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RESEARCH ARTICLE

(T, S)-INTUITIONISTIC FUZZY NORMAL SUBNEARRING OF A NEARRING

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of a (T, S)-intuitionistic fuzzy normal subnearring of a nearring.

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KEY WORDS: T-fuzzy subnearring, (T, S)-intuitionistic fuzzy subnearring, (T, S)-intuitionistic fuzzy normal subnearring, product.

INTRODUCTION

After the introduction of fuzzy sets by L.A.Zadeh[16], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K.T.Atanassov[4, 5], as a generalization of the notion of fuzzy set. Azriel Rosenfeld[6] defined the fuzzy groups. Asok Kumer Ray[3] defined a product of fuzzy subgroups. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan [13, 14]. In this paper, we introduce the some Theorems in (T, S)-intuitionistic fuzzy normal subnearring of a nearring.

1.PRELIMINARIES:

1.1 Definition: A (T, S)-norm is a binary operations T: $[0, 1]\times[0, 1] \to [0, 1]$ and S: $[0, 1]\times[0, 1] \to [0, 1]$ satisfying the following requirements;

(i) T(0, x)= 0, T(1, x) = x (boundary condition)

(ii) T(x, y) = T(y, x) (commutativity)

(iii) T(x, T(y, z))= T (T(x,y), z)(associativity)

(iv) if $x \leq y$ and $w \leq z$, then T(x, w) $\leq$ T (y, z)(monotonicity).

(v) S(0, x) = x, S (1, x) = 1 (boundary condition)

(vi) S(x, y) = S (y, x)(commutativity)

(vii) S (x, S(y, z))= S (S(x, y), z) (associativity)

(viii) if $x \leq y$ and $w \leq z$, then S (x, w) $\leq$ S (y, z)(monotonicity).

1.2 Definition: Let (R, +, .) be a nearring. A fuzzy subset A of R is said to be a T-fuzzy subnearring (fuzzy subnearring with respect to T-norm) of R if it satisfies the following conditions:

(i) $\mu_A(x-y) \geq T(\mu_A(x), \mu_A(y))$

(ii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$ for all x and y in R.

1.3 Definition: Let (R, +, .) be a nearring. An intuitionistic fuzzy subset A of R is said to be an (T, S)-intuitionistic fuzzy subnearring (intuitionistic fuzzy subnearring with respect to (T, S)-norm) of R if it satisfies the following conditions:

(i) $\mu_A(x - y) \geq T(\mu_A(x), \mu_A(y))$

(ii) $\mu_A(xy) \geq T(\mu_A(x), \mu_A(y))$

(iii) $\nu_A(x - y) \leq S(\nu_A(x), \nu_A(y))$

(iv) $\nu_A(xy) \leq S(\nu_A(x), \nu_A(y))$ for all x and y in R.

1.4 Definition: Let A and B be intuitionistic fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{\langle (x, y), \mu_{A\times B}(x, y), \nu_{A\times B}(x, y) \rangle$ / for all x in G and y in H }, where $\mu_{A\times B}(x, y) = \min\{ \mu_A(x), \mu_B(y) \}$ and $\nu_{A\times B}(x, y) = \max\{ \nu_A(x), \nu_B(y) \}$.

1.5 Definition: Let A be an intuitionistic fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is an intuitionistic fuzzy relation on A is V given by $\mu_V(x, y) = \min\{ \mu_A(x), \mu_A(y) \}$ and $\nu_V(x, y) = \max\{ \nu_A(x), \nu_A(y) \}$, for all x and y in S.

1.6 Definition: Let (R, +, .) and (R^l, +, .) be any two nearrings. Let f : R → R^l be any function and A be an (T, S)-intuitionistic fuzzy subnearring in R, V be an (T, S)-intuitionistic fuzzy subnearring in f(R)= R^l, defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let (R, +, .) be a nearring. An (T, S)-intuitionistic fuzzy subnearring A of R is said to be an (T, S)-intuitionistic fuzzy normal subnearring of R if it satisfies the following conditions:

(i) $\mu_A(x+y) = \mu_A(y+x)$

(ii) $\mu_A(xy) = \mu_A(yx)$

(iii) $\nu_A(x+y) = \nu_A(y+x)$

(iv) $\nu_A(xy) = \nu_A(yx)$ for all x and y in R.

2. PROPERTIES:

2.1 Theorem: Intersection of any two (T, S)-intuitionistic fuzzy subnearrings of a nearring R is a (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.2 Theorem: The intersection of a family of (T, S)-intuitionistic fuzzy subnearrings of nearring R is an (T, S)-intuitionistic fuzzy subnearring of a nearring R.

2.3 Theorem: If A and B are any two (T, S)-intuitionistic fuzzy subnearrings of the nearrings R_1 and R_2 respectively, then A×B is an (T, S)-intuitionistic fuzzy subnearring of $R_1\times R_2$.

