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COMMON FIXED POINT THEOREMS FO FOUR WEAKLY COMPATIBLE SELF-MAPPINGS IN FUZZY METRIC SPACES

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ABSTRACT

In this paper we prove common fixed point theorems for four weakly compatible self-mappings in fuzzy metric space using different contractive conditions. At the end we provide an example satisfying the main theorem.

Key Words

Fuzzy Metric Spaces, Weakly Compatible Mappings, t-nom Hadzic-type, E.A.property, (CLR) Property

1.INTRODUCTION

Introduction of fuzzy set by Zadeh[6] proved a turning point in the development of fuzzy mathematics. Fuzzy set theory has many applications in applied sciences such as neural network, stability theory, modeling theory, mathematical programming, engineering sciences image processing, control theory, communication etc.) medical sciences medical genetics, nervous systems). Using fuzzy sets, fuzzy metric space was defined by Kramosil and Michalek [5] and it was modified by George and Veeramani [3]. Using fuzzy metric space many researchers have proved fixed point theorems for different contractive conditions. See for example Deng [1], Erceg [2], Kaleva and seikkala [4].

2. Preliminaries

2.1 Fuzzy Metric Space

A fuzzy metric space is a triple (X,M,T) where X is a nonempty set, T is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ and the following conditions are satisfied for all $x, y \in X$ and $t, s > 0$:

(FM-1) $M(x, y, t) > 0$;

(FM-2) $M(x, y, t) = 1 \Leftrightarrow x = y$;

(FM-3) $M(x, y, t) = M(y, x, t)$

(FM-4) M (x, y, .) : $(0,\infty) \rightarrow [0, 1]$ is continuous.

(FM-5) M (x, z, t + s) ≥ T (M(x, y, t), M(y ,z, s)).

2.2 Weakly Compatible Mappings

Two self-mappings f and g of a non – empty set X are said to be weakly compatible (o coincidentally commuting) if they commute at their coincidence points. i.e., if fz = gz for some z ϵ X, then *fgz = gfz.*

2.3 Hudzic – Type

Let T be a t – norm and $T^n: [0, 1] \rightarrow [0, 1]$ be defined by $T^1(x) = T(x)$, $T^{n+1}(x) = T(T^n(x), x)$, for all n ϵ N, and x ϵ (0, 1). Then we say that the t – norm T is of Hadzic – Type if the family { $T^n(x)$; n ϵ N } is equicontinuous at x = 1 if for every λ ϵ (0,1) there exists δ(λ) ϵ (0,1) such that

$x > 1 - \delta(\lambda) \Rightarrow T^n(x) > 1 - \lambda$

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$$x > 1 - \delta(\lambda) \Rightarrow T^n(x) > 1 - \lambda$$

2.4 E.A Property

Two self – mappings f and g of a fuzzy metric space (X, M, T) are said to satisfy E.A. property, if there exists a sequence { xn } in X such that $\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = u$ for some u ϵ X.

2.5 (CLRg) Property

qproperty (common limit in the range of g property) if there exist a sequence { xn } in X such that that $\lim_{n\to\infty} x_n = x$ and that $\lim_{n\to\infty} f x_n = \lim_{n\to\infty} g x_n = gx$.

2.6 Lemma

Let (X, M, T) be a fuzzy metric space. If there exists k ϵ (0, 1) such that

M(x, y, kt) ≥ M(x, y, t) for all x, y ϵ X and t > 0, then x = y.

3. Some Common Fixed Point Theorems for Four Weakly Compatible Self – Mappings in Fuzzy Metric Spaces

Now we prove a fixed point theorem for four weakly compatible self – mappings in complete fuzzy metric space.

3.1 Theorem

Let (X, M, T) be a complete fuzzy metric space with continuous t – norm of Hadzic – type. Let *f, g, p* and *q* be self – mappings on X satisfy a the following conditions:

(C1) $f(X) \subset q(x)$, $g(x) \subset p(x)$,

(C2) The pairs f, p) and (g, q) are weakly compatible,

(C3) There exists $K \in (0, 1)$ such that

$$M(fx, gy, kt) \geq \min \left\{ \begin{array}{c} M(px, qy, t), M(fx, px, t), \\ M(gy, qy, t), M(fx, qy, t) \end{array} \right\}$$

For all $x, y \in X$ and $t > 0$,

(C4) One of f(X), g(X), p(X) or q(X) is a closed subset of X.

If $x_0, x_1, x_2 \in X$ thee exists $\mu \in (k, 1)$ such that $y_1 = fx_0 = qx_1$, $y_2 = gx_1 = px_2$ and $\lim T_{i=n}^{\infty} M(y_1, y_2, 1/\mu^i) = 1$.

