U $\cap$ (A $\cap$ B)$\neq \phi$**.** Hence x$\in$G$\subset$ (i,j)-δ-Cl(A $\cap$ B). Thus A $\cap$ B$\subset$ iInt((i,j)-δ-Cl(A $\cap$ B)).

Lemma 3.2: Let A $\subset$ B $\subset$ X. If B $\in$ (i,j)-δ O(X)and A $\in$ (i,j)-δ PO(B) , then A $\in$ (i,j)-δ PO(X)(Raychoudhuri and Mukherjee 1993[9]).

Theorem 3.2: Iff:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) is a (i,j)-contra-δ-precontinuous function and A is any (i,j)-δ-open subset of X , then the restriction $f|_A$: A $\rightarrow$ Y is (i,j)-contra-δ-precontinuous.

Proof: Let F be a σ_{i} closed set in Y. Then by Theorem 3.2, f^{-1}(F) $\in$ (i,j)-δ PO(X). Since A is (i,j)-δ-open in X , it follows from Lemma 3.5 , that $\left(f|_A\right)^{-1}$(F) =A $\cap$ f^{-1}(F) $\in$ (i,j)-δ PO(A). Hence $f|_A$ is a (i,j)-contra-δ-precontinuous.

Theorem 3.3: Letf:(X,T$_1$,T$_2$)$\rightarrow$(Y,σ_1,σ_2) be a function and $\{U_\alpha : \alpha \in I\}$ be a (i,j)- δ-open cover of X . If for each $\alpha \in I$, **f|**$_{U_\alpha}$ is (i,j)-contra-δ-precontinuous then f:(X,T$_1$,T$_2$)$\rightarrow$(Y,σ_1,σ_2) is a (i,j)-contra-δ-precontinuous function.

Proof: Let F be a σ_{i} closed set in Y. Since for each $\alpha \in I$, **f|**$_{U_\alpha}$ is (i,j)-contra-δ-precontinuous ,$\left(f|_{U_\alpha}\right)^{-1}$(F) $\in$ (i,j)-δ PO(U_α). Since $U_\alpha \in$ (i,j)-δ O(X), by Lemma 3.6 , $\left(f|_{U_\alpha}\right)^{-1}$(F) $\in$ (i,j)-δ PO(X), for each $\alpha \in I$. Then f^{-1}(F) = $\bigcup_{\alpha \in I}\left[\left(f|_{U_\alpha}\right)^{-1}(F)\right] \in$ (i,j)-δ O(X) . This shows that f is a (i,j)-contra-δ-precontinuous function.

Definition 3.3:Let (X,T$_1$,T$_2$) be a bitopological space. The collection of all (i,j)- regular open sets forms a base for topology T* . It is called the semi-regularization. If T$_1$= T$_2$=T* then (X,T$_1$,T$_2$) is called semi-regular bitopological space.

Theorem 3.4: Letf:(X,T$_1$,T$_2$)$\rightarrow$(Y,σ_1,σ_2) be a function and g:X$\rightarrow$X$\times$Y the graph function of f, defined by g(x) = (x,f(x)) for every x $\in$X . If g is (i,j)-contra-δ-precontinuous then f is (i,j)-contra-δ-precontinuous .

Proof: Let U be a σ_{i} open set in Y, then X$\times$U is a σ_{i} open set in X$\times$Y. It follows from Theorem 3.1 that f^{-1}(U) = g^{-1}(X$\times$U)$\in$ (i,j)-δ PC(X). Thus f is (i,j)-contra-δ-precontinuous .

Lemma 3.3: Let A be a subset of abitopological space(X,T$_1$,T$_2$). Then A $\in$ (i,j)-δ **P**O(X) iff A $\cap$ U$\in$ (i,j)-δ **P**O(X) for each (i,j)- regular open ((i,j)-δ-open) set U of X (Raychoudhuri and Mukherjee 1993[9]).

Definition 3.4:A function f:(X,T$_1$,T$_2$)$\rightarrow$(Y,σ_1,σ_2) is called (i,j)-contra-super-continuous for every x $\in$X and each F $\in$(i,j)-C(Y,f(x)), there exists a (i,j)- regular open set U in X containing x such that f(U) $\subset$ F (Jafari and Noiri 1999[5]).

