



THE RATIONAL VALUED CHARACTERS OF THE GROUP ($Q_{2m} \times C_5$) WHEN M= 2P (SUCH THAT P IS A PRIME NUMBER, P ≠ 2)

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ABSTRACT

The purpose of this work is to find the rational valued characters table of the group $(Q_{2m} \times C_5)$, when m= 2p, p is prime number, which is denoted by $\equiv^*(Q_{4p} \times C_5)$, where Q_{2m} is denoted to Quaternion group of order 4m , and C_5 is the cyclic group of order 5. Moreover we have found the general form of the rational valued characters table of the group $(Q_{4p} \times C_5)$.

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1.INTRODUCTION

Let $x, y \in G$ and G be a finite group, then x and y are said to be **σ -conjugate** if the cyclic subgroups they generate are conjugate in G . This process defines an equivalence relation on G ; its classes are called **Γ -classes**.

A finitely generated abelian group $cf(G, Z)$ of a rank equal to the number of **Γ -classes**. Is formed by the Z -valued class functions on the group G , which is constant on the **Γ -classes**. If $cf(G, Z)$ intersects with the group of all generalized characters of G , where $R(G)$ is a normal subgroup of $cf(G, Z)$ denoted by $\overline{R}(G)$, then, $cf(G, Z)/\overline{R}(G)$ is a finite abelian factor group denoted by $K(G)$. If χ is an irreducible character of the group G and σ is any element in Galois group $Gal(Q(\chi)/Q)$. Then each element in $\overline{R}(G)$ can be written as $v_1\vartheta_1 + v_2\vartheta_2 + \dots + v_r\vartheta_r$, where r is the number of **σ -classes**, $v_1, v_2, \dots, v_r \in Z$ and $\vartheta_i = \sum_{\sigma \in Gal(Q(\chi)/Q)} \sigma(\chi_i)$. Let $\equiv^*(G)$ denotes the $r \times r$ matrix which the rows corresponds

to the ϑ_i 's and the columns correspond to the **σ -classes** of G . to find the cyclic decomposition of $K(G)$, We can use the theory of invariant factors to obtain the direct sum of the cyclic Z -module of

orders the distinct invariant factors of $\equiv^*(G)$ depending on the matrix expressing $\overline{R}(G)$ basis in terms of the $cf(G, Z)$ basis is $\equiv^*(G)$.

In 1982, M. S. Kirdar [2] studied the $K(C_n)$. In 1995, N. R. Mahmood [3] studied the factor group $K(Q_{2m})$.

The aim of this paper is to find $\equiv^*(Q_{4p} \times C_5)$ and determine general (16×16) matrix from of the rational valued characters table of the group $(Q_{4p} \times C_5)$.

2. Preliminaries

The Generalized Quaternion Group Q_{2m} (3.1) [3]

In general, the generalized quaternion group of order $4m$ it can be written as :

$$Q_{2m} = \{x^k y^h, 0 \leq k \leq 2m-1, h=0,1\}$$

Which has the following properties

$$\{x^{2m} = y^4 = 1, x^m = y^2, y x^r y^{-1} = x^{-r} = x^{2m-r}\}$$

The Characters Table of the Quaternion Group Q_{2m} when $m = 2p$, and p is prime number (3.2) [3]

There are two types of irreducible characters. One of them is the character of the linear representations R_1, R_2, R_3 and R_4 which are denoted by ψ_1, ψ_2, ψ_3 and ψ_4 respectively as in the following table:

| | x^k | $x^k y$ |
|----------|----------|--------------|
| ψ_1 | 1 | 1 |
| ψ_2 | 1 | -1 |
| ψ_3 | $(-1)^k$ | $(-1)^k$ |
| ψ_4 | $(-1)^k$ | $(-1)^{k+1}$ |

Table (1)

where $0 \leq k \leq 2m-1$.

The other characters of irreducible representations T_h of degree 2 are denoted by χ_h such that:

$$\begin{aligned} \chi_h(x^k) &= \omega^{hk} + \omega^{-hk} \\ &= e^{\pi i hk/m} + e^{-\pi i hk/m} = 2\cos(\pi hk/m) \end{aligned}$$

We are denote to $(\omega^{hk} + \omega^{-hk})$ by V_{hk} , thus $V_{hk} = V_{2m-hk}, V_m = -2, V_{2m} = 2$, also we will write $V_{J(hk)}$ such that $J(hk) = \min\{hk \pmod{2m}, 2m-hk \pmod{2m}\}$ in the character table of the quaternion group Q_{2m} when m is an even number, such that:

$$V_{J(hk)} = 2\cos(\pi J(hk)/m), \chi_h(x^k y) = 0$$

where $0 \leq k \leq 2m-1, 1 \leq h \leq m-1$ and $\omega = e^{2\pi i/2m} = e^{\pi i/m}$.

So, there are $m+3$ irreducible characters of Q_{2m} . Then the general form of the characters table of Q_{2m} when $m=2p$, p is prime number, $p \neq 2$ is given in the table(2)

| CLa | [1] | [X^2] | [X^4] | ... | [x^{m-2}] | [x^m] | [x] | [x^3] | ... | [x^{m-1}] | [y] | [xy] |
|----------|-----|------------|------------|-----|---------------|-----------|-------|------------|-----|---------------|-----|------|
| CLa | 1 | 2 | 2 | ... | 2 | 1 | 2 | 2 | ... | 2 | m | m |
| ψ_1 | 1 | 1 | 1 | ... | 1 | 1 | 1 | 1 | ... | 1 | 1 | 1 |
| ψ_4 | 1 | 1 | 1 | ... | 1 | 1 | -1 | -1 | ... | -1 | -1 | 1 |
| χ_2 | 2 | $V_{J(4)}$ | $V_{J(8)}$ | ... | $V_{J(2m-4)}$ | 2 | v_2 | $V_{J(6)}$ | ... | $V_{J(2m-2)}$ | 0 | 0 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

| | | | | | | | | | | | | |
|--------------|----------|---------------|---------------|----------|--------------------|----------|--------------|---------------|----------|--------------------|----------|----------|
| χ_{m-2} | 2 | $V_{j(2m-4)}$ | $V_{j(4m-8)}$ | ... | $V_{((m-2)(m-2))}$ | 2 | $V_{j(m-2)}$ | $V_{j(3m-6)}$ | ... | $V_{((m-2)(m-1))}$ | 0 | 0 |
| χ_1 | 2 | v_2 | $V_{j(4)}$ | ... | $V_{j(m-4)}$ | -2 | V_1 | $V_{j(3)}$ | ... | $V_{j(m-1)}$ | 0 | 0 |
| \vdots | \vdots | \vdots | \vdots | \ddots | \vdots | \vdots | \vdots | \vdots | \ddots | \vdots | \vdots | \vdots |
| χ_{m-1} | 2 | $V_{j(2m-2)}$ | $V_{j(4m-4)}$ | ... | $V_{((m-1)(m-2))}$ | -2 | $V_{j(m-1)}$ | $V_{j(3m-3)}$ | ... | $V_{((m-1)(m-1))}$ | 0 | 0 |
| ψ_2 | 1 | 1 | 1 | ... | 1 | 1 | 1 | 1 | ... | 1 | -1 | -1 |
| ψ_3 | 1 | 1 | 1 | ... | 1 | 1 | -1 | -1 | ... | -1 | 1 | -1 |

Table (2)

Theorem(3.3)[1]:

Let $T_1: G_1 \rightarrow GL(n, F)$ and $T_2: G_2 \rightarrow GL(m, F)$ be two irreducible representations of the groups G_1 and G_2 with characters χ_1 and χ_2 respectively then : $T_1 \otimes T_2$ is irreducible representation of the group $G_1 \times G_2$ with the character $\chi_1 \cdot \chi_2$.

.The Group $Q_{2m} \times C_5$ (3.4)

Let Q_{2m} is the quaternion group of order $4m$, and C_5 is the cyclic group of order 5, then the group ($Q_{2m} \times C_5$) is the direct product group of order $20m$ therefore the irreducible representations of the group ($Q_{2m} \times C_5$) is a tensor products of each an irreducible representation of Q_{2m} and an irreducible representation of the group C_5 . The group C_5 has five irreducible representations; their characters $\delta_1, \delta_2, \delta_3, \delta_4$ and δ_5 are given in the table(3)

$$\equiv C_5 =$$

| CL_α | [1] | [r] | [r^2] | [r^3] | [r^4] |
|-----------------|-----|------------|------------|------------|------------|
| $ CL_\alpha $ | 1 | 1 | 1 | 1 | 1 |
| δ_1 | 1 | 1 | 1 | 1 | 1 |
| δ_2 | 1 | α | α^2 | α^3 | α^4 |
| δ_3 | 1 | α^2 | α^4 | α | α^3 |
| δ_4 | 1 | α^3 | α | α^4 | α^2 |
| δ_5 | 1 | α^4 | α^3 | α^2 | α |

Where $\alpha = e^{2\pi i/5} = \cos 2\pi/5 + i \sin 2\pi/5$

According to proposition (3.3), each irreducible character χ_i of Q_{2m} defines five characters $\chi_{i1}, \chi_{i2}, \chi_{i3}, \chi_{i4}$ and χ_{i5} such that $\chi_{i1} = \chi_i \delta_1, \chi_{i2} = \chi_i \delta_2, \chi_{i3} = \chi_i \delta_3, \chi_{i4} = \chi_i \delta_4, \chi_{i5} = \chi_i \delta_5$ Then $\equiv (Q_{2m} \times C_5) \equiv (Q_{2m}) \otimes \equiv (C_5)$.