Then f, g, p and q have a unique common fixed point in X.

Proof: Since $g(x) \subset p(x)$, thee exist $x_1, x_2 \in X$ such that $gx_1 = px_2$.

Inductively, we can construct two sequences { xn } and { yn } of X such that

$Y_{2}n - 1 = qx_{2n+1} = fx_{2n-2}$, $y_2n = px_{2n} = gx_{2n-1}$, n= 1, 2, …..

Putting $x = x_{2n}$ and $y = y_{n+1}$ in (C3), we have that for all $t > 0$

$M(y_2n_{+1}, y_2n_{+2}, kt) = M(fx_2n, gx_2n_{+1}, kt)$

$\geq \min \{ M(px_2n, qx_2n_{+1}, t), M(fx_2n, px_2n, t), M(gx_2n, qx_2n_{+1}, t), M(fx_2n, qx_2n_{+1}, t)\}$

$= \min \{ M(y_2n, y_2n_{+1}, kt), M(y_2n_{+1}, y_2n_{+2}, t), M(y_2n_{+2}, y_2n_{+1}, kt), M(y_2n_{+1}, y_2n_{+1}, t)\}$

If we take $M(y_2n_{+1}, y_2n_{+2}, kt) \geq M(fx_2n, gx_2n_{+1}, kt)$,

Which is contraction by Lemma 2.6 since $k \in (0,1)$. Therefore we hae

$M(y_2n_{+1}, y_2n_{+2}, kt) \geq M(y_2n, y_2n_{+1}, kt)$.

Also, taking $x = x_2n_{+1}$ and $y = x_2n_{+2}$ in (C3), we have that fo all $t > 0$

$M(y_2n_{+2}, y_2n_{+3}, kt) \geq M(yn_{+1}, y\, n_{+2}, t)$.

In general, for any $n \in N$, we have

$M(yn_{+1}, yn, kt) \geq M(y_2n_{+1}, y\, _2n_{-1}, t)$.

It follows that

$M(yn_{+1}, yn, t) \geq M(yn_{+1}, y\, n_{-1}, t/k)$.

$\geq$ M (y_{n+1}, y_{n-2}, t/k^2).

....

$\geq$ M (y_1, y_2, t/k^{n-1}).

Thus for all t > 0 and n = 1, 2, 3, ...

M (y_{n+1}, y_n, t) $\geq$ M (y_1, y_2, t/k^{n-1}).

Now, we show that { yn } is a Cauchy Sequence in X.

Let $\sigma = \frac{k}{\mu}$. Since $0 < \sigma < 1$, the series $\sum_{i=1}^{\infty} \sigma^i$ is convergent and there exists $m_0 \in N$ such that $\sum_{i=m_0}^{\infty} \sigma^i$ < 1. Hence for every $m = m_0 + 1$ and $s \in N$.

$$t > t\sum_{i=m_0}^{\infty} \sigma^i > \sum_{i=m-1}^{m+s-1} \sigma^i$$

Now

M (y_{m+s+1}, y_m, t)

$\geq$ M (y_{m+s+1}, y_m, $t\sum_{i=m-1}^{m+s-1} \sigma^i$)

$\geq$ M (y_{m+s+1}, y_m, $t\sigma^{m-1} + t\sigma^{m-1+1} + t\sigma^{m-1+2} + \ldots + t\sigma^{m-1+s}$)

$\geq$ M (y_{m+s+1}, y_m, $t\sigma^{m-1+1} + t\sigma^{m-1+2} + \ldots + t\sigma^{m-1+s} + \ldots + t\sigma^{m-1}$)

$\geq$ T(M (y_{m+s+1}, y_{m+1}, $t\sigma^{m-1+1} + t\sigma^{m-1+2} + \ldots + t\sigma^{m-1+s}$), M($y_{m+1}, y_m$, $t\sigma^{m-1}$))

$\geq$ T (T(M (y_{m+s+1}, y_{m+2}, $t\sigma^{m-1+2} + \ldots + t\sigma^{m-1+s}$), M($y_{m+2}, y_{m+1}$, $t\sigma^{m-1}$),

M(y_{m+1}, y_m, $t\sigma^{m-1}$)))

$\geq$ T(T (T(M (y_{m+s+1}, y_{m+3}, $t\sigma^{m-1+3} + \ldots + t\sigma^{m-1+s}$), M($y_{m+3}, y_{m+2}$, t , $t\sigma^{m+1}$)

M(y_{m+2}, y_{m+1}, $t\sigma^{m}$), M(y_{m+1}, y_m, $t\sigma^{m-1}$))))