Theorem 3.5: If f:(X,T$_1$,T$_2$)$\rightarrow$(Y,σ_1,σ_2) is (i,j)-contra-super-continuous , g:X$\rightarrow$Y is (i,j)-contra-δ-precontinuous and Y is Urisohn , then E = {x $\in$ X: f(x) = g(x)} is (i,j)-δ-preclosed in X.

Proof: If x $\in$ X\E, then it follows that f(x) $\neq$ g(x) . Since Y is Urisohn , there exist σ_{i} open set V and σ_{j} open set W such that f(x) $\in$ V ,g(x) $\in$ W and iCl(V) $\cap$jCl(W) = ϕ. Since f is (i,j)-contra-super-continuous and g is (i,j)-contra-δ-precontinuous , there exists a (i,j)- regular open set U containing x and there exists a (i,j)-δ-preopen set G containing x such that f(U) $\subset$iCl(V) and g(G) $\subset$jCl(W) . Set O = U $\cap$ G. By the previous Lemma, O is (i,j)-δ-preopen in X . Hence f(O) $\cap$ g(O) = ϕ and it follows that x$\notin$ (i,j)-δ PC(E) . This shows that E is (i,j)-δ-preclosed in X.

Definition 3.5:A filter base $\wedge$ is said to be (i,j)-δ-preconvergent (resp. (i,j)-**C**-convergent) to a point x in X if for any U $\in$ (i,j)-δ **P**O(X) containing x (resp. U $\in$(i,j)-C(X) containing x) , there exists a B $\in \wedge$ such that B $\subset$ U .

Theorem 3.6: Iff:(X,T_1,T_2)$\longrightarrow$(Y,σ_1,σ_2) is a (i,j)-contra-δ-precontinuous , then for each x $\in$X and each filter base $\wedge$ in X which is (i,j)-δ-preconvergent to x ,the filter base f($\wedge$) is (i,j)-**C**-convergent to f(x) .

Proof: Let x $\in$X and $\wedge$ be any filter base in X which is (i,j)-δ-preconvergent to x . Since f is (i,j)-contra-δ-precontinuous, then for any V $\in$ C(Y) containing f(x) , there exists U $\in$ (i,j)-δ **P**O(X) containing x such that f(U) $\subset$ V . Since $\wedge$ is (i,j)-δ-preconvergent to x there exists a B $\in \wedge$ such that B $\subset$ U . It follows that f(B) $\subset$ V and hence the filter base f($\wedge$) is (i,j)-**C**-convergent to f(x) .

Theorem 3.7: Letf:(X,T_1,T_2)$\longrightarrow$(Y,σ_1,σ_2) be a function and x $\in$X . If there exists U $\in$ (i,j)-δ O(X) such that x $\in$ U and the restriction of f to U is a (i,j)-contra-δ-precontinuous function at x , then f is (i,j)-contra-δ-precontinuous at x .

Proof: Suppose that F$\in$ C(Y) containing f(x). Since $f|_U$ is (i,j)-contra-δ-precontinuous at x , there exists V $\in$ (i,j)-δ **P**O(U) containing x such that f(V) = ($f|_U$)(V)$\subset$ F . Since U $\in$ (i,j)-δ O(X) containing x , it follows from Lemma 3.6 that V $\in$ (i,j)-δ **P**O(X) containing x . This shows clearly that f is (i,j)-contra-δ-precontinuous at x .

Definition 3.6:A function f:(X,T_1,T_2)$\longrightarrow$(Y,σ_1,σ_2) is said to be (i,j)-δ **-**preirresolute if for each x $\in$X and each V $\in$ (i,j)-δ **P**O(Y,f(x)), there exists a (i,j)-δ-preopen set U in X containing x such that f(U) $\subset$ V .

Theorem 3.8: Letf:(X,T_1,T_2)$\longrightarrow$(Y,σ_1,σ_2) and g:(Y,σ_1,σ_2)$\longrightarrow$(Z,Ω_1,Ω_2) be functions. Then the following properties hold :

(i) If f is (i,j)-δ-preirresolute and g is (i,j)-contra-δ-precontinuous , then gof:X$\longrightarrow$Z is (i,j)-contra-δ-precontinuous .