Example (3.5)

To find characters table $Q_{28} \times C_5$ by Theorem (3.3) we have the characters table Q_{28} as Table (4), where $v_i = 2 \cos(\pi i / 14)$, $v_m = 2$, $v_7 = -2$, $v_7 = 2 \cos(7\pi / 14) = 2 \cos(\pi / 2) = 0$,

| CL_β | [1] | $[x^2]$ | $[x^4]$ | $[x^6]$ | $[x^8]$ | $[x^{10}]$ | $[x^{12}]$ | $[x^{14}]$ | [x] | $[x^3]$ | $[x^5]$ | $[x^7]$ | $[x^9]$ | $[x^{11}]$ | $[x^{13}]$ | [y] | $[xy]$ |
|--------------|-----|----------|----------|----------|----------|------------|------------|------------|----------|----------|----------|---------|----------|------------|------------|-----|--------|
| $ CL_\beta $ | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 14 | 14 |
| Ψ_1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Ψ_4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 |
| χ_2 | 2 | V_4 | V_8 | V_{12} | V_{12} | V_8 | V_4 | 2 | V_2 | V_6 | V_{10} | -2 | V_{10} | V_6 | V_2 | 0 | 0 |
| χ_4 | 2 | V_8 | V_{12} | V_4 | V_4 | V_{12} | V_8 | 2 | V_4 | V_{12} | V_8 | 2 | V_8 | V_{12} | V_4 | 0 | 0 |
| χ_6 | 2 | V_{12} | V_4 | V_8 | V_8 | V_4 | V_{12} | 2 | V_6 | V_{10} | V_2 | -2 | V_2 | V_{10} | V_6 | 0 | 0 |
| χ_8 | 2 | V_{12} | V_4 | V_8 | V_8 | V_4 | V_{12} | 2 | V_8 | V_4 | V_{12} | 2 | V_{12} | V_4 | V_8 | 0 | 0 |
| χ_{10} | 2 | V_8 | V_{12} | V_4 | V_4 | V_{12} | V_8 | 2 | V_{10} | V_2 | V_6 | -2 | V_6 | V_2 | V_{10} | 0 | 0 |
| χ_{12} | 2 | V_4 | V_8 | V_{12} | V_{12} | V_8 | V_4 | 2 | V_{12} | V_8 | V_4 | 2 | V_4 | V_8 | V_{12} | 0 | 0 |
| χ_1 | 2 | V_2 | V_4 | V_6 | V_8 | V_{10} | V_{12} | -2 | V_1 | V_3 | V_5 | 0 | V_9 | V_{11} | V_{13} | 0 | 0 |
| χ_3 | 2 | V_6 | V_{12} | V_{10} | V_4 | V_2 | V_8 | -2 | V_3 | V_9 | V_{13} | 0 | V_1 | V_5 | V_{11} | 0 | 0 |
| χ_5 | 2 | V_{10} | V_8 | V_2 | V_{12} | V_6 | V_4 | -2 | V_5 | V_{13} | V_3 | 0 | V_{11} | V_1 | V_9 | 0 | 0 |
| χ_7 | 2 | -2 | 2 | -2 | 2 | -2 | 2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| χ_9 | 2 | V_{10} | V_8 | V_2 | V_{12} | V_6 | V_4 | -2 | V_9 | V_1 | V_{11} | 0 | V_3 | V_{13} | V_5 | 0 | 0 |
| χ_{11} | 2 | V_6 | V_{12} | V_{10} | V_4 | V_2 | V_8 | -2 | V_{11} | V_5 | V_1 | 0 | V_{13} | V_9 | V_3 | 0 | 0 |
| χ_{13} | 2 | V_2 | V_4 | V_6 | V_8 | V_{10} | V_{12} | -2 | V_{13} | V_{11} | V_9 | 0 | V_5 | V_3 | V_1 | 0 | 0 |
| Ψ_2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 |
| Ψ_3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | -1 | -1 |

Table(4)

And the characters table C_5 as table (3), and from(3.3) the characters table $Q_{28} \times C_5$ can be written as follows :

$$\equiv(Q_{28} \times C_5) = \equiv(Q_{28}) \otimes \equiv(C_5),$$

Then $\equiv(Q_{28} \times C_5)$ is given in the table (5) .

where $v_i = 2 \cos(\pi i / 14)$, $v_{2m} = 2$, $v_m = -2$, $v_7 = 2 \cos(\pi / 2) = 0$.

4. The main results

Proposition(4.1)[2]

The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$ form basis for $\overline{R}(G)$, wher χ_i are the irreducible characters of G , and the numbers of θ_i are equal to the number of all distinct Γ -classes of G .

Proposition(4.2)[2]

The rational valued characters table of the cyclic group C_{p^s} of rank $s+1$ where p is a prime number which is denoted by $\equiv^*(C_{p^s})$, is given as follows:

| Γ -classes | [r] | [r^p] | [r^{p^2}] | ... | [$r^{p^{s-3}}$] | [$r^{p^{s-2}}$] | [$r^{p^{s-1}}$] | [I] |
|-------------------|----------|-----------|---------------|----------|-------------------|-------------------|-------------------|----------------|
| θ_1 | 1 | 1 | 1 | ... | 1 | 1 | 1 | 1 |
| θ_2 | -1 | $p-1$ | $p-1$ | ... | $p-1$ | $p-1$ | $p-1$ | $p-1$ |
| θ_3 | 0 | $-p$ | $p(p-1)$ | ... | $p(p-1)$ | $p(p-1)$ | $p(p-1)$ | $p(p-1)$ |
| \vdots | \vdots | \vdots | \vdots | \ddots | \vdots | \vdots | \vdots | \vdots |
| θ_{s-1} | 0 | 0 | 0 | ... | $-p^{s-3}$ | $p^{s-3}(p-1)$ | $p^{s-3}(p-1)$ | $p^{s-3}(p-1)$ |
| θ_s | 0 | 0 | 0 | ... | 0 | $-p^{s-2}$ | $p^{s-2}(p-1)$ | $p^{s-2}(p-1)$ |
| θ_{s+1} | 0 | 0 | 0 | ... | 0 | 0 | $-p^{s-1}$ | $p^{s-1}(p-1)$ |

Table(5)

where its rank $s+1$ represents the number of all distinct Γ -classes

The rational character table of the quaternion group Q_{2m} when $m=2p$, p is prime number , $p \neq 2$ (4.3) [3]

The rational characters table of Q_{2m} when $m = 2p$, p is prime number , $p \neq 2$ is given in the following table (after change order the rows and the columns) $\equiv^*(Q_{4p}) =$

| | | | | | | | |
|----|----|----|----|----|----|--------|--------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 |
| -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | -2 | 2 | -2 | 2 |
| 0 | 0 | 1 | -2 | -1 | -1 | $p-1$ | $p-1$ |
| 0 | 0 | -1 | 2 | -1 | -1 | $p-1$ | $p-1$ |
| 0 | 0 | 0 | 0 | 2 | 2 | $2p-2$ | $2p-2$ |

Table(6)

Example(4.4)

To construct the rational valued characterstable of $(Q_{28} \times C_5)$ when havetodothe following :

From Example(3.5) we have the characterstable of $Q_{28} \times C_5$

By the definition of $Q_{28} \times C_5$:

$$\equiv(Q_{28} \times C_5) = \equiv(Q_{28}) \otimes \equiv(C_5)$$

To calculate the rational valued characterstable of the group

$$(Q_{28} \times C_5) = \equiv^*(Q_{28} \times C_5)$$

$$\theta_{11} = \Psi_{11}, \theta_{12} = \Psi_{12} + \Psi_{13} + \Psi_{14} + \Psi_{15}$$

$$\theta_{21} = \Psi_{41}, \theta_{22} = \Psi_{42} + \Psi_{43} + \Psi_{44} + \Psi_{45}$$

$$\theta_{71} = \Psi_{21}, \theta_{72} = \Psi_{22} + \Psi_{23} + \Psi_{24} + \Psi_{25}$$

$$\theta_{81} = \Psi_{31}, \theta_{82} = \Psi_{32} + \Psi_{33} + \Psi_{34} + \Psi_{35}$$

$$\theta_{51} = \chi_{71}, \theta_{52} = \chi_{72} + \chi_{73} + \chi_{74} + \chi_{75}$$

The elements of Gal(ϕ_{1i}) / Q are :

$$\{\sigma_{1i}, \sigma_{2i}, \sigma_{3i}, \sigma_{4i}, \sigma_{5i}, \sigma_{6i}, \sigma_{8i}, \sigma_{9i}, \sigma_{10i}, \sigma_{11i}, \sigma_{12i}, \sigma_{13i}\}$$

$$\sigma_{1i}(\phi_{1i}) = \phi_{1i}, \sigma_{2i}(\phi_{1i}) = \phi_{2i}, \sigma_{3i}(\phi_{1i}) = \phi_{3i}, \sigma_{4i}(\phi_{1i}) = \phi_{4i}, \sigma_{5i}(\phi_{1i}) = \phi_{5i}$$

$$\sigma_{6i}(\phi_{1i}) = \phi_{6i}, \sigma_{8i}(\phi_{1i}) = \phi_{8i}, \sigma_{9i}(\phi_{1i}) = \phi_{9i}, \sigma_{10i}(\phi_{1i}) = \phi_{10i}, \sigma_{11i}(\phi_{1i})$$

$$= \phi_{11i}, \sigma_{12i}(\phi_{1i}) = \phi_{12i}, \sigma_{13i}(\phi_{1i}) = \phi_{13i},$$

and i=1,2,3,4,5

By proposition (4. 1)