2.4 Theorem: Let A be an intuitionistic fuzzy subset of a nearring R and V be the strongest intuitionistic fuzzy relation of R. Then A is an (T, S)-intuitionistic fuzzy subnearring of R if and only if V is an (T, S)-intuitionistic fuzzy subnearring of R×R.

2.5 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H and f is an isomorphism from a nearring R onto H. Then A◦f is an (T, S)-intuitionistic fuzzy subnearring of R.

2.6 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H and f is an anti-isomorphism from a nearring R onto H. Then A◦f is an (T, S)-intuitionistic fuzzy subnearring of R.

2.7 Theorem: Let (R, +, .) and (R^l, +, .) be any two nearrings. The homomorphic image of an (T, S)-intuitionistic fuzzy subnearring of R is an (T, S)-intuitionistic fuzzy subnearring of R^l.

2.8 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two nearrings. The homomorphic preimage of an (T, S)-intuitionistic fuzzy subnearring of R^{l} is a (T, S)-intuitionistic fuzzy subnearring of R.
2.9 Theorem: Let (R,+, .) and (R^{l}, +, .) be any two nearrings. The anti-homomorphic image of an (T, S)-intuitionistic fuzzy subnearring of R is an (T, S)-intuitionistic fuzzy subnearring of R^{l}.
2.10 Theorem: Let (R, +, .) and (R^{l}, +, .) be any two nearrings. The anti-homomorphic preimage of an (T, S)-intuitionistic fuzzy subnearring of R^{l} is an (T, S)-intuitionistic fuzzy subnearring of R.
2.11 Theorem: Let (R, +, .) be a nearring. If A and B are two (T, S)-intuitionistic fuzzy normal subnearrings of R, then A∩B is an (T, S)-intuitionistic fuzzy normal subnearring of R.
Proof: Let x and y∈R. Let A = { ⟨ x, $\mu_A(x)$, $\nu_A(x)$ ⟩ / x∈R } and B = { ⟨ x, $\mu_B(x)$, $\nu_B(x)$ ⟩ / x∈R } be an (T, S)-intuitionistic fuzzy normal subnearrings of a nearring R. Let C = A∩B and C = { ⟨ x, $\mu_C(x)$, $\nu_C(x)$ ⟩ / x∈R }, where $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$ and $\nu_C(x) = \max\{\nu_A(x), \nu_B(x)\}$. Then clearly C is an (T, S)-intuitionistic fuzzy subnearring of a nearring R, since A and B are two (T, S)-intuitionistic fuzzy subnearrings of a nearring R. And $\mu_C(x+y) = \min\{\mu_A(x+y), \mu_B(x+y)\} = \min\{\mu_A(y+x), \mu_B(y+x)\} = \mu_C(y+x)$ for all x and y in R. Therefore $\mu_C(x+y) = \mu_C(y+x)$ for all x and y in R. Also $\mu_C(xy) = \min\{\mu_A(xy), \mu_B(xy)\} = \min\{\mu_A(yx), \mu_B(yx)\} = \mu_C(yx)$ for all x and y in R. Therefore $\mu_C(xy) = \mu_C(yx)$ for all x and y in R. And $\nu_C(x+y) = \max\{\nu_A(x+y), \nu_B(x+y)\} = \max\{\nu_A(y+x), \nu_B(y+x)\} = \nu_C(y+x)$ for all x and y in R. Therefore $\nu_C(x+y) = \nu_C(y+x)$ for all x and y in R. Also $\nu_C(xy) = \max\{\nu_A(xy), \nu_B(xy)\} = \max\{\nu_A(yx), \nu_B(yx)\} = \nu_C(yx)$ for all x and y in R. Therefore $\nu_C(xy) = \nu_C(yx)$ for all x and y in R. Hence A∩B is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring R.
2.12 Theorem: Let (R, +, .) be a nearring. The intersection of a family of (T, S)-intuitionistic fuzzy normal subnearrings of R is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring R.
Proof: It is trivial.
2.13 Theorem: Let A and B be (T, S)-intuitionistic fuzzy subnearring of the nearrings G and H, respectively. If A and B are (T, S)-intuitionistic fuzzy normal subnearrings, then A×B is an (T, S)-intuitionistic fuzzy normal subnearring of G×H.
Proof: Let A and B be (T, S)-intuitionistic fuzzy normal subnearrings of the nearrings G and H respectively. Clearly A×B is an (T, S)-intuitionistic fuzzy subnearring of G×H. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in G×H. Now $\mu_{A\times B}[(x_1, y_1)+(x_2, y_2)] = \mu_{A\times B}(x_1+x_2, y_1+y_2) = \min\{\mu_A(x_1+x_2), \mu_B(y_1+y_2)\} = \min\{\mu_A(x_2+x_1), \mu_B(y_2+y_1)\} = \mu_{A\times B}(x_2+x_1, y_2+y_1) = \mu_{A\times B}[(x_2, y_2)+(x_1, y_1)]$. Therefore $\mu_{A\times B}[(x_1, y_1)+(x_2, y_2)] = \mu_{A\times B}[(x_2, y_2)+(x_1, y_1)]$. And $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \mu_{A\times B}(x_1x_2, y_1y_2) = \min\{\mu_A(x_1x_2), \mu_B(y_1y_2)\} = \min\{\mu_A(x_2x_1), \mu_B(y_2y_1)\} = \mu_{A\times B}(x_2x_1, y_2y_1) = \mu_{A\times B}[(x_2, y_2)(x_1, y_1)]$. Therefore $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \mu_{A\times B}[(x_2, y_2)(x_1, y_1)]$. Also $\nu_{A\times B}[(x_1, y_1)+(x_2, y_2)] = \nu_{A\times B}(x_1+x_2, y_1+y_2) = \max\{\nu_A(x_1+x_2), \nu_B(y_1+y_2)\} = \max\{\nu_A(x_2+x_1), \nu_B(y_2+y_1)\} = \nu_{A\times B}(x_2+x_1, y_2+y_1) = \nu_{A\times B}[(x_2, y_2)+(x_1, y_1)]$. Therefore $\nu_{A\times B}[(x_1, y_1)+(x_2, y_2)] = \nu_{A\times B}[(x_2, y_2)+(x_1, y_1)]$. And $\nu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \nu_{A\times B}(x_1x_2, y_1y_2) = \max\{\nu_A(x_1x_2), \nu_B(y_1y_2)\} = \max\{\nu_A(x_2x_1), \nu_B(y_2y_1)\} = \nu_{A\times B}(x_2x_1, y_2y_1) = \nu_{A\times B}[(x_2, y_2)(x_1, y_1)]$. Therefore $\nu_{A\times B}[(x_1, y_1)(x_2, y_2)] = \nu_{A\times B}[(x_2, y_2)(x_1, y_1)]$. Hence A×B is an (T, S)-intuitionistic fuzzy normal subnearring of G×H.
2.14 Theorem: Let A be an intuitionistic fuzzy subset in a nearring R and V be the strongest intuitionistic fuzzy relation on R. Then A is an (T, S)-intuitionistic fuzzy normal subnearring of R if and only if V is an (T, S)-intuitionistic fuzzy normal subnearring of R×R.
Proof: Suppose that A is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R. Clearly V is an (T, S)-intuitionistic fuzzy subnearring