(ii) If f is (i,j)-contra-δ-precontinuous and g is (i,j)-continuous , then gof:X$\longrightarrow$Z is (i,j)-contra-δ-precontinuous .

Proof: (i) Let x $\in$X and W$\in$(Z,(gof)(x)) , since g is (i,j)-contra-δ-precontinuous , there exists a (i,j)-δ-preopen set V in Y containing f(x) such that g(V) $\subset$ W . Since f is (i,j)-δ-preirresolute, there exists a (i,j)-δ-preopen set U in X containing x such that f(U) $\subset$ V . This shows that (gof)(U)$\subset$ W . Hence gof is (i,j)-contra-δ-precontinuous .

(ii) Let x $\in$X and W$\in$(Z,(gof)(x)) , since g is (i,j)-continuous , V = g^{-1}(V) is (i,j)-closed. Since f is (i,j)-contra-δ-precontinuous, there exists a (i,j)-δ-preopen set U in X containing x such that f(U) $\subset$ V. Therefore (gof)(U)$\subset$ W . This shows that gof is (i,j)-contra-δ-precontinuous .

Definition 3.7:A function f:(X,T_1,T_2)$\longrightarrow$(Y,σ_1,σ_2) is called (i,j)-δ-preopen if image of each (i,j)-δ-preopen set is (i,j)-δ-preopen .

Theorem 3.9: Iff:(X,T_1,T_2)$\longrightarrow$(Y,σ_1,σ_2) is a surjective (i,j)-δ-preopen function and g:(Y,σ_1,σ_2)$\longrightarrow$(Z,Ω_1,Ω_2) is a function such that gof:(X,T_1,T_2)$\longrightarrow$(Z,Ω_1,Ω_2) is (i,j)-contra-δ-precontinuous , then g is (i,j)-contra-δ-precontinuous .

Proof: Let x $\in$X and y $\in$Y such that f(x) = y. Let V$\in$C(Z,(gof)(x)) . Then there exists a (i,j)-δ -preopen set U in X containing x such that g(f(U)) $\subset$ V . Since f is (i,j)-δ -preopen, f(U) is a (i,j)-δ -preopen set in Ycontaining y such that g(f(U)) $\subset$ V. This shows that g is (i,j)-contra-δ -precontinuous .

Corollary 3.1: Letf:(X,T_1,T_2)$\to$(Y,σ_1,σ_2) be a surjective (i,j)-δ -preirresolute and (i,j)-δ -preopen function and let g:(Y,σ_1,σ_2)$\to$(Z,Ω_1,Ω_2) be a function . Then gof:X$\to$Z is (i,j)-contra-δ -precontinuousiff g is (i,j)-contra-δ -precontinuous .

Proof: It can be obtained from Theorem 3.18 and Theorem 3.20.

Definition 3.8:A function f:(X,T_1,T_2)$\to$(Y,σ_1,σ_2) is said to be (i,j)-weakly contra-δ -precontinuous if for each x $\in$X and each σ_i-closed set F, I = 1,2 of Ycontaining f(x) ,$\exists$ a (i,j)-δ -preopen set U in X containing x such that iInt(jClf(U))$\subset$ V.

Definition 3.9:A function f:(X,T_1,T_2)$\to$(Y,σ_1,σ_2) is called (i,j)-δ -pre-semiopenif the image of each (i,j)-δ -preopen set is (i,j)-semi-open .

Theorem 3.10: If afunction f:(X,T_1,T_2)$\to$(Y,σ_1,σ_2) is (i,j)-weakly contra-δ -precontinuous and (i,j)-δ -pre-semi-open , then f is (i,j)-contra-δ -precontinuous .