1- (I) if i =1

$$\theta_{61} = \sigma_{(1,1)}(\varphi_{11}) + \sigma_{(3,1)}(\varphi_{11}) + \sigma_{(5,1)}(\varphi_{11}) + \sigma_{(9,1)}(\varphi_{11}) + \sigma_{(11,1)}(\varphi_{11}) + \sigma_{(13,1)}(\varphi_{11})$$

$$= \varphi_{(1,1)} + \varphi_{(3,1)} + \varphi_{(5,1)} + \varphi_{(9,1)} + \varphi_{(11,1)} + \varphi_{(13,1)}$$

$$\theta_{61}([I, I]) = \theta_{61}([I, r]) = 2+2+2+2+2+2=12$$

$$\theta_{61}([x^2, I]) = \theta_{61}([x^2, r]) = (V_2 + V_6 + V_{10}) + (V_{10} + V_6 + V_2) = 1+1=2$$

$$\theta_{61}([x^4, I]) = \theta_{61}([x^4, r]) = (V_4 + V_{12} + V_8) + (V_8 + V_{12} + V_4) = (-1) + (-1) = -2$$

$$\theta_{61}([x^{14}, I]) = \theta_{61}([x^{14}, r]) = (-2) + (-2) + (-2) + (-2) + (-2) + (-2) = -12$$

$$\theta_{61}([x, I]) = \theta_{61}([x, r]) = V_1 + V_3 + V_5 + V_9 + V_{11} + V_{13} = 0$$

$$\theta_{61}([x^7, I]) = \theta_{61}([x^7, r]) = 0+0+0+0+0+0 = 0$$

$$\theta_{61}([y, I]) = \theta_{61}([y, r]) = 0+0+0+0+0+0 = 0$$

$$\theta_{61}([xy, I]) = \theta_{61}([xy, r]) = 0+0+0+0+0+0 = 0$$

$$\theta_{4i} = \varphi_{2i} + \varphi_{6i} + \varphi_{10i}$$

$$\theta_{41} = \varphi_{(2,1)} + \varphi_{(6,1)} + \varphi_{(10,1)}$$

$$\theta_{41}([I, I]) = \theta_{41}([I, r]) = 2+2+2=6$$

$$\theta_{41}([x^2, I]) = \theta_{41}([x^2, r]) = V_4 + V_{12} + V_8 = -1$$

$$\theta_{41}([x^4, I]) = [x^4, r] = V_8 + V_4 + V_{12} = -1$$

$$\theta_{41}([x^{14}, I]) = \theta_{41}([x^{14}, r]) = 2+2+2=6$$

$$\theta_{41}([x, I]) = \theta_{41}([x, r]) = V_2 + V_6 + V_{10} = 1$$

$$\theta_{41}([x^7, I]) = \theta_{41}([x^7, r]) = (-2) + (-2) + (-2) = -6$$

$$\theta_{41}([y, I]) = \theta_{41}([y, r]) = 0+0+0+0+0+0 = 0$$

$$\theta_{41}([xy, I]) = \theta_{41}([xy, r]) = 0+0+0+0+0+0 = 0$$

$$\theta_{3i} = \varphi_{4i} + \varphi_{8i} + \varphi_{12i}$$

$$\theta_{(3,1)} = \varphi_{(4,1)} + \varphi_{(8,1)} + \varphi_{(12,1)}$$

$$\theta_{31}([I, I]) = \theta_{31}([I, r]) = 2+2+2=6$$

$$\theta_{31}([x^2, I]) = \theta_{31}([x^2, r]) = V_8 + V_{12} + V_4 = -1$$

$$\theta_{31}([x^4, I]) = \theta_{31}([x^4, r]) = V_{12} + V_4 + V_8 = -1$$

$$\theta_{31}([x^{14}, I]) = \theta_{31}([x^{14}, r]) = 2+2+2=6$$

$$\theta_{31}([x, I]) = \theta_{31}([x, r]) = V_8 + V_4 + V_{12} = -1$$

$$\theta_{31}([x^7, I]) = \theta_{31}([x^7, r]) = 2+2+2=6$$

$$\theta_{31}([y, I]) = \theta_{31}([y, r]) = 0+0+0+0+0+0 = 0$$

$$\theta_{31}([xy, I]) = \theta_{31}([xy, r]) = 0+0+0+0+0+0 = 0$$

2- (II) if i =2,3,4,5

$$\theta_{62} = \sigma_{(1,2)}(\varphi_{12}) + \sigma_{(1,3)}(\varphi_{13}) + \sigma_{(1,4)}(\varphi_{14}) + \sigma_{(1,5)}(\varphi_{15}) + \sigma_{(3,2)}(\varphi_{12}) + \sigma_{(3,3)}(\varphi_{13}) + \sigma_{(3,4)}(\varphi_{14})$$

$$+ \sigma_{(3,5)}(\varphi_{15}) + \sigma_{(5,2)}(\varphi_{12}) + \sigma_{(5,3)}(\varphi_{13}) + \sigma_{(5,4)}(\varphi_{14}) + \sigma_{(5,5)}(\varphi_{15}) + \sigma_{(9,2)}(\varphi_{12}) + \sigma_{(9,3)}(\varphi_{13}) + \sigma_{94}(\varphi_{14})$$

There are sixteen Γ -classes in $\equiv^*(Q_{28} \times C_5)$

$$[I, I] = \{[I, I]\},$$

$$[I, r] = \{[I, r], [I, r^2], [I, r^3], [I, r^4]\},$$

$$[x^2, I] = \{[x^2, I], [x^6, I], [x^{10}, I]\},$$

$$[x^2, r] = \{$$

$$[x^2, r], [x^2, r^2], [x^2, r^3], [x^2, r^4], [x^6, r], [x^6, r^2], [x^6, r^3], [x^6, r^4], [x^{10}, r], [x^{10}, r^2],$$

$$[x^{10}, r^3], [x^{10}, r^4]\},$$

$$[x^4, I] = \{[x^4, I], [x^8, I], [x^{12}, I]\},$$

$$[x^4, r] = \{[x^4, r], [x^4, r^2], [x^4, r^3], [x^4, r^4],$$

$$[x^8, r], [x^8, r^2], [x^8, r^3], [x^8, r^4], [x^{12}, r], [x^{12}, r^2], [x^{12}, r^3], [x^{12}, r^4]\},$$

$$[x^{14}, I] = \{[x^{14}, I]\},$$

$$[x^{14}, r] = \{[x^{14}, r], [x^{14}, r^2], [x^{14}, r^3], [x^{14}, r^4]\},$$

$$[x, I] = \{[x, I], [x^3, I], [x^5, I], [x^9, I], [x^{11}, I], [x^{13}, I]\},$$

$$[x, r] = \{[x, r], [x, r^2], [x, r^3], [x, r^4], \{[x^3, r], [x^3, r^2], [x^3, r^3], [x^3, r^4],$$

$$[x^5, r], [x^5, r^2], [x^5, r^3], [x^5, r^4], [x^9, r], [x^9, r^2], [x^9, r^3], [x^9, r^4]\},$$

$$[x^{11}, r], [x^{11}, r^2], [x^{11}, r^3], [x^{11}, r^4], [x^{13}, r], [x^{13}, r^2], [x^{13}, r^3], [x^{13}, r^4]\},$$

$$[x^7, I] = \{[x^7, I]\},$$

$$[x^7, r] = \{[x^7, r], [x^7, r^2], [x^7, r^3], [x^7, r^4]\},$$

$$[y, I] = \{[y, I], [xy, I]\}$$

$$[y, r] = \{[y, r], [y, r^3], [y, r^4], \{[xy, r], [xy, r^2], [xy, r^3], [xy, r^4]\}\}$$

can write the rational character table of $(Q_{28} \times C_5)$ as table(8).

| Γ -classes | [I, I] | [I, r] | [x ² , I] | [x ² , r] | [x ⁴ , I] | [x ⁴ , r] | [x ¹⁴ , I] | [x ¹⁴ , r] | [x, I] | [x, r] | [x ⁷ , I] | [x ⁷ , r] | [y, I] | [y, r] | [xy, I] | [xy, r] |
|-------------------|--------|--------|----------------------|----------------------|----------------------|----------------------|-----------------------|-----------------------|--------|--------|----------------------|----------------------|--------|--------|---------|---------|
| CLa | 1 | 1 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 2 | 2 | 2 | 14 | 14 | 14 | 14 |
| θ_{11} | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| θ_{12} | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 |
| θ_{21} | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 |
| θ_{22} | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | -4 | 1 | -4 | 1 | -4 | 1 | 4 | -1 |
| θ_{31} | 6 | 6 | -1 | -1 | -1 | -1 | 6 | 6 | -1 | -1 | 6 | 6 | 0 | 0 | 0 | 0 |
| θ_{32} | 24 | -6 | -4 | 1 | -4 | 1 | 24 | -6 | -4 | -1 | 24 | -6 | 0 | 0 | 0 | 0 |
| θ_{41} | 6 | 6 | -1 | -1 | -1 | -1 | 6 | 6 | 1 | 1 | -6 | -6 | 0 | 0 | 0 | 0 |
| θ_{42} | 24 | -6 | -4 | 1 | -4 | 1 | 24 | -6 | 4 | -1 | -24 | 6 | 0 | 0 | 0 | 0 |
| θ_{51} | 2 | 2 | -2 | -2 | 2 | 2 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| θ_{52} | 8 | -2 | -8 | 2 | 8 | -2 | -8 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| θ_{61} | 12 | 12 | 2 | 2 | -2 | -2 | -12 | -12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| θ_{62} | 48 | -12 | 8 | -2 | -8 | 2 | -48 | 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| θ_{71} | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 |
| θ_{72} | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | -4 | 1 | -4 | 1 |
| θ_{81} | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 |
| θ_{82} | 4 | -1 | 4 | -1 | 4 | -1 | 4 | -1 | -4 | 1 | -4 | 1 | 4 | -1 | -4 | 1 |

Table(8)

Theorem (4.5)

If m is an even number then:

$$\equiv^*(Q_{2m} \times C_5) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_5)$$

where $\equiv^*(Q_{2m} \times C_5)$ is the rational valued characters table of the group $Q_{2m} \times C_5$. **Proof:-**

Let φ_i be irreducible character of Q_{2m} , ϕ_j is an irreducible character of C_5

