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RESEARCH ARTICLE

AN EULER – MARUYAMA METHOD TO THE BLACK SCHOLES SDE TO THE PRESERVED INCRETIA ACTIVITY OF GLP-1 WITH TYPE -2 DIABETES PATIENTS

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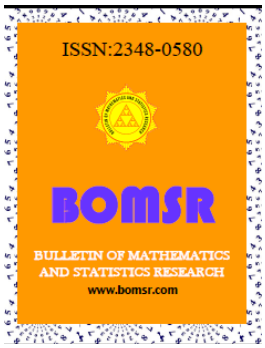

ABSTRACT

The aim is to evaluate in type-2 diabetes, the over all incretion effect is reduced. The present investigation was designed to compare insulinotropic actions of exogenous incretion hormones (GIP) and glucagons like peptide 1 (GLP -1) in nine type-2 diabetic patients and in nine age- and weight-matched normal subjects. Plasma GIP and GLP-1 concentrations were comparable to those after oral glucose with the low, and clearly supraphysiological with the high infusion rates. Both GIP and GLP-1 dose – dependently augmented insulin secretion in both groups. With GLP-1 type -2 diabetic patients reached 71% of the increments in C- peptide of normal subjects.

This paper introduces the survey of numerical solution methods for stochastic differential equations. The solution will be continuous stochastic processes that represent diffusion dynamics. We include a review of fundamental concepts, a description of elementary numerical methods and the concepts of convergence and order for stochastic differential equation solvers. The Euler – Maruyama method to the Black scholes SDE.

$w_{i+1} = w_i + \mu w_i \Delta t_i + \sigma w_i \Delta W_i$

Key Words : Glucagon like peptide -1 (GLP-1), Stochastic models, type -2 diabetics, Euler Maruyama method.

2000 Mathematics Subject Classification: 60H10.

1.1. INTRODUCTION

In normal subjects oral glucose enhances insulin secretion more than does intravenous glucose infusion. This augmentation of insulin secretion is due to the secretion and action of gut hormones[6] with insulinotropic activity, namely, gastric inhibitory polypeptide (GIP) from the upper

gut and glucagons- like peptide 1 (GLP-1) from the lower gut. In type-2 diabetic patients, the incretia effect is reduced or lost. The present study was designed to compare, the insulinotropic and glucagon - lowering actions of both synthetic human GIP and GLP -1, infused at both approximately physiological and pharmacological concentrations, in matched groups of type -2 diabetic patients and normal subjects.

2.1. NOTATIONS

μ = Shape parameter

σ =Scale parameter

ΔW_i = Random number

W_t = Wiener process.

3.1. Stochastic differential equations

Unlike deterministic models, ordinary differential equations of which have a unique solution for much appropriate initial condition, SDE's have solutions that are continuous - time stochastic processes. Methods for the computational solution of stochastic differential equation are based on similar techniques for ordinary differential equations.

We will begin with a quick survey of the most fundamental concepts from stochastic calculus that are needed to proceed with our description of numerical methods. For full details, the reader may consult [4],[9],[11].

A set of random variables X_t indexed by real numbers $t \geq 0$ is called a continuous-time stochastic process. Each instance, of the stochastic process is a choice from the random variable X_t for each t.

Any (deterministic) function f (t) can be trivially considered as a stochastic process, with variance V(f(t)) = 0 . Here the Wiener process w_t, a continuous-time stochastic process with the following three properties.

Property 1. For each t, the random variable W_t normally distributed with mean 0 and variance t.

Property 2. For each $t_1<t_2$, the normal random variable $W_{t2} - W_{t1}$ is independent of the random variable w_t and is fact independent of all W_t,

$0 \leq t \leq t_1$

Property 3. The Wiener process w_t can be represented by continuous paths.

A typical diffusion process is modeled as a differential equation involving deterministic and stochastic, the latter represented by a Wiener process, as in the equation

$dX = a(t,X)\,dt + b(t,X)dW_t$ (1)

The SDE (1) is given in a differential form, unlike the derivative form of an ODE. That is because many interesting stochastic processes, like Brownian motion, are continuous but not differentiable. Therefore the meaning of the SDE (1) is by definition, the integral equation

$$X(t) = X(0) + \int_0^t a(s,y)ds + \int_0^t b(s,y)dW_s$$

where the meaning of the last integral, called an Ito integral.