Proof: Let x $\in$X and F be a (i,j)-closed set containing f(x) . Since f is (i,j)-weakly contra-δ -precontinuous , $\exists$ a (i,j)-δ -preopen set U in X containing x such that iInt(jCl(f(U)))$\subset$ F. Since f is (i,j)-δ -pre-semiopen , f(U)$\in$(i,j)-SO(Y) and f(U) $\subset$iCl(jInt(f(U)))$\subset$ F. This shows that f is (i,j)-contra-δ -precontinuous.

4. Several theorems in bitopological spaces

In this section, graphs and preservation theorems of (i,j)-contra-δ -precontinuity are studied.

Definition 4.1:A bitopological space (X,T_1,T_2) is said to be

(i) (i,j)-weakly Hausdorff(Soundararajan , 1971[10]) if each element of X is an intersection of (i,j)-regular closed sets.

(ii) (i,j)-δ -pre-Hausdorff if for each pair of distinct points x and y in X , $\exists$ U$\in$(i,j)-δ PO(X,x) and V$\in$(i,j)-δ PO(X,y) such that U$\cap$V = ϕ .

(iii) (i,j)-δ -pre-T_1 if for each pair of distinct points x and y in X,$\exists$ (i,j)-δ -preopen set U and V containing x and y respectively such that y$\notin$ U and x$\notin$ V.

Here we have given the following examples:

Example 4.1: Consider the topologies on X = {a, b, c} be

T_1 = {X, ϕ ,{a},{b}, {c}, {a, b},{b, c},{a,c}}and T_2={X, ϕ ,{a},{b},{a,b},{b,c}}

and let A = {b}, B={b, c}, C={a, c}and D={a, b}be subsets of X, then we have A, B, C, D are (1, 2)-regular closed. Also we have A$\cap$B={b}, B$\cap$C={c} and C$\cap$D={a}. Therefore, X is (1, 2)-weakly Hausdorff.

Example 4.2: Consider the topologies on X = {a, b, c} be

T_1 = {X, ϕ ,{a},{b}, {a, b}}and T_2={X, ϕ ,{c},{a, c},{b, c}}. Then we have

(1, 2)-δ **-**preopen sets are X, ϕ , {a},{b},{a, b} and

(2, 1)-δ -preopen sets are X, ϕ , {c},{b, c},{a, c}. Hence (X, T_1, T_2) is a (i,j)-δ -pre-Hausdorff space.

Example 4.3: Same as example 4.2.

Remark 4.1: Thefollowingimplications are hold for a bitopological space (X,T_1,T_2):

(i) Pairwise T_1 $\Rightarrow$ (i,j)-δ -pre-T_1

(ii) PairwiseT_2 $\Rightarrow$ (i,j)-δ -pre-T_2

These implications are not reversible.

Example 4.4: Let X = {a,b,c,d} with topologies T_1={X, ϕ ,{a},{b,c}}, T_2={X, ϕ ,{b},{c,d}}. Then (X,T_1,T_2) is (i,j)- δ -pre- T_2 but not T_2.

Definition 4.2: For a function f:(X,T_1,T_2) $\rightarrow$ (Y, σ_1, σ_2) , the subset {(x,f(x)): x $\in$X}$\subset$ X$\times$ Y is called the graph of f and is denoted by G(f).

Definition 4.3: The graph G(f) of a function f:(X,T_1,T_2) $\rightarrow$ (Y, σ_1, σ_2) is said to be (i,j)-contra- δ -preclosed if for each (x,y)$\in$(X$\times$ Y)\G(f) , $\exists$ (i,j)- δ -preopen set U in X containing x and V$\in$(x,y) such that (U$\times$ V)$\cap$ G(f) = ϕ .

Lemma 4.1: The following properties are equivalent for the graph G(f) of a function f :

(i) G(f) is (i,j)-contra- δ -preclosed

(ii) for each (x,y)$\in$(X$\times$ Y)\G(f), $\exists$ (i,j)- δ -preopen set U in X containing x and V$\in$(i,j)- (Y,y) such that f(U)$\cap$V = ϕ .

Proof: Obvious.

Theorem 4.1: Iff:(X,T_1,T_2) $\rightarrow$ (Y, σ_1, σ_2) is (i,j)- contra- δ -precontinuous and Y is Urysohn , G(f) is (i,j)-contra- δ -preclosed in X$\times$ Y .