$i=1,2,3,\dots,m+3, j=1,2$

From the definition (3.4) of $(Q_{2m} \times C_5)$ and theorem (3.3)

$$\equiv(Q_{2m} \times C_5) = \equiv(Q_{2m}) \otimes \equiv(C_5)$$

each element in $(Q_{2m} \times C_5)$

is $u \in (Q_{2m} \times C_5), u = (q, c)$

$q \in Q_{2m}$ and $c \in C_5, c = r^i, i = 0, \dots, 4$

$$q = x^s y^k, 0 \leq s \leq 2m-1, k=0,1$$

since

| CL_α | $[I]$ | $[r]$ | $[r^2]$ | $[r^3]$ | $[r^4]$ |
|---------------|-------|------------|------------|------------|------------|
| $ CL_\alpha $ | 1 | 1 | 1 | 1 | 1 |
| ϕ_1 | 1 | 1 | 1 | 1 | 1 |
| ϕ_2 | 1 | α | α^2 | α^3 | α^4 |
| ϕ_3 | 1 | α^2 | α^4 | α | α^3 |
| ϕ_4 | 1 | α^3 | α | α^4 | α^2 |
| ϕ_5 | 1 | α^4 | α^3 | α^2 | α |

Then, $\phi_1(I) = \theta_1(r) = 1$

$$\phi_1(r) = \phi_1(r^2) = \phi_1(r^3) = \phi_1(r^4) = \theta_1(r^i) = 1, i = 1, \dots, 4$$

$$\phi_2(I) + \phi_3(I) + \phi_4(I) + \phi_5(I) = \theta_2(I) = 1 + 1 + 1 + 1 = 4$$

$$\phi_2(r) + \phi_3(r) + \phi_4(r) + \phi_5(r) = \theta_2(r) = \alpha + \alpha^2 + \alpha^3 + \alpha^4 = -1$$

$$\text{Where } \alpha + \alpha^2 + \alpha^3 + \alpha^4 = -1$$

And from (theorem (4.1)), each irreducible character of $(Q_{2m} \times C_5)$ is

$$\varphi_{ij}(u) = \varphi_{ij}(q, c) = \varphi_i(q) \cdot \phi_j(c)$$

Then,

$$\varphi_{ij}(u) = \varphi_{ij}(q, c) = \varphi_i(q) \cdot \phi_j(c) = \begin{cases} \varphi_i(q) \text{ if } j=1 \text{ for all } c \\ 4\varphi_i(q) \text{ if } j=2, 3, 4, 5 \text{ and } c=I \\ -\varphi_i(q) \text{ if } j=2, 3, 4, 5 \text{ and } c \neq I \end{cases}$$

And by proposition (4.2)

| \bar{r} - classes | $[I]$ | $[r]$ |
|---------------------|-------|-------|
| $\bar{\theta}$ | 1 | 1 |
| $\bar{\theta}$ | 4 | -1 |

Then,

$$\theta_1(l) = \hat{\theta}_1(l) = 1, \hat{\theta}_2(l) = 4 \text{ and } \hat{\theta}_2(r) = -1$$

From proposition (4.1)

$$\theta_{ij} = \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_{ij})$$

where θ_{ij} is the rational valued characters table of $(Q_{2m} \times C_5)$ then,

$$\theta_{ij}(u) = \theta_{ij} = \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_{ij}(u)) = \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_i(q) \cdot \varphi_j(c))$$

$$(I) \quad \text{if } j=1, \varphi_j(c) = 1$$

$$\theta_{ij}(u) = \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_i(q) \cdot \varphi_j(c)) = \theta_i(q) \cdot 1 = \theta_i(q) \cdot \hat{\theta}(l)$$

where θ_l is the rational valued characters table of Q_{2m}

$$(II) a - \text{if } j=2, 3, 4, 5 \text{ and } c=l$$

$$\theta_{ij}(u) = \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_i(q) \cdot \varphi_j(l)) = \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_i(q) \cdot [\sum_{\sigma \in C_5} \sigma \varphi_j(l)])$$

$$= \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_i(q) [1+1+1+1]) = \theta_l(q) \cdot 4 = \theta_l(q) \cdot \hat{\theta}_j(l)$$

$$b - \text{if } j=2, 3, 4, 5 \text{ and } c \neq l$$

$$\theta_{ij}(u) = \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_i(q) \cdot [\sum_{\sigma \in C_5} \sigma \varphi_j(c)])$$

$$= \sum_{\sigma \in \text{Gal}(Q(\varphi_{ij})/Q)} \sigma(\varphi_i(q) [\alpha + \alpha^2 + \alpha^3 + \alpha^4])$$

$$= -\sum_{\sigma} (\varphi_l(q)) = \sum_{\sigma} (\varphi_l(q)) \cdot -1 = \theta_l(q) \cdot \hat{\theta}_j(c)$$

$$\sigma \in \text{Gal}(Q(\varphi_l(q))/Q)$$

$$\sigma \in \text{Gal}(Q(\varphi_l(q))/Q)$$

From (I) and (II) we have $\theta_{ij} = \theta_l \cdot \hat{\theta}_j$

$$\text{Then } \equiv^*(Q_{2m} \times C_5) = \equiv^*(Q_{2m}) \otimes \equiv^*(C_5)$$

The rational character table of the quaternion group $(Q_{2m} \times C_5)$ when $m = 2p$, p is prime number, $p \neq 2$ (4.6)

From Theorem (4.5) and form of $\equiv^*(Q_{4p})$ in then table (7) then the rational character table of the quaternion group $(Q_{2m} \times C_5)$ when $m = 2p$, p is prime number given in the general (16×16)

matrix form $\equiv^*(P_{4m} \times C_5)$ as table (9).

| | | | | | | | | | | | | | | | | | | | |
|----|----|----|----|----|----|------|------|----|----|----|----|----|----|------|------|------|------|------|------|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | -2 | 2 | -2 | 2 | 0 | 0 | 0 | 0 | -2 | 2 | -2 | 2 | -2 | 2 | -2 | 2 |
| 0 | 0 | 1 | -2 | -1 | -1 | p-1 | p-1 | 0 | 0 | 1 | -2 | -1 | -1 | p-1 | p-1 | p-1 | p-1 | p-1 | p-1 |
| 0 | 0 | -1 | 2 | -1 | -1 | p-1 | p-1 | 0 | 0 | -1 | 2 | -1 | -1 | p-1 | p-1 | p-1 | p-1 | p-1 | p-1 |
| 0 | 0 | 0 | 0 | 2 | -2 | 2-2p | 2p-2 | 0 | 0 | 0 | 0 | 2 | -2 | 2-2p | 2p-2 | 2p-2 | 2p-2 | 2p-2 | 2p-2 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| -4 | 4 | -4 | -4 | 4 | 4 | 4 | 4 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| -4 | -4 | 4 | 4 | 4 | 4 | 4 | 4 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 4 | -4 | -4 | -4 | 4 | 4 | 4 | 4 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 0 | 0 | 0 | 0 | -8 | 8 | -8 | 8 | 0 | 0 | 0 | 0 | 2 | -2 | 2 | -2 | 2 | -2 | 2 | -2 |
| 0 | 0 | 4 | -8 | -4 | -4 | 4p-4 | 4p-4 | 0 | 0 | -1 | 2 | 1 | 1 | 1-p | 1-p | 1-p | 1-p | 1-p | 1-p |
| 0 | 0 | -4 | 8 | -4 | -4 | 4p-4 | 4p-4 | 0 | 0 | 1 | -2 | 1 | 1 | 1-p | 1-p | 1-p | 1-p | 1-p | 1-p |
| 0 | 0 | 0 | 0 | 8 | -8 | 8-8p | 8p-8 | 0 | 0 | 0 | 0 | -2 | -2 | 2p-2 | 2p-2 | 2p-2 | 2p-2 | 2p-2 | 2p-2 |

Table(5)

| $[x^{10},l]$ | $[x^{10},r]$ | $[x^{10},r^2]$ | $[x^{10},r^3]$ | $[x^{10},r^4]$ | $[x^{12},l]$ | $[x^{12},r]$ | $[x^{12},r^2]$ | $[x^{12},r^3]$ | $[x^{12},r^4]$ | $[x^{14},l]$ | $[x^{14},r]$ | $[x^{14},r^2]$ | $[x^{14},r^3]$ | $[x^{14},r^4]$ | $[x,l]$ | $[x,r]$ | $[x,r^2]$ | $[x,r^3]$ | $[x,r^4]$ |
|--------------|-------------------|-------------------|-------------------|-------------------|--------------|-------------------|-------------------|-------------------|-------------------|--------------|--------------|----------------|----------------|----------------|---------|----------------|----------------|----------------|----------------|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 |
| 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ |
| 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ |
| v_8 | v_8 | v_8 | v_8 | v_8 | v_4 | v_4 | v_4 | v_4 | v_4 | 2 | 2 | 2 | 2 | 2 | v_2 | v_2 | v_2 | v_2 | |
| v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ |
| v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ |
| v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ |
| v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 |
| v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | v_8 | v_8 | v_8 | v_8 | v_8 | 2 | 2 | 2 | 2 | 2 | v_4 | v_4 | v_4 | v_4 | |
| v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | 2 | 2α | $2\alpha^2$ | $2\alpha^2$ | $2\alpha^4$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ |
| v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ |
| v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ |
| v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 |
| v_4 | v_4 | v_4 | v_4 | v_4 | v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | 2 | 2 | 2 | 2 | 2 | v_6 | v_6 | v_6 | v_6 | |
| v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | 2 | 2α | $2\alpha^2$ | $2\alpha^2$ | $2\alpha^4$ | v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ |
| v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^2$ | v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ |
| v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | 2 | $2\alpha^2$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ |
| v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | 2 | $2\alpha^4$ | $2\alpha^2$ | 2α | $2\alpha^2$ | v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 |