of a nearring R. We have $\mu_V(x+y) = \mu_V[(x_1, x_2)+(y_1, y_2)] = \mu_V(x_1+y_1, x_2+y_2) = \min\{\mu_A(x_1+y_1), \mu_A(x_2+y_2)\} = \min\{\mu_A(y_1+x_1), \mu_A(y_2+x_2)\} = \mu_V(y_1+x_1, y_2+x_2) = \mu_V[(y_1, y_2)+(x_1, x_2)] = \mu_V(y+x)$. Therefore $\mu_V(x+y) = \mu_V(y+x)$ for all x and y in R×R. And $\mu_V(xy) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(x_1y_1, x_2y_2) = \min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} = \min\{\mu_A(y_1x_1), \mu_A(y_2x_2)\} = \mu_V(y_1x_1, y_2x_2) = \mu_V[(y_1, y_2)(x_1, x_2)] = \mu_V(yx)$. Therefore $\mu_V(xy) = \mu_V(yx)$ for all x and y in R×R. Also $\nu_V(x+y) = \nu_V[(x_1, x_2)+(y_1, y_2)] = \nu_V(x_1+y_1, x_2+y_2) = \max\{\nu_A(x_1+y_1), \nu_A(x_2+y_2)\} = \max\{\nu_A(y_1+x_1), \nu_A(y_2+x_2)\} = \nu_V(y_1+x_1, y_2+x_2) = \nu_V[(y_1, y_2) + (x_1, x_2)] = \nu_V(y+x)$. Therefore $\nu_V(x+y) = \nu_V(y+x)$ for all x and y in R×R. And $\nu_V(xy) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(x_1y_1, x_2y_2) = \max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} = \max\{\nu_A(y_1x_1), \nu_A(y_2x_2)\} = \nu_V(y_1x_1, y_2x_2) = \nu_V[(y_1, y_2)(x_1, x_2)] = \nu_V(yx)$. Therefore $\nu_V(xy) = \nu_V(yx)$ for all x and y in R×R. This proves that V is an (T, S)-intuitionistic fuzzy normal subnearring of R×R. Conversely assume that V is an (T, S)-intuitionistic fuzzy normal subnearring of R×R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R, we know that A is an (T, S)-intuitionistic fuzzy subnearring of R, then $\mu_A(x_1+y_1) = \min\{\mu_A(x_1+y_1), \mu_A(x_2+y_2)\} = \mu_V(x_1+y_1, x_2+y_2) = \mu_V[(x_1, x_2)+(y_1, y_2)] = \mu_V(x+y) = \mu_V(y+x) = \mu_V[(y_1, y_2)+(x_1, x_2)] = \mu_V(y_1+x_1, y_2+x_2) = \min\{\mu_A(y_1+x_1), \mu_A(y_2+x_2)\} = \mu_A(y_1+x_1)$. If $x_2= 0$, $y_2= 0$, we get $\mu_A(x_1+y_1) = \mu_A(y_1+x_1)$ for all x_1 and y_1 in R. And $\mu_A(x_1y_1) = \min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} = \mu_V(x_1y_1, x_2y_2) = \mu_V[(x_1, x_2)(y_1, y_2)] = \mu_V(xy) = \mu_V(yx) = \mu_V[(y_1, y_2)(x_1, x_2)] = \mu_V(y_1x_1, y_2x_2) = \min\{\mu_A(y_1x_1), \mu_A(y_2x_2)\} = \mu_A(y_1x_1)$. If $x_2= 0$, $y_2= 0$, we get $\mu_A(x_1y_1) = \mu_A(y_1x_1)$ for all x_1 and y_1 in R. Also $\nu_A(x_1+y_1) = \max\{\nu_A(x_1+y_1), \nu_A(x_2+y_2)\} = \nu_V(x_1+y_1, x_2+y_2) = \nu_V[(x_1, x_2)+(y_1, y_2)] = \nu_V(x+y) = \nu_V(y+x) = \nu_V[(y_1, y_2)+(x_1, x_2)] = \nu_V(y_1+x_1, y_2+x_2) = \max\{\nu_A(y_1+x_1), \nu_A(y_2+x_2)\} = \nu_A(y_1+x_1)$. If $x_2 = 0$, $y_2 = 0$, we get $\nu_A(x_1+y_1) = \nu_A(y_1+x_1)$ for all x_1 and y_1 in R. And $\nu_A(x_1y_1) = \max\{\nu_A(x_1y_1), \nu_A(x_2y_2)\} = \nu_V(x_1y_1, x_2y_2) = \nu_V[(x_1, x_2)(y_1, y_2)] = \nu_V(xy) = \nu_V(yx) = \nu_V[(y_1, y_2)(x_1, x_2)] = \nu_V(y_1x_1, y_2x_2) = \max\{\nu_A(y_1x_1), \nu_A(y_2x_2)\} = \nu_A(y_1x_1)$. If $x_2 = 0$, $y_2 = 0$, we get $\nu_A(x_1y_1) = \nu_A(y_1x_1)$ for all x_1 and y_1 in R. Therefore A is an (T, S)-intuitionistic fuzzy normal subnearring of R.