Proof: Suppose that Y is Urysohn . Let (x,y)$\in$(X$\times$ Y)\G(f). It follows that f(x)$\neq$ y. Since Y is Urysohn ,$\exists$ σ_{i-} open set V and σ_{j-} open set W such that f(x)$\in$ V, y $\in$ W and iCl(V) $\cap$jCl(W) = ϕ . Since fis (i,j)- contra- δ -precontinuous , $\exists$ (i,j)- δ -preopen set U in X containing x such that f(U)$\subset$iCl(V). Therefore f(U) $\cap$jCl(W) = ϕ and G(f) is (i,j)-contra- δ -preclosed in X$\times$ Y .

Theorem 4.2: Let f:(X,T_1,T_2) $\rightarrow$ (Y, σ_1, σ_2) have a (i,j)-contra- δ -preclosed graph. If f is injective, then X is (i,j)- δ -pre- T_1 .

Proof: Let x and y be any two distinct points of X. Then we have (x,f(y)) $\in$ (X$\times$ Y)\G(f). By Lemma 4.5, $\exists$ (i,j)- δ -preopen set U in X containing x and F$\in$C(Y,f(y)) such that f(U)$\cap$F = ϕ . Hence U$\cap$ f^{-1}(F)= ϕ . Therefore we have y$\notin$ U. This implies that X is (i,j)- δ -pre- T_1 .

Definition 4.4:A bitopological space (X,T_1,T_2) is called (i,j)- δ -preconnected provided that X is not the union of two disjoint non-empty (i,j)- δ -preopen sets .

Theorem 4.3: Iff:(X,T_1,T_2) $\rightarrow$ (Y, σ_1, σ_2) is (i,j)- contra- δ -precontinuous surjection and X is (i,j)- δ -preconnected , then Y is (i,j)-connected .

Proof: SupposeY is not (i,j)-connected space . There exist disjoint σ_{i-} open set V_1 and σ_{j-} open set V_2 such that Y = $V_1 \cup V_2$. Therefore V_1 and V_2 are (i,j)-clopen in Y. Since f is (i,j)- contra- δ -precontinuous , $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are (i,j)- δ -preopen in X . Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are non-empty disjoint and X = i $f^{-1}(V_1)$ $\cup$ j $f^{-1}(V_2)$. This shows that X is not (i,j)- δ -pre-connected, which is a contradiction. Hence Y is (i,j)-connected .

Definition 4.5:A bitopological space (X,T_1,T_2) is called

(i) (i,j)- δ -pre-ultra-connectedif every two non-empty (i,j)- δ -preclosed subsets of X intersect ,

(ii) (i,j)-hyperconnected (Steen and Seebach 1970[11]) if everyi-open set is j- dense.

Here we have given the following examples:

Example 4.5: Consider the topologies on X = {a, b, c} be

T_1 = {X, ϕ ,{a},{b}, {a, b}}and T_2={X, ϕ ,{c},{a, c},{b, c}}. Then we have

(1, 2)- δ -preclosed subsets are X, ϕ , {b, c},{a, c},{c} and we see that any two non-empty subsets are intersect, hence (X, T_1, T_2) is (1, 2)- δ -pre-ultra-connected.

Example 4.6: Consider the topologies on X = {a, b, c} be

T_1 = {X, ϕ ,{b}, {b, c}}and T_2={X, ϕ ,{b},{a, b}}. Then we have

T_2-Cl{b,c}= X and T_2-Cl{b}= X.

Again, T_1-Cl{a, b}= X and T_1-Cl{b}= X.

Hence (X, T_1, T_2)is (i,j)-hyperconnected.

Theorem 4.4: If X is (i,j)- δ -pre-ultra-connected and f:(X,T_1,T_2)$\longrightarrow$(Y, σ_1, σ_2) is (i,j)- contra- δ -precontinuous and surjective , then Y is (i,j)-hyperconnected.