Let $c = t_o < t_1 \ldots < t_{n-1} < t_n = d$ be a grid of points on the interval [c,d].

The Riemann integral is defined as a limit

$$\int_c^d f(x)dx = \lim_{\Delta t \to 0} \sum_{i=1}^{n} f(t_i^{'})\Delta t_i \,,$$

Where $\Delta t_i = t_i - t_{i-1}$ and $t_{i-1} \leq t_i^{'} \leq t_i$.

Similarly, the Ito integral is the limit

$$\int_c^d f(t)dW_t = \lim_{\Delta t \to 0} \sum_{i=1}^{n} f(t_i - 1)\Delta W_i$$

Where $\Delta W_i = W_{t_i} - W_{t_{i-1}}$ a step of Brownian motion across the interval.

Because f and W_t are random variables, so is the Ito integral

$I = \int_c^d f(t)dW_t$. The differential dI is a notational convenience; thus

$$I = \int_c^d f \; dW_t$$

is expressed in differential form as

$dI = fdW_t$

To solve SDEs analytically, we need to introduce the chain rule for stochastic differentials, called the Ito formula [8].

If Y = f(t,X), then

$$dY = \frac{\partial f}{\partial t}(t, X)dt + \frac{\partial f}{\partial x}(t, X)dx + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(t, X)dxdx \qquad (2)$$

where the dx dx term is interpreted by using the identities

dt dt = 0

$dt\, dW_t = dW_t\, dt = 0$

$dW_t\, dW_t = dt$ (3)

The Ito formula is the stochastic analogue to the chain rule of conventional calculus.

The important features of typical stochastic differential equations can be illustrated using the Black Scholes diffusion equation:[1],[2],[3].

$dX = \mu X\, dt + \sigma X\, dW_t$

$X(0) = X_o$ (4)

The solution of the Block Scholes stochastic differential equation is geometric Brownian motion

$$X(t) = X_0 e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t} \qquad (5)$$

To check this, write X =f (t, Y) = X_0e^Y , where $Y = (\mu - \frac{1}{2}\sigma^2)t + \sigma W_t$ By the Ito formula,

$$dX = X_0 e^Y dY + \frac{1}{2} e^Y dY$$

Where $dY = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW_t$ dt. Using the differential identities from the Ito formula [2][3]

$dY\, dY = \sigma^2 dt$

and therefore

$$dX = X_0 e^Y (r - \frac{1}{2}\sigma^2)dt + X_0 e^Y \sigma dW_t + \frac{1}{2}\sigma^2 e^Y dt$$

$$= X_0 e^Y \mu dt + X_0 e^Y \sigma dW_t$$

$$= \mu X dt + \sigma X dW_t$$

as claimed.

4.1. NUMERICAL METHODS FOR SDEs

The Euler Maruyama method is the analogue of the Euler method for ordinary differential equations [6],[10]. To develop an approximate solution on the interval [c, d], assign a grid of points

$$c = t_0 < t_1 < t_2 < \ldots\ldots < t_n = d$$

Approximate x values

$$\omega_0 < \omega_1 < \omega_2 < \ldots\ldots < \omega_n$$

will be determined at the respective t points. Given the SDE initial value problem

$$dX(t) = a(t, x)dt + b(t, x)dW_t \quad (6)$$

$$x(c) = X_c$$

We compute the approximate solution as follows:

5.1. Euler – Maruyama Method

$$\omega_0 = X_0$$

$$\omega_{i+1} = \omega_i + a(t_i, \omega_i)\Delta t_{i+1} + b(t_i, \omega_i)\Delta W_{i+1} \quad (7)$$

Where

$$\Delta t_{i+1} = t_{i+1} - t_i$$

$$\Delta W_{i+1} = W(t_{i+1}) - W(t_i) \quad (8)$$

Define N(0,1) to be the standard random variable that is normally distributed with mean 0 and standard deviation 1. Each random number ΔW_i is computed [12], where z_i is chosen from N (0, 1). Each set of $\{\omega_0, \ldots, \omega_n\}$ produced by the Euler Maruyama method is an approximate realization of the solution stochastic process X(t) which depends on the random numbers Z_i that were chosen.