Table(5)

| $[x^3, l]$ | $[x^3, r]$ | $[x^3, r^2]$ | $[x^3, r^3]$ | $[x^3, r^4]$ | $[x^5, l]$ | $[x^5, r]$ | $[x^5, r^2]$ | $[x^5, r^3]$ | $[x^5, r^4]$ | $[x^7, l]$ | $[x^7, r]$ | $[x^7, r^2]$ | $[x^7, r^3]$ | $[x^7, r^4]$ | $[x^9, l]$ | $[x^9, r]$ | $[x^9, r^2]$ | $[x^9, r^3]$ | $[x^9, r^4]$ |
|------------|-------------------|-------------------|-------------------|-------------------|------------|-------------------|-------------------|-------------------|-------------------|------------|---------------|---------------|---------------|---------------|------------|-------------------|-------------------|-------------------|-------------------|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 |
| 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | |
| -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ |
| -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ |
| -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ |
| -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ |
| v_6 | v_6 | v_6 | v_6 | v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | -2 | -2 | -2 | -2 | v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | |
| v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | -2 | -2 α | -2 α^2 | -2 α^3 | -2 α^4 | v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ |
| v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | -2 | -2 α^2 | -2 α^4 | -2 α | -2 α^3 | v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ |
| v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | -2 | -2 α^3 | -2 α | -2 α^4 | -2 α^2 | v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ |
| v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | -2 | -2 α^4 | -2 α^3 | -2 α^2 | -2 α | v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} |
| v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | v_8 | v_8 | v_8 | v_8 | v_8 | 2 | 2 | 2 | 2 | v_8 | v_8 | v_8 | v_8 | v_8 | |
| v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | 2 | 2 α | 2 α^2 | 2 α^2 | 2 α^4 | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ |
| v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | 2 | 2 α^2 | 2 α^4 | 2 α | 2 α^3 | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ |
| v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | 2 | 2 α^3 | 2 α | 2 α^4 | 2 α^2 | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ |
| v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | 2 | 2 α^4 | 2 α^3 | 2 α^2 | 2 α | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 |
| v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | v_2 | v_2 | v_2 | v_2 | v_2 | -2 | -2 | -2 | -2 | v_2 | v_2 | v_2 | v_2 | v_2 | |
| v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | -2 | -2 α | -2 α^2 | -2 α^2 | -2 α^4 | v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ |
| v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ | -2 | -2 α^2 | -2 α^4 | -2 α | -2 α^2 | v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ |
| v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | -2 | -2 α^2 | -2 α | -2 α^4 | -2 α^2 | v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ |
| v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | -2 | -2 α^4 | -2 α^2 | -2 α^2 | -2 α | v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 |

Table(5)

| $[x^{11},.]$ | $[x^{11},r]$ | $[x^{11},r^2]$ | $[x^{11},r^3]$ | $[x^{11},r^4]$ | $[x^{13},.]$ | $[x^{13},r]$ | $[x^{13},r^2]$ | $[x^{13},r^3]$ | $[x^{13},r^4]$ | $[y,.]$ | $[y,r]$ | $[y,r^2]$ | $[y,r^3]$ | $[y,r^4]$ | $[xy,.]$ | $[xy,r]$ | $[[xy,r^2]$ | $[xy,r^3]$ | $[xy,r^4]$ |
|--------------|-------------------|-------------------|-------------------|-------------------|--------------|----------------|-------------------|----------------|----------------|---------|-------------|-------------|-------------|-------------|----------|------------|-------------|------------|------------|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 |
| 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | |
| -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | 1 | α | α^2 | α^3 | α^4 |
| -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | 1 | α^2 | α^4 | α | α^3 |
| -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | 1 | α^3 | α | α^4 | α^2 |
| -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | 1 | α^4 | α^3 | α^2 | α |
| v_6 | v_6 | v_6 | v_6 | v_6 | v_2 | v_2 | v_2 | v_2 | v_2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_8 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_8 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | v_2 | $\alpha^2 v_2$ | $\alpha^4 v_{10}$ | αv_2 | $\alpha^3 v_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_8 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | v_4 | v_4 | v_4 | v_4 | v_4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | v_6 | v_6 | v_6 | v_6 | v_6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

Table(5)

Table(5)

| | | | | | | | | | | | | | | | | | | |
|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|-----------|--------------|--------------|--------------|--------------|--------------|-------------------|-------------------|-------------------|
| v_4 | v_4 | v_4 | v_4 | v_4 | v_{12} | v_{12} | v_{12} | v_{12} | 2 | 2 | 2 | 2 | v_8 | v_8 | v_8 | v_8 | | |
| v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | |
| v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | $\alpha^3 v_8$ |
| v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ |
| v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ |
| v_{12} | v_{12} | v_{12} | v_{12} | v_8 | v_8 | v_8 | v_8 | v_8 | 2 | 2 | 2 | 2 | 2 | v_{10} | v_{10} | v_{10} | v_{10} | |
| v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ |
| v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ |
| v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ |
| v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ |
| v_8 | v_8 | v_8 | v_8 | v_8 | v_4 | v_4 | v_4 | v_4 | 2 | 2 | 2 | 2 | 2 | v_{12} | v_{12} | v_{12} | v_{12} | |
| v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ |
| v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ |
| v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ |
| v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ |
| v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | -2 | -2 | -2 | -2 | v_1 | v_1 | v_1 | v_1 | |
| v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | v_1 | αv_1 | $\alpha^2 v_1$ | $\alpha^3 v_1$ |
| v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | v_1 | $\alpha^2 v_1$ | $\alpha^4 v_1$ | $\alpha^3 v_1$ |
| v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | v_1 | $\alpha^3 v_1$ | αv_1 | $\alpha^4 v_1$ |
| v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | v_1 | $\alpha^4 v_1$ | $\alpha^3 v_1$ | $\alpha^2 v_1$ |
| v_2 | v_2 | v_2 | v_2 | v_2 | v_8 | v_8 | v_8 | v_8 | v_8 | -2 | -2 | -2 | -2 | v_3 | v_3 | v_3 | v_3 | |
| v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | v_3 | αv_3 | $\alpha^2 v_3$ | $\alpha^3 v_3$ |
| v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | v_3 | $\alpha^4 v_3$ | $\alpha^3 v_3$ | $\alpha^4 v_3$ |
| v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | v_3 | $\alpha^3 v_3$ | αv_3 | $\alpha^4 v_3$ |
| v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | v_3 | $\alpha^4 v_3$ | $\alpha^3 v_3$ | $\alpha^2 v_3$ |
| v_6 | v_6 | v_6 | v_6 | v_6 | v_4 | v_4 | v_4 | v_4 | v_4 | -2 | -2 | -2 | -2 | v_5 | v_5 | v_5 | v_5 | |
| v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | v_5 | αv_5 | $\alpha^2 v_5$ | $\alpha^3 v_5$ |
| v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | v_5 | $\alpha^2 v_5$ | $\alpha^4 v_5$ | αv_5 |
| v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | v_5 | $\alpha^3 v_5$ | αv_5 | $\alpha^4 v_5$ |
| v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | v_5 | $\alpha^4 v_5$ | $\alpha^3 v_5$ | $\alpha^2 v_5$ |

| | | | | | | | | | | | | | | | | | |
|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|----|--------------|--------------|--------------|--------------|----------|-------------------|-------------------|
| v_4 | v_4 | v_4 | v_4 | v_4 | v_{12} | v_{12} | v_{12} | v_{12} | 2 | 2 | 2 | 2 | v_{12} | v_{12} | v_{12} | v_{12} | |
| v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ |
| v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ |
| v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} |
| v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ |
| v_2 | v_2 | v_2 | v_2 | v_6 | v_6 | v_6 | v_6 | v_6 | -2 | -2 | -2 | -2 | -2 | v_6 | v_6 | v_6 | |
| v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | v_6 | αv_6 | $\alpha^2 v_6$ |
| v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ | v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ |
| v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | v_6 | $\alpha^3 v_6$ | αv_6 |
| v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ |
| v_8 | v_8 | v_8 | v_8 | v_8 | v_4 | v_4 | v_4 | v_4 | 2 | 2 | 2 | 2 | 2 | v_4 | v_4 | v_4 | |
| v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_4 | αv_4 | $\alpha^2 v_4$ |
| v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ |
| v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_4 | $\alpha^3 v_4$ | αv_4 |
| v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ |
| v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ |
| v_3 | v_3 | v_3 | v_3 | v_5 | v_5 | v_5 | v_5 | v_5 | 0 | 0 | 0 | 0 | 0 | v_9 | v_9 | v_9 | |
| v_3 | αv_3 | $\alpha^2 v_3$ | $\alpha^3 v_3$ | $\alpha^4 v_3$ | v_5 | αv_5 | $\alpha^2 v_5$ | $\alpha^3 v_5$ | $\alpha^4 v_5$ | 0 | 0 | 0 | 0 | 0 | v_9 | αv_9 | $\alpha^2 v_9$ |
| v_3 | $\alpha^2 v_3$ | $\alpha^4 v_3$ | αv_3 | $\alpha^3 v_3$ | v_5 | $\alpha^2 v_5$ | $\alpha^4 v_5$ | αv_5 | $\alpha^3 v_5$ | 0 | 0 | 0 | 0 | 0 | v_9 | $\alpha^2 v_9$ | $\alpha^4 v_9$ |
| v_3 | $\alpha^3 v_3$ | αv_3 | $\alpha^4 v_3$ | $\alpha^2 v_3$ | v_5 | $\alpha^3 v_5$ | αv_5 | $\alpha^4 v_5$ | $\alpha^2 v_5$ | 0 | 0 | 0 | 0 | 0 | v_9 | $\alpha^3 v_9$ | αv_9 |
| v_3 | $\alpha^4 v_3$ | $\alpha^3 v_3$ | $\alpha^2 v_3$ | αv_3 | v_5 | $\alpha^4 v_5$ | $\alpha^3 v_5$ | $\alpha^2 v_5$ | αv_5 | 0 | 0 | 0 | 0 | 0 | v_9 | $\alpha^4 v_9$ | $\alpha^3 v_9$ |
| v_9 | v_9 | v_9 | v_9 | v_9 | v_{13} | v_{13} | v_{13} | v_{13} | v_{13} | 0 | 0 | 0 | 0 | 0 | v_1 | v_1 | v_1 |
| v_9 | αv_9 | $\alpha^2 v_9$ | $\alpha^3 v_9$ | $\alpha^4 v_9$ | v_{13} | αv_{13} | $\alpha^2 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^4 v_{13}$ | 0 | 0 | 0 | 0 | 0 | v_1 | αv_1 | $\alpha^2 v_1$ |
| v_9 | $\alpha^2 v_9$ | $\alpha^4 v_9$ | αv_9 | $\alpha^3 v_9$ | v_{13} | $\alpha^2 v_{13}$ | $\alpha^4 v_{13}$ | αv_{13} | $\alpha^3 v_{13}$ | 0 | 0 | 0 | 0 | 0 | v_1 | $\alpha^2 v_1$ | $\alpha^4 v_1$ |
| v_9 | $\alpha^3 v_9$ | αv_9 | $\alpha^4 v_9$ | $\alpha^2 v_9$ | v_{13} | $\alpha^3 v_{13}$ | αv_{13} | $\alpha^4 v_{13}$ | $\alpha^2 v_{13}$ | 0 | 0 | 0 | 0 | 0 | v_1 | $\alpha^3 v_1$ | αv_1 |
| v_9 | $\alpha^4 v_9$ | $\alpha^3 v_9$ | $\alpha^2 v_9$ | αv_9 | v_{13} | $\alpha^4 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^2 v_{13}$ | αv_{13} | 0 | 0 | 0 | 0 | 0 | v_1 | $\alpha^4 v_1$ | $\alpha^2 v_1$ |
| v_{13} | v_{13} | v_{13} | v_{13} | v_3 | v_3 | v_3 | v_3 | v_3 | 0 | 0 | 0 | 0 | 0 | v_{11} | v_{11} | v_{11} | |
| v_{13} | αv_{13} | $\alpha^2 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^4 v_{13}$ | v_3 | αv_3 | $\alpha^2 v_3$ | $\alpha^3 v_3$ | $\alpha^4 v_3$ | 0 | 0 | 0 | 0 | 0 | v_{11} | αv_{11} | $\alpha^2 v_{11}$ |
| v_{13} | $\alpha^2 v_{13}$ | $\alpha^4 v_{13}$ | αv_{13} | $\alpha^3 v_{13}$ | v_3 | $\alpha^2 v_3$ | $\alpha^4 v_3$ | αv_3 | $\alpha^3 v_3$ | 0 | 0 | 0 | 0 | 0 | v_{11} | $\alpha^2 v_{11}$ | $\alpha^4 v_{11}$ |
| v_{13} | $\alpha^3 v_{13}$ | αv_{13} | $\alpha^4 v_{13}$ | $\alpha^2 v_{13}$ | v_3 | $\alpha^3 v_3$ | αv_3 | $\alpha^4 v_3$ | $\alpha^2 v_3$ | 0 | 0 | 0 | 0 | 0 | v_{11} | $\alpha^3 v_{11}$ | αv_{11} |
| v_{13} | $\alpha^4 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^2 v_{13}$ | αv_{13} | v_3 | $\alpha^4 v_3$ | $\alpha^3 v_3$ | $\alpha^2 v_3$ | αv_3 | 0 | 0 | 0 | 0 | 0 | v_{11} | $\alpha^4 v_{11}$ | $\alpha^3 v_{11}$ |