Proof: Let us suppose that Y is not (i,j)-hyperconnected. Then $\exists \sigma_{i}$ – open set V such that V is not j-dense in Y. Then $\exists$ disjoint non-empty σ_{i} – open subset B_1 and σ_{j} – open subset B_2 in Y, such that B_1 = iInt(jCl(V)) and B_2 = Y\jCl(V). Since f is (i,j)- contra- δ -precontinuous and onto, by Theorem 3.2, A_1 = f^{-1}(B_1) and A_2 = f^{-1}(B_2) are disjoint non-empty (i,j)-preclosed subsets of X . By assumption, the (i,j)- δ -pre-ultra-connectedness of X implies that A_1 and A_2 must intersect, which is a contradiction. Hence Y is (i,j)-hyperconnected.

Theorem 4.5: Iff:(X,T_1,T_2)$\longrightarrow$(Y, σ_1, σ_2) is (i,j)- contra- δ -precontinuous injection and Y is Urysoshn , then X is (i,j)- δ -pre-Hausdorff.

Proof: Suppose that Y is Urysohn. By the injectivityof f , it follows that f(x)$\neq$ f(y) for any distinct points x,y$\in$X. Since Y is Urysohn ,$\exists \sigma_{i}$ – open set V and σ_{j} – open set W such that f(x)$\in$ V, f(y) $\in$ W and iCl(V) $\cap$jCl(W) = ϕ . Since fis (i,j)- contra- δ -precontinuous , $\exists$ (i,j)- δ -preopen set U and G in X containing x and y respectively such that f(U)$\subset$iCl(V) and f(G)$\subset$jCl(W) . Hence U$\cap$ G = ϕ . This shows that X is (i,j)- δ -pre-Hausdorff.

Theorem 4.6: Iff:(X,T_1,T_2)$\longrightarrow$(Y, σ_1, σ_2) is (i,j)- contra- δ -precontinuous injection and Y is (i,j)-weakly Hausdorff then X is (i,j)- δ -pre- T_1 .

Proof: Suppose that Y is (i,j)-weakly Hausdorff. For any distinct points x,y$\in$X, $\exists$ (i,j)-regular closed sets V, W in Y such that f(x)$\in$ V, f(y)$\notin$V , f(x)$\notin$W and f(y)$\in$ W. Since f is (i,j)- contra- δ -precontinuous, by Theorem 3.1, f^{-1}(V) and f^{-1}(W) are (i,j)- δ -preopen subsets of X such that x$\in$ f^{-1}(V), y$\notin$ f^{-1}(V), x$\notin$ f^{-1}(W) and y$\in$ f^{-1}(W). This shows that X is (i,j)- δ -pre- T_1 .

Definition 4.6: Abitopological space (X,T_1,T_2) is said to be

(i) (i,j)- δ -pre-compact (Dontchev 1996[1]) if every (i,j)- δ -preopen (resp. (i,j)-closed) cover of X has a finite subcover

(ii) (i,j)-countably δ -pre-compact ((i,j)-strongly countably S-closed) if every countable cover of X by (i,j)- δ -preopen (resp. (i,j)-closed) sets has a finite subcover.

(iii) (i,j)- δ -pre-Lindelöf ((i,j)-strongly S-Lindelöf) if every (i,j)-δ -preopen (resp. (i,j)-closed) cover of X has a countablesubcover.

Theorem 4.7: The (i,j)- contra-δ -precontinuous image of (i,j)-δ -pre-compact((i,j)- δ -pre-Lindelöf,(i,j)-countablyδ -pre-compact) space are (i,j)-strongly S-closed (resp.(i,j)-strongly S-Lindelöf, (i,j)-strongly countably S-closed).

Proof: Suppose that f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) is (i,j)- contra-δ -precontinuous surjection. Let $\{V_\alpha : \alpha \in I\}$be any closed cover of Y. Since f is (i,j)- contra-δ -precontinuous, then $\{ f^{-1}(V_\alpha): \alpha \in I \}$ is a (i,j)-δ -preopen cover of X and hence ∃a finite subset I_0 of I such that X = $\cup\{ f^{-1}(V_\alpha): \alpha \in I_0 \}$. Hence we have Y = $\cup \{V_\alpha : \alpha \in I_0\}$and Y is (i,j)-strongly S-closed. Similarly, the other proof can be obtained.