$\geq$..................

$$\geq \overbrace{T(T (\ldots\ldots(T}^{\text{s-times}}(M (y_{m+s+1}, y_{m+s},\ t\sigma^{m-1+s} + \ldots + t\sigma^{m-1+s}), M(y_{m+3}, y_{m+s-1}, t\ \sigma^{m-s+2}),$$

$$\ldots\ldots\ldots, M(y_{m+1}, y_m, t\sigma^{m-1})))\ldots\ldots))$$

$$\geq \overbrace{T(T (\ldots\ldots(T}^{\text{s-times}}(M (y_1, y_2,\ t\sigma^{m-1+s} + \ldots + t\sigma^{m-1+s} / k^{m-1+s}), M(y_1, y_2, t\ \sigma^{m-s+2} / k^{m-2+s}),$$

$$\ldots\ldots\ldots, M(y_{m+1}, y_m, t\sigma^{m-1} k^{m-1})))\ldots\ldots))$$

s-times

$\geq$ T(T (.......(T(M (y_1, y_2, t /$\mu^{m-1=s}$), M(y_1, y_2, t /μ^{m-2+s}),

......................., M(y_{m+1}, y_m, t t /μ^{m-1})))........))

$\geq T_{i=m-1}^{m+s-1}$M (y_1, y_2, t /μ^i)

$\geq T_{i=m-1}^{\infty}$M (y_1, y_2, t /μ^i)

It is obvious that $\lim_{n\to\infty} T_{i=m-1}^{\infty}$ M (y_1, y_2, t /μ^i) = I implies $\lim_{n\to\infty} T_{i=m-1}^{\infty}$ M (y_1, y_2, t /μ^i) = 1

For every t > 0. Now for every t > 0 and λ ϵ (0, 1), there exists m_1 (t, λ) such that

M (y_{m+s+1}, y_m, t) > 1 – λ for every m ≥ m_1 (t,λ) and s ϵ N . Hence { y_n } is a Cauchy sequence in X.

Since X is complete, there exists a point z in X such that $\lim_{n\to\infty} x_n$ = x and this gives

$$\lim_{n\to\infty} p\,x_{2n} = \lim_{n\to\infty} qx_{2n-1} = \lim_{n\to\infty} fx_{2n-2} = \lim_{n\to\infty} gx_{2n-1} = z$$

For all n ϵ N. Without loss of generality, we assume that p (X) is a closed subset of X. Then z = pu for some

u ϵ X. Subsequently, we have

$$\lim_{n\to\infty} p\,x_n = \lim_{n\to\infty} qx_n = \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z = pu.$$

Next, we claim that fu = pu . For this purpose, if we put z = u and y = x_n in (C3), then this gives

M(fu, x_n, kt) ≥ min {M (pu, qx_n , t), M(fu, qu, t), M(gx_n,qx_n,t), M(fu,qx_n,t)}

Taking limit as n →∞ , we have

M(fu, z, kt) ≥ min { M(z, z, t), M(fu, z, t), M(z, z, t), M(fu, z, t)}

= M(fu, z, t)

By Lemma 2.6, we have fu = z and hence fu = qu = z. Since f(X) ⊂ q (X), there exists a points v ϵ X such that fu = z = qv

Next, we claim that qv = gv. Putting x = u and y = v in (C3), we have

M(fu, gv, kt) ≥ min { M(pu, qv, t) M(fu, pu,t) M(gv, qv, t) , M(fu, pv, t)}

= min{ (M(z, z, t), M(z, z, t), M(gv, qv, t), M(z, z, t)}

= M(gv , q, t)

Therefore we have qv = gv. Thus fu = pu = qv = gv = z . Since the pairs (f, p) (g , p) are weakly compatible u and v are their coincidence points, respectively, we obtain fz = f(pu) = p (fu) = pz and gz = g (qv) = qz.

Now , we poe that z is a common fixed point of f, g, p and q. Fo ths purpose, putting x = z and y = v in (C3), we get

$$M(fz, gv, kt) \geq \min \{M(pz, p, t), M(fz,pz, t), M(g, qv, t), M(fz, q, t)\}$$

$$= \min \{M(fz, gv, t), M(fz, pz, t), M(fz, pz, t), M(fz, gv, t)\}$$

$$= M(fz, g, t)$$

Which implies that fz = gv. Hence fz = gv = z and z = fz = pz and z is a common fixed point f and p.