The Euler- Maruyama equations (7) to the Black Scholes SDE (4) have form

$$w_0 = X_o \quad (9)$$

$$w_{i+1} = w_i + \mu w_i \Delta t_i + \sigma w_i \Delta W_i$$

The simulations results of the example of fig 2 are applied in the equation (9) are shown in fig 1.

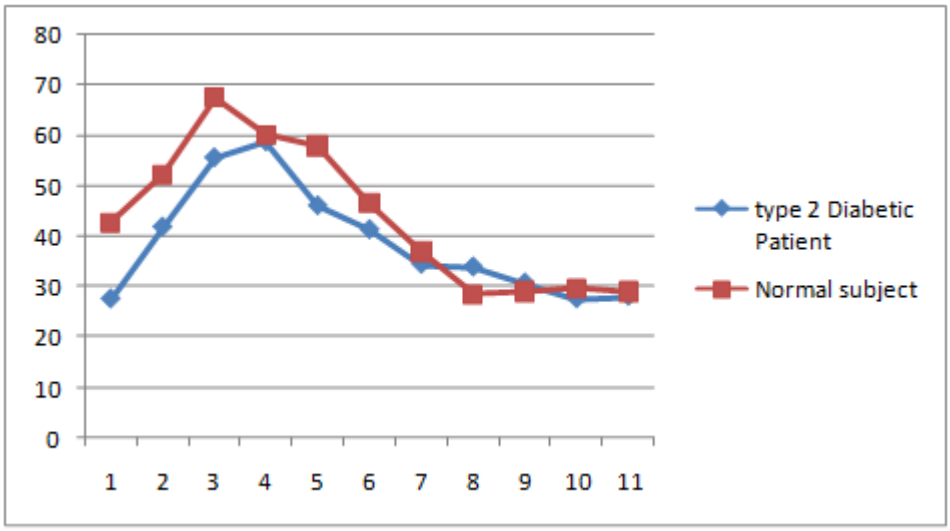

Figure 1

6.1. Example

Nine type -2 diabetic patients and nine subjects with normal glucose tolerance participated in the study. Each participant took part in examination in an oral glucose challenge (75g/30ml) [5],[7]&[8]. The tests were performed in the morning after an overnight fast. After drawing basal blood specimens at 0 minutes, oral glucose was administered.

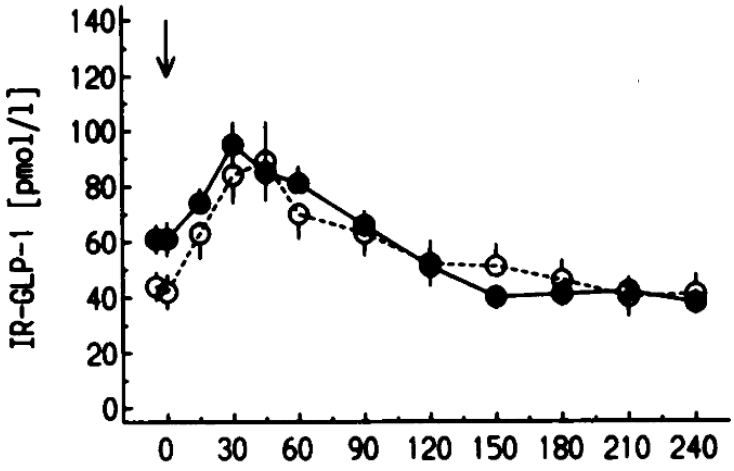

Figure 2

○ Normal Subjects

● Type -2 Diabetic Patients

Basal concentrations of immunoreactive GLP-1 were higher in type –2 diabetic patients, whereas GLP- 1 integrated incremental responses after oral glucose were lower than in normal subjects. The peak concentrations reached, however, where similar in type-2 diabetic patients and normal subjects in Fig 2.

CONCLUSION

The Mathematical Model also stresses the same cumulative effects of type-2 diabetic patients and in normal subjects which are beautifully fitted with Euler –Maruyama method to the Black Scholes SDE. (Fig-1)

The Results of these analyses indicate that in Glucagon like peptide -1 integrated incremental responses after oral glucose, the type-2 diabetic patients and normal subjects are similar in the peak concentrations(Fig.1&Fig.2). The results exactly related with the mathematical and medical report.

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