Definition 4.7: Abitopological space (X,T_1,T_2) is said to be

(i) (i,j)-δ -preclosed-compact if every (i,j)-δ -preclosed cover of X has a finite subcover

(ii) (i,j)-countablyδ -preclosed-compact if every (i,j)-countable cover of X by (i,j)-δ -preclosed sets has a finite subcover.

(iii) (i,j)- δ -preclosed-Lindelöf if every cover of X by (i,j)- δ -preclosed set has a countable subcover.

Theorem 4.8: The (i,j)- contra-δ -precontinuous image of (i,j)-δ -preclosed-compact((i,j)- δ -preclosed-Lindelöf,(i,j)-countablyδ -preclosed-compact) space are pairwise compact (resp. pairwise Lindelöf,pairwisecountably compact).

Proof: Suppose that f:(X,T_1,T_2)$\rightarrow$(Y,σ_1,σ_2) is (i,j)- contra-δ -precontinuous surjection. Let $\{V_\alpha : \alpha \in I\}$be any open cover of Y. Since f is (i,j)- contra-δ -precontinuous, then $\{ f^{-1}(V_\alpha): \alpha \in I \}$ is a (i,j)-δ -preclosed cover of X. Since X is (i,j)-δ -preclosed-compact , ∃a finite subset I_0 of I such that X = $\cup\{ f^{-1}(V_\alpha): \alpha \in I_0 \}$. Hence we have Y = $\cup \{V_\alpha : \alpha \in I_0\}$and Y is pairwise compact.Similarly, the other proof can be obtained.

5. REFERENCES

[1] Dontchev, J. (1986): Contra-continuous functions and stongly S-closed spaces, Internat J. Math. Sci., 19, 303.

[2] Dontchev, J. and Noiri,T. (1999): Contra-semicontinuous functions, Math. Pannon., 10,159.

[3] Ekici, E. (2004): Almost Contra-precontinuous functions, Bull. Mlaysian Math. Sc. Soc.(Second series), 27,53.

[4] El-Deeb,S.N.,Hasanein,I.A., Mashhour, A.S. and Noiri,T. (1983): On p-rgular spaces, Bull.Math. Soc. Sci. Math. R.S. Roumania,27(75),311.

[5] Jafari,S. and Noiri,T. (1999): Contra-super-continuous functions, Ann.Univ.Sci. Budapest Eotvos Sect. Math., 42,27.

[6] Jafari,S. and Noiri,T. (2002): On contra-precontinuous functions, Bull. Mlaysian Math. Sci. Soc., (2),25,115.

[7] Levine, N.(1960): Stong continuity in topological spaces, Amer. Math. Monthly,67,269.

[8] Noiri,T. (1984a): Super-continuity and some strong forms of continuity, Indian J.Pure Appl. Math., 15,241.

[9] Raychaudhuri, S. and Mukherjee, N. (1993): On δ-almost continuity and δ-preopen sets, Bull. Inst. Math.Acad.Sinica,21,357.

[10] Soundararajan, T. (1968): Weakly Hausdorff spaces and the cardinality of topological spaces, General Topology and its Relation to Modern Analysis and Algebra III, Proc. Conf. Kanpur, Academia, Prague 1971,301.

[11] Steen, L.A. and Seebach Jr., J.A. (1970): Counterexamples in Topology, Holt, Rinerhart and Winston, New York.

[12] Velicko, N.V. (1968): H-closed topological spaces, Amer. Math. Soc. Transl., 78,103.

[13] G. K. Banerjee, On pairwise almost strongly θ-continuous mappings, Bull. Calcutta Math. Soc., 79(1987), 314-320.

[14] S. Bose, Semi-open sets, semi-continuity and semi-open mapping in bitopological spaces, Bull. Calcutta Math. Soc., 73(1981), 237-246.

[15] M. Jelic, A decomposition of pairwise continuity, J. Inst. Math. Comput. Sci. Math. Ser., 3(1990), 25-29.

[16] Ekici, E. and Noiri, T. (2006): Contra δ-precontinuous functions, Bull. Cal. Math. Soc., 98,(3) 275-284.