One can prove that z is also a common fixed point of g and q . Finally , in order to prove the uniqueness, suppose that w (≠ z) be another fixed point of f, g, p and q . Then, fo all t > 0, we have

$$M(z, w, kt) = M(fz, gw, kt) \geq \min \begin{Bmatrix} M(pz, qw, t), M(fz, pz, t) M(gw, qw, t) \\ M(fz, qw, t) \end{Bmatrix}$$

$$= \min \{M(z, w, t), M(z, z, t), M(w, w, t), M z, w, t)\}$$

$$= M(z, w, t)$$

Thus we have z = w. Hence z is a unique common fixed point f, g, p and q.

Next, we prove a fixed point theorem for weakly compatible self – mappings with E.A. property.

3.2 Theorem

Let (X, M, T) be a fuzzy metric space with continuous t – norm o Hadzic – type. Let f, g, p and q be self – mappings on X satisfying (C1), (C2), (C4) and the following condition:

(C5) the pairs (f, g) or (g,p) satisfy E.A. property

(C6) thee exists k ϵ X and t > 0.

$$M(fx, gy, kt) \geq \min \begin{Bmatrix} M(px, qy, t), M(fz, pz, t) M(fx, px, t), M(gy, py, t) \\ M(fx, qy, t), M(gy, px, t) \end{Bmatrix}$$

For all x, y ϵ X and t > 0

Then f, g, p and q have a unique common fixed point in X.

Proof:

With out loss of generality, we assume that the pair (g, q) satisfies E.A property. Then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} gx_n = \lim_{n\to\infty} qx_n$ = z fo some z ϵ X. Since g(X) ⊂ p (X), there exists a sequence $\{y_n\}$ in X such that gx_n = p y_n. Hence $\lim_{n\to\infty} px_n$ = z. Also since f (X) ⊂ g (y), there exists a sequence $\{y'_n\}$ in X such that f$y_n{}'$ = q x_n. Hence $\lim_{n\to\infty} f\, y_n{}'$ = z. Suppose that p (X) is a closed subset of X. Then z = pu fo some u ϵ X. Subsequently, we have that $\lim_{n\to\infty} gx_n$ = $\lim_{n\to\infty} f y_n{}'$ = $\lim_{n\to\infty} p\, y_n$ = z = pu. for some u ϵ X.

Next, we claim that fu = pu. For this purpose , if we put x = u and y = x_n in (C6), then this gives

$$M(fu, gx_n, kt) \geq \min \left\{ \begin{array}{l} M(pu, qx_n, t), M(fu, pu, t) M(fx_n, px_n, t), \\ M(fu, qx_n, t), M(z, z, t) \end{array} \right\}$$

Taking limit as n → ∞, we have

$$M(fu, z, kt) \geq \min \left\{ \begin{array}{l} M(z, z, t), M(fu, z, t), M(z, z, t), \\ M(fu, z, t), M(z, z, t) \end{array} \right\}$$

$$= M(fu, z, t)$$

Thus by Lemma 2.6 we have fu = z and hence fu = pu = z. Since f(X) ⊂ q (X), there exists a point y ϵ X such that fu = z = qy

Next, we claim that qv = gv. Putting x = u and y = v in (C6), we hae

$$M(fu, gv, kt) \geq \min \left\{ \begin{array}{l} M(pu, qv, t), M(fu, pu, t) M(fx_n, px_n, t), \\ M(fu, qx_n, t), M(z, z, t) \end{array} \right\}$$

$$M(fu, z, kt) \geq \min \left\{ \begin{array}{l} M(z, z, t), M(z, z, t), M(gv, qv, t), \\ M(fu, z, t), M(gv, qv, t) \end{array} \right\}$$

$$= M(gv, qv, t)$$

Hence we have qv = gv. Thus fu = pu = qv = gv = z. Since the pairs (f, g) and (g, q) are weakly compatible and u and v are their coincidence points, respectively, we obtain

Fz = f(pu) = p (fu) = pz and gz = g (qv) = qz.

Now, we prove that z is a common fixed point of f , g p and q. For purpose, if we put x = z and y = in (C6), then this gives

$$M(fu, z , kt) \geq \min \begin{Bmatrix} M(pz, qv, t), M (fz, pz, t) , M(gv, qv , t), \\ M(fz, qv, t), M (gv, pz, t) \end{Bmatrix}$$

$$M(fu, z , kt) \geq \min \begin{Bmatrix} M(fz, gv, t), M (fz, fz, t) , M(gv, gv , t), \\ M(fz, gv, t), M (fz, gv, t) \end{Bmatrix}$$

$$= M (fz, gv, t)$$

Then we have fz = gv and hence z = fz = pz and z is a common fixed point of f and p. One can easily prove that z is also a common fixed point of g and q.