| | | | | | | | | | | | | | | | | | | | | |
|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|---|---|---|---|---|---|---|---|---|---|---|
| v_4 | v_4 | v_4 | v_4 | v_4 | v_8 | v_8 | v_8 | v_8 | v_8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_2 | v_2 | v_2 | v_2 | v_2 | v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ | v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_8 | v_8 | v_8 | v_8 | v_8 | v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{11} | v_{11} | v_{11} | v_{11} | v_{11} | v_{13} | v_{13} | v_{13} | v_{13} | v_{13} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{11} | αv_{11} | $\alpha^2 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^4 v_{11}$ | v_{13} | αv_{13} | $\alpha^2 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^4 v_{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{11} | $\alpha^2 v_{11}$ | $\alpha^4 v_{11}$ | αv_{11} | $\alpha^3 v_{11}$ | v_{13} | $\alpha^2 v_{13}$ | $\alpha^4 v_{13}$ | αv_{13} | $\alpha^3 v_{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{11} | $\alpha^3 v_{11}$ | αv_{11} | $\alpha^4 v_{11}$ | $\alpha^2 v_{11}$ | v_{13} | $\alpha^3 v_{13}$ | αv_{13} | $\alpha^4 v_{13}$ | $\alpha^2 v_{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{11} | $\alpha^4 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^2 v_{11}$ | αv_{11} | v_{13} | $\alpha^4 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^2 v_{13}$ | αv_{13} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_5 | v_5 | v_5 | v_5 | v_5 | v_{11} | v_{11} | v_{11} | v_{11} | v_{11} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_5 | αv_5 | $\alpha^2 v_5$ | $\alpha^3 v_5$ | $\alpha^4 v_5$ | v_{11} | αv_{11} | $\alpha^2 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^4 v_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_5 | $\alpha^2 v_5$ | $\alpha^4 v_5$ | αv_5 | $\alpha^3 v_5$ | v_{11} | $\alpha^2 v_{11}$ | $\alpha^4 v_{11}$ | αv_{11} | $\alpha^3 v_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_5 | $\alpha^3 v_5$ | αv_5 | $\alpha^4 v_5$ | $\alpha^2 v_5$ | v_{11} | $\alpha^3 v_{11}$ | αv_{11} | $\alpha^4 v_{11}$ | $\alpha^2 v_{11}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_5 | $\alpha^4 v_5$ | $\alpha^3 v_5$ | $\alpha^2 v_5$ | αv_5 | v_{11} | $\alpha^4 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^2 v_{11}$ | αv_{11} | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_1 | v_1 | v_1 | v_1 | v_1 | v_9 | v_9 | v_9 | v_9 | v_9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_1 | αv_1 | $\alpha^2 v_1$ | $\alpha^3 v_1$ | $\alpha^4 v_1$ | v_9 | αv_9 | $\alpha^2 v_9$ | $\alpha^3 v_9$ | $\alpha^4 v_9$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_1 | $\alpha^2 v_1$ | $\alpha^4 v_1$ | αv_1 | $\alpha^3 v_1$ | v_9 | $\alpha^2 v_9$ | $\alpha^4 v_9$ | αv_9 | $\alpha^3 v_9$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_1 | $\alpha^3 v_1$ | αv_1 | $\alpha^4 v_1$ | $\alpha^2 v_1$ | v_9 | $\alpha^3 v_9$ | αv_9 | $\alpha^4 v_9$ | $\alpha^2 v_9$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_1 | $\alpha^4 v_1$ | $\alpha^3 v_1$ | $\alpha^2 v_1$ | αv_1 | v_9 | $\alpha^4 v_9$ | $\alpha^3 v_9$ | $\alpha^2 v_9$ | αv_9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 5

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----------------|---|-------------|-------------|-------------|-------------|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|
| $\chi_{(7,1)}$ | 2 | 2 | 2 | 2 | 2 | -2 | -2 | -2 | -2 | 2 | 2 | 2 | 2 | -2 | -2 | -2 | -2 | -2 | 2 | 2 | 2 | 2 | 2 | 2 | |
| $\chi_{(7,2)}$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ |
| $\chi_{(7,3)}$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ |
| $\chi_{(7,4)}$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ |
| $\chi_{(7,5)}$ | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α |
| $\chi_{(9,1)}$ | 2 | 2 | 2 | 2 | 2 | v_{10} | v_{10} | v_{10} | v_{10} | v_8 | v_8 | v_8 | v_8 | v_8 | v_2 | v_2 | v_2 | v_2 | v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | |
| $\chi_{(9,2)}$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ |
| $\chi_{(9,3)}$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ |
| $\chi_{(9,4)}$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ |
| $\chi_{(9,5)}$ | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} |
| $\chi_{(11,1)}$ | 2 | 2 | 2 | 2 | 2 | v_6 | v_6 | v_6 | v_6 | v_6 | v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | v_4 | v_4 | v_4 | v_4 | v_4 |
| $\chi_{(11,2)}$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ |
| $\chi_{(11,3)}$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ |
| $\chi_{(11,4)}$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ |
| $\chi_{(11,5)}$ | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 |
| $\chi_{(13,1)}$ | 2 | 2 | 2 | 2 | 2 | v_2 | v_2 | v_2 | v_2 | v_2 | v_4 | v_4 | v_4 | v_4 | v_6 | v_6 | v_6 | v_6 | v_6 | v_8 | v_8 | v_8 | v_8 | v_8 | |
| $\chi_{(13,2)}$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ |
| $\chi_{(13,3)}$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ |
| $\chi_{(13,4)}$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | v_2 | $\alpha^3 v_2$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ |
| $\chi_{(13,5)}$ | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 |
| $\psi_{(2,1)}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| $\psi_{(2,2)}$ | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 |
| $\psi_{(2,3)}$ | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 |
| $\psi_{(2,4)}$ | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 |
| $\psi_{(2,5)}$ | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α |
| $\psi_{(3,1)}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| $\psi_{(3,2)}$ | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 |
| $\psi_{(3,3)}$ | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 |
| $\psi_{(3,4)}$ | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 |
| $\psi_{(3,5)}$ | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α |