Finally, in order to prove the uniqueness, let w (≠ z) be another fixed point of f, g, p and q. Then, for all t < 0, we have

$$M(fu, z , kt) = M (fz, gw, kt)$$

$$\geq \min \begin{Bmatrix} M(pz, qw, t), M (fz, pz, t) , M(gw, qw , t), \\ M(fz, qw, t), M (gw, pz, t) \end{Bmatrix}$$

$$= M (z, w, t)$$

This we have z = w. Hence z is a unique common fixed point of f, g, p and q.

Finally, we prove a fixed point theorem for weakly compatible mappings with CLRs popety.

3.3 Theorem

Let (X, M, T) be a fuzzy metric space with continuous t – norm of Hadzic – type. Let f, g, p and q be self – mappings on X satisfying

(C1), (C2), (C6) and the following conditions.

(C7) the pairs (f,g) or (g, q) satisfy CLRg property,

(C8) one of f (X), g (X), p (X) or q (X) is a closed subset of X.

Then f, g p and q have a unique common fixed point in X.

Proof:

If the pair (f, p) satisfies the CLRs property, then there exists a sequence { x_n } in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} px_n$ = z, z ϵp(X). Therefore their exists a point u ϵ X such that pu = z. Since q (X) is a closed subset of X and f (X) ⊂ g (X), so for each sequence $\{x_n\}$ in X, there corresponds a sequence { y_n }in X such that fx_n = qy_n. Therefore $\lim_{n\to\infty} qx_n = \lim_{n\to\infty} fx_n$ = z, where z ϵ p(X). Thus we have

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} p\,y_n = z.$$

Now, we show that $\lim_{n\to\infty} gy_n$ = z. Putting x = x_n and y = y_n in (C6), we get

$$M(fx_n, qy_n , kt) \ \geq \min \left\{ \begin{array}{c} M(px_n, qy_n, t), M (fx_n, px_n, t) , M(gy_n, qy_n , t), \\ M(fx_n, qy_n, t), M (gy_n, px_n, t) \end{array} \right\}$$

Let $\lim_{n\to\infty} gy_n = l$ for t >0. Then taking limit as n → ∞, we have

$$M(z, l , kt) \geq \min \left\{ \begin{array}{c} M (z, z, t), M (z, z, t) , M(l, z , t), \\ M(z, z, t), M (l, z, t) \end{array} \right\}$$

$$= M (z, l, t)$$

By Lemma 2.6, we hae z = l and hence $\lim_{n\to\infty} gy_n$ = z. Therefore

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} px_n = \lim_{n\to\infty} qy_n = \lim_{n\to\infty} gy_n = z = pu$$

For some u ϵ X. Using Theoem 3.2 and Lemma 2.6, we can easily prove that z is a unique common fixed point of f, g, p and q.

3. 4 Example

Let X =[0, 2] equipped with the Euclidian metric and the fuzzy metric space induced by (X, d), i.e,

M (x, y, t) = $\frac{t}{t+d\,(x\,,y\,)}$ for every x, y ϵ X and t > 0. Clearly (X, M, T) is a fuzzy metric space with continuous t – norm Hadzic – type with T (a, b) = min { a, b }. Define the self – mappings f, g, p and q : X → X by

$$fx = \begin{cases} 0 & \text{if } x = 0 \\ 0.25 & \text{if } x > 0 \end{cases} \qquad gx = \begin{cases} 0 & \text{if } x = 0 \\ 0.45 & \text{if } x > 0 \end{cases}$$

$$px = \begin{cases} 0 & \text{if } x = 0 \\ 0.45 & \text{if } 0 < x \leq 0.6 \\ x - 0.45 & \text{if } x > 0.6 \end{cases} \qquad px = \begin{cases} 0 & \text{if } x = 0 \\ 0.25 & \text{if } 0 < x \leq 0.6 \\ x - 0.25 & \text{if } x > 0.6 \end{cases}$$

fX = { 0} ∪ { 0.25 }, gX = { 0 } ∪ { 0.45 }, pX = { 0 } ∪ (0.55, 1.55) and

qX = { 0 } ∪ { 0.25 } ∪ (0.35, 1.75)

Consider the sequence { x_n } = { 0.60+ $1/n$}. Then fx_n → 0.25, gx_n → 0.45, px_n → 0.15, qx_n → 0.35, fpx_n → 0.25, px_n → 0.45, gqx_n → 0.45 and gqx_n → 0.25. The pairs (f, p) and g, q) are compatible at coincidence point. If w take q = 0.6 and t = 1, then f, g , p and q satisfy the conditions of Theorems 3.1 and zero is the unique common fixed point f, g, p and q. Moreover, f, g, p and q are discontinuous at the fixed point zero.

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