| | | | | | | | | | | | | | | | | | | | |
|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|----|--------------|--------------|--------------|--------------|----------|-------------------|-------------------|-------------------|-------------------|
| -2 | -2 | -2 | -2 | -2 | 2 | 2 | 2 | 2 | 2 | -2 | -2 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | |
| -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | 2 | 2α | $2\alpha^2$ | $2\alpha^3$ | $2\alpha^4$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | 0 | 0 | 0 | 0 | 0 |
| -2 | $-2\alpha^2$ | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^3$ | 2 | $2\alpha^2$ | $2\alpha^4$ | 2α | $2\alpha^3$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | 0 | 0 | 0 | 0 | 0 |
| -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | 2 | $2\alpha^3$ | 2α | $2\alpha^4$ | $2\alpha^2$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | 0 | 0 | 0 | 0 | 0 |
| -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | 2 | $2\alpha^4$ | $2\alpha^3$ | $2\alpha^2$ | 2α | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | 0 | 0 | 0 | 0 | 0 |
| v_6 | v_6 | v_6 | v_6 | v_6 | v_4 | v_4 | v_4 | v_4 | v_4 | -2 | -2 | -2 | -2 | v_9 | v_9 | v_9 | v_9 | v_9 | |
| v_6 | αv_6 | $\alpha^2 v_6$ | $\alpha^3 v_6$ | $\alpha^4 v_6$ | v_4 | αv_4 | $\alpha^2 v_4$ | $\alpha^3 v_4$ | $\alpha^4 v_4$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | v_9 | αv_9 | $\alpha^2 v_9$ | $\alpha^3 v_9$ | $\alpha^4 v_9$ |
| v_6 | $\alpha^2 v_6$ | $\alpha^4 v_6$ | αv_6 | $\alpha^3 v_6$ | v_4 | $\alpha^2 v_4$ | $\alpha^4 v_4$ | αv_4 | $\alpha^3 v_4$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | v_9 | $\alpha^2 v_9$ | $\alpha^4 v_9$ | $\alpha^2 v_9$ | $\alpha^3 v_9$ |
| v_6 | $\alpha^3 v_6$ | αv_6 | $\alpha^4 v_6$ | $\alpha^2 v_6$ | v_4 | $\alpha^3 v_4$ | αv_4 | $\alpha^4 v_4$ | $\alpha^2 v_4$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | v_9 | $\alpha^3 v_9$ | $\alpha^2 v_9$ | $\alpha^4 v_9$ | $\alpha^2 v_9$ |
| v_6 | $\alpha^4 v_6$ | $\alpha^3 v_6$ | $\alpha^2 v_6$ | αv_6 | v_4 | $\alpha^4 v_4$ | $\alpha^3 v_4$ | $\alpha^2 v_4$ | αv_4 | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | v_9 | $\alpha^4 v_9$ | $\alpha^3 v_9$ | $\alpha^2 v_9$ | $\alpha^2 v_9$ |
| v_2 | v_2 | v_2 | v_2 | v_2 | v_8 | v_8 | v_8 | v_8 | v_8 | -2 | -2 | -2 | -2 | v_{11} | v_{11} | v_{11} | v_{11} | v_{11} | |
| v_2 | αv_2 | $\alpha^2 v_2$ | $\alpha^3 v_2$ | $\alpha^4 v_2$ | v_8 | αv_8 | $\alpha^2 v_8$ | $\alpha^3 v_8$ | $\alpha^4 v_8$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | v_{11} | αv_{11} | $\alpha^2 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^4 v_{11}$ |
| v_2 | $\alpha^2 v_2$ | $\alpha^4 v_2$ | αv_2 | $\alpha^3 v_2$ | v_8 | $\alpha^2 v_8$ | $\alpha^4 v_8$ | αv_8 | $\alpha^3 v_8$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | v_{11} | $\alpha^2 v_{11}$ | $\alpha^4 v_{11}$ | αv_{11} | $\alpha^3 v_{11}$ |
| v_2 | $\alpha^3 v_{10}$ | αv_2 | $\alpha^4 v_2$ | $\alpha^2 v_2$ | v_8 | $\alpha^3 v_8$ | αv_8 | $\alpha^4 v_8$ | $\alpha^2 v_8$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | v_{11} | $\alpha^3 v_{11}$ | αv_{11} | $\alpha^4 v_{11}$ | $\alpha^2 v_{11}$ |
| v_2 | $\alpha^4 v_2$ | $\alpha^3 v_2$ | $\alpha^2 v_2$ | αv_2 | v_8 | $\alpha^4 v_8$ | $\alpha^3 v_8$ | $\alpha^2 v_8$ | αv_8 | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | v_{11} | $\alpha^4 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^2 v_{11}$ | αv_{11} |
| v_{10} | v_{10} | v_{10} | v_{10} | v_{10} | v_{12} | v_{12} | v_{12} | v_{12} | v_{12} | -2 | -2 | -2 | -2 | v_{13} | v_{13} | v_{13} | v_{13} | v_{13} | |
| v_{10} | αv_{10} | $\alpha^2 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^4 v_{10}$ | v_{12} | αv_{12} | $\alpha^2 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^4 v_{12}$ | -2 | -2α | $-2\alpha^2$ | $-2\alpha^3$ | $-2\alpha^4$ | v_{13} | αv_{13} | $\alpha^2 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^4 v_{13}$ |
| v_{10} | $\alpha^2 v_{10}$ | $\alpha^4 v_{10}$ | αv_{10} | $\alpha^3 v_{10}$ | v_{12} | $\alpha^2 v_{12}$ | $\alpha^4 v_{12}$ | αv_{12} | $\alpha^3 v_{12}$ | -2 | $-2\alpha^2$ | $-2\alpha^4$ | -2α | $-2\alpha^3$ | v_{13} | $\alpha^2 v_{13}$ | $\alpha^4 v_{13}$ | αv_{13} | $\alpha^3 v_{13}$ |
| v_{10} | $\alpha^3 v_{10}$ | αv_{10} | $\alpha^4 v_{10}$ | $\alpha^2 v_{10}$ | v_{12} | $\alpha^3 v_{12}$ | αv_{12} | $\alpha^4 v_{12}$ | $\alpha^2 v_{12}$ | -2 | $-2\alpha^3$ | -2α | $-2\alpha^4$ | $-2\alpha^2$ | v_{13} | $\alpha^3 v_{13}$ | αv_{13} | $\alpha^4 v_{13}$ | $\alpha^2 v_{13}$ |
| v_{10} | $\alpha^4 v_{10}$ | $\alpha^3 v_{10}$ | $\alpha^2 v_{10}$ | αv_{10} | v_{12} | $\alpha^4 v_{12}$ | $\alpha^3 v_{12}$ | $\alpha^2 v_{12}$ | αv_{12} | -2 | $-2\alpha^4$ | $-2\alpha^3$ | $-2\alpha^2$ | -2α | v_{13} | $\alpha^4 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^2 v_{13}$ | αv_{13} |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 |
| 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ |
| 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | -1 | $-\alpha^3$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^2$ |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ |

| | | | | | | | | | | | | | | | | | | | |
|----------|-------------------|-------------------|-------------------|-------------------|----------|-------------------|-------------------|-------------------|-------------------|----|-------------|-------------|-------------|-------------|-------------------|-------------------|-------------------|-------------------|-------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| v_1 | v_1 | v_1 | v_1 | v_1 | v_{11} | v_{11} | v_{11} | v_{11} | v_{11} | 0 | 0 | 0 | 0 | v_3 | v_3 | v_3 | v_3 | v_3 | |
| v_1 | αv_1 | $\alpha^2 v_1$ | $\alpha^3 v_1$ | $\alpha^4 v_1$ | v_{11} | αv_{11} | $\alpha^2 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^4 v_{11}$ | 0 | 0 | 0 | 0 | v_3 | αv_3 | $\alpha^2 v_3$ | $\alpha^3 v_3$ | $\alpha^4 v_3$ | |
| v_1 | $\alpha^2 v_1$ | $\alpha^4 v_1$ | αv_1 | $\alpha^3 v_1$ | v_{11} | $\alpha^2 v_{11}$ | $\alpha^4 v_{11}$ | αv_{11} | $\alpha^3 v_{11}$ | 0 | 0 | 0 | 0 | v_3 | $\alpha^2 v_3$ | $\alpha^4 v_3$ | αv_3 | $\alpha^3 v_3$ | |
| v_1 | $\alpha^3 v_1$ | αv_1 | $\alpha^4 v_1$ | $\alpha^2 v_1$ | v_{11} | $\alpha^3 v_{11}$ | αv_{11} | $\alpha^4 v_{11}$ | $\alpha^2 v_{11}$ | 0 | 0 | 0 | 0 | v_3 | $\alpha^3 v_3$ | αv_3 | $\alpha^4 v_3$ | $\alpha^2 v_3$ | |
| v_1 | $\alpha^4 v_1$ | $\alpha^3 v_1$ | $\alpha^2 v_1$ | αv_1 | v_{11} | $\alpha^4 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^2 v_{11}$ | αv_{11} | 0 | 0 | 0 | 0 | v_3 | $\alpha^4 v_3$ | $\alpha^3 v_3$ | $\alpha^2 v_3$ | αv_3 | |
| v_5 | v_5 | v_5 | v_5 | v_5 | v_1 | v_1 | v_1 | v_1 | v_1 | 0 | 0 | 0 | 0 | v_{13} | v_{13} | v_{13} | v_{13} | v_{13} | |
| v_5 | αv_5 | $\alpha^2 v_5$ | $\alpha^3 v_5$ | $\alpha^4 v_5$ | v_1 | αv_1 | $\alpha^2 v_1$ | $\alpha^3 v_1$ | $\alpha^4 v_1$ | 0 | 0 | 0 | 0 | v_{13} | αv_{13} | $\alpha^2 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^4 v_{13}$ | |
| v_5 | $\alpha^2 v_5$ | $\alpha^4 v_5$ | αv_5 | $\alpha^3 v_5$ | v_1 | $\alpha^2 v_1$ | $\alpha^4 v_1$ | αv_1 | $\alpha^3 v_1$ | 0 | 0 | 0 | 0 | v_{13} | $\alpha^2 v_{13}$ | $\alpha^4 v_{13}$ | αv_{13} | $\alpha^3 v_{13}$ | |
| v_5 | $\alpha^3 v_5$ | αv_5 | $\alpha^4 v_5$ | $\alpha^2 v_5$ | v_1 | $\alpha^3 v_1$ | αv_1 | $\alpha^4 v_1$ | $\alpha^2 v_1$ | 0 | 0 | 0 | 0 | v_{13} | $\alpha^3 v_{13}$ | αv_{13} | $\alpha^4 v_{13}$ | $\alpha^2 v_{13}$ | |
| v_5 | $\alpha^4 v_5$ | $\alpha^3 v_5$ | $\alpha^2 v_5$ | αv_5 | v_1 | $\alpha^4 v_1$ | $\alpha^3 v_1$ | $\alpha^2 v_1$ | αv_1 | 0 | 0 | 0 | 0 | v_{13} | $\alpha^4 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^2 v_{13}$ | αv_{13} | |
| v_{11} | v_{11} | v_{11} | v_{11} | v_9 | v_9 | v_9 | v_9 | v_9 | v_9 | 0 | 0 | 0 | 0 | v_5 | v_5 | v_5 | v_5 | v_5 | |
| v_{11} | αv_{11} | $\alpha^2 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^4 v_{11}$ | v_9 | αv_9 | $\alpha^2 v_9$ | $\alpha^3 v_9$ | $\alpha^4 v_9$ | 0 | 0 | 0 | 0 | v_5 | αv_5 | $\alpha^2 v_5$ | $\alpha^3 v_5$ | $\alpha^4 v_5$ | |
| v_{11} | $\alpha^2 v_{11}$ | $\alpha^4 v_{11}$ | αv_{11} | $\alpha^3 v_{11}$ | v_9 | $\alpha^2 v_9$ | $\alpha^4 v_9$ | αv_9 | $\alpha^3 v_9$ | 0 | 0 | 0 | 0 | v_5 | $\alpha^2 v_5$ | $\alpha^4 v_5$ | αv_5 | $\alpha^3 v_5$ | |
| v_{11} | $\alpha^3 v_{11}$ | αv_{11} | $\alpha^4 v_{11}$ | $\alpha^2 v_{11}$ | v_9 | $\alpha^3 v_9$ | αv_9 | $\alpha^4 v_9$ | $\alpha^2 v_9$ | 0 | 0 | 0 | 0 | v_5 | $\alpha^3 v_5$ | αv_5 | $\alpha^4 v_5$ | $\alpha^2 v_5$ | |
| v_{11} | $\alpha^4 v_{11}$ | $\alpha^3 v_{11}$ | $\alpha^2 v_{11}$ | αv_{11} | v_9 | $\alpha^4 v_9$ | $\alpha^3 v_9$ | $\alpha^2 v_9$ | αv_9 | 0 | 0 | 0 | 0 | v_5 | $\alpha^4 v_5$ | $\alpha^3 v_5$ | $\alpha^2 v_5$ | αv_5 | |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 |
| 1 | α^2 | α^4 | α^2 | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | |
| -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ |
| -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ |
| -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ |
| -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ |

Table 5

| | | | | | | | | | | | | | | | | | | | |
|-----------------|-------------------|-------------------|-------------------|-------------------|--------------|----------------|----------------|----------------|----------------|----|-------------|-------------|-------------|-------------|----|--------------|-------------|-------------|-------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{13} | v_{13} | v_{13} | v_{13} | v_5 | v_5 | v_5 | v_5 | v_5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| αv_{13} | $\alpha^2 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^4 v_{13}$ | v_5 | αv_5 | $\alpha^2 v_5$ | $\alpha^3 v_5$ | $\alpha^4 v_5$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{13} | $\alpha^2 v_{13}$ | $\alpha^4 v_{13}$ | αv_{13} | $\alpha^3 v_{13}$ | v_5 | $\alpha^2 v_5$ | $\alpha^4 v_5$ | αv_5 | $\alpha^3 v_5$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{13} | $\alpha^3 v_{13}$ | αv_{13} | $\alpha^4 v_{13}$ | $\alpha^2 v_{13}$ | v_5 | $\alpha^3 v_5$ | αv_5 | $\alpha^4 v_5$ | $\alpha^2 v_5$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_{13} | $\alpha^4 v_{13}$ | $\alpha^3 v_{13}$ | $\alpha^2 v_{13}$ | αv_{13} | v_5 | $\alpha^4 v_5$ | $\alpha^3 v_5$ | $\alpha^2 v_5$ | αv_5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_9 | v_9 | v_9 | v_9 | v_9 | v_3 | v_3 | v_3 | v_3 | v_3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_9 | αv_9 | $\alpha^2 v_9$ | $\alpha^3 v_9$ | $\alpha^4 v_9$ | v_3 | αv_3 | $\alpha^2 v_3$ | $\alpha^3 v_3$ | $\alpha^4 v_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_9 | $\alpha^2 v_9$ | $\alpha^4 v_9$ | αv_9 | $\alpha^3 v_9$ | v_3 | $\alpha^2 v_3$ | $\alpha^4 v_3$ | αv_3 | $\alpha^3 v_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_9 | $\alpha^3 v_9$ | αv_9 | $\alpha^4 v_9$ | $\alpha^2 v_9$ | v_3 | $\alpha^3 v_3$ | αv_3 | $\alpha^4 v_3$ | $\alpha^2 v_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_9 | $\alpha^4 v_9$ | $\alpha^3 v_9$ | $\alpha^2 v_9$ | αv_9 | v_3 | $\alpha^4 v_3$ | $\alpha^3 v_3$ | $\alpha^2 v_3$ | αv_3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_3 | v_3 | v_3 | v_3 | v_3 | v_1 | v_1 | v_1 | v_1 | v_1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_3 | αv_3 | $\alpha^2 v_3$ | $\alpha^3 v_3$ | $\alpha^4 v_3$ | v_1 | αv_1 | $\alpha^2 v_1$ | $\alpha^3 v_1$ | $\alpha^4 v_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_3 | $\alpha^2 v_3$ | $\alpha^4 v_3$ | αv_3 | $\alpha^3 v_3$ | v_1 | $\alpha^2 v_1$ | $\alpha^4 v_1$ | αv_1 | $\alpha^3 v_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_3 | $\alpha^3 v_3$ | αv_3 | $\alpha^4 v_3$ | $\alpha^2 v_3$ | v_1 | $\alpha^3 v_1$ | αv_1 | $\alpha^4 v_1$ | $\alpha^2 v_1$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| v_3 | $\alpha^4 v_3$ | $\alpha^3 v_3$ | $\alpha^2 v_3$ | αv_3 | v_1 | $\alpha^4 v_1$ | $\alpha^3 v_1$ | $\alpha^2 v_1$ | αv_1 | 0 | 0 | 0 | 0 | 0 | 0 | - α^3 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 1 | α | α^2 | α^3 | α^4 | 1 | α | α^2 | α^3 | α^4 | -1 | - α | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ |
| 1 | α^2 | α^4 | α | α^3 | 1 | α^2 | α^4 | α | α^3 | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ |
| 1 | α^3 | α | α^4 | α^2 | 1 | α^3 | α | α^4 | α^2 | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ |
| 1 | α^4 | α^3 | α^2 | α | 1 | α^4 | α^3 | α^2 | α | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ |
| -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 |
| -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ | 1 | α | α^2 | α^3 | α^4 | -1 | $-\alpha$ | $-\alpha^2$ | $-\alpha^3$ | $-\alpha^4$ |
| -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | $-\alpha^3$ | 1 | α^2 | α^4 | α | α^3 | -1 | $-\alpha^2$ | $-\alpha^4$ | $-\alpha$ | |
| -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ | 1 | α^3 | α | α^4 | α^2 | -1 | $-\alpha^3$ | $-\alpha$ | $-\alpha^4$ | $-\alpha^2$ |
| -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ | 1 | α^4 | α^3 | α^2 | α | -1 | $-\alpha^4$ | $-\alpha^3$ | $-\alpha^2$ | $-\alpha$ |

By using Theorem (4.5) the rational valued characters table of $Q_{28} \times C_5$, Since $p=7$, it is same to the table Example(4.3), (after change order the rows and the columns) as Table(10).

| CL_β | $[y, I]$ | $[xy, I]$ | $[x, I]$ | $[x^7, I]$ | $[x^2, I]$ | $[x^4, I]$ | $[x^{14}, I]$ | $[I, I]$ | $[y, r]$ | $[xy, r]$ | $[x, r]$ | $[x^7, r]$ | $[x^2, r]$ | $[x^4, r]$ | $[x^{14}, r]$ | $[I, r]$ |
|---------------|----------|-----------|----------|------------|------------|------------|---------------|----------|----------|-----------|----------|------------|------------|------------|---------------|----------|
| $ CL_\beta $ | 14 | 14 | 2 | 2 | 2 | 2 | 1 | 1 | 14 | 14 | 2 | 2 | 2 | 2 | 1 | 1 |
| θ_{11} | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| θ_{12} | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| θ_{21} | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 |
| θ_{22} | -4 | 4 | -4 | -4 | 4 | 4 | 4 | 4 | 1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 |
| θ_{31} | 0 | 0 | -1 | 6 | -1 | -1 | 6 | 6 | 0 | 0 | -1 | 6 | -1 | -1 | 6 | 6 |
| θ_{32} | 0 | 0 | -4 | 24 | -4 | -4 | 24 | 24 | 0 | 0 | 1 | -6 | 1 | 1 | -6 | -6 |
| θ_{41} | 0 | 0 | 1 | -6 | -1 | -1 | 6 | 6 | 0 | 0 | 1 | -6 | -1 | -1 | 6 | 6 |
| θ_{42} | 0 | 0 | 4 | -24 | -4 | -4 | 24 | 24 | 0 | 0 | -1 | 6 | 1 | 1 | -6 | -6 |
| θ_{51} | 0 | 0 | 0 | 0 | -2 | 2 | -2 | 2 | 0 | 0 | 0 | 0 | -2 | 2 | -2 | 2 |
| θ_{52} | 0 | 0 | 0 | 0 | -8 | 8 | -8 | 8 | 0 | 0 | 0 | 0 | 2 | -2 | 2 | -2 |
| θ_{61} | 0 | 0 | 0 | 0 | 2 | -2 | -12 | 12 | 0 | 0 | 0 | 0 | 2 | -2 | -12 | 12 |
| θ_{62} | 0 | 0 | 0 | 0 | 8 | -8 | -48 | 48 | 0 | 0 | 0 | 0 | -2 | 2 | 12 | -12 |
| θ_{71} | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 |
| θ_{72} | -4 | -4 | 4 | 4 | 4 | 4 | 4 | 4 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 |
| θ_{81} | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| θ_{82} | 4 | -4 | -4 | -4 | 4 | 4 | 4 | 4 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |

Table(10)

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