2.15 Theorem: Let (R, +, .) and (R^I, +, .) be any two nearrings. The homomorphic image of an (T, S)-intuitionistic fuzzy normal subnearring of R is an (T, S)-intuitionistic fuzzy normal subnearring of R^I.

Proof: Let $f : R \to R^I$ be a homomorphism. Let V= f(A) where A is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R. We have to prove that V is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R^I. Now for f(x), f(y) in R^I, clearly V is an (T, S)-intuitionistic fuzzy subnearring of a nearring R^I, since A is an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Now $\mu_v(f(x)+f(y)) = \mu_v(f(x+y)) \geq \mu_A(x+y) = \mu_A(y+x) \leq \mu_v(f(y+x)) = \mu_v(f(y)+f(x))$ which implies that $\mu_v(f(x)+f(y)) = \mu_v(f(y)+f(x))$ for all f(x) and f(y) in R^I. And $\mu_v(f(x)f(y)) = \mu_v(f(xy)) \geq \mu_A(xy) = \mu_A(yx) \leq \mu_v(f(yx)) = \mu_v(f(y)f(x))$ which implies that $\mu_v(f(x)f(y)) = \mu_v(f(y) f(x))$ for all f(x) and f(y) in R^I. Also $\nu_v(f(x)+f(y)) = \nu_v(f(x+y)) \leq \nu_A(x+y) = \nu_A(y+x) \geq \nu_v(f(y+x)) = \nu_v(f(y)+f(x))$ which implies that $\nu_v(f(x)+f(y)) = \nu_v(f(y)+f(x))$ for all f(x) and f(y) in R^I. And $\nu_v(f(x)f(y)) = \nu_v(f(xy)) \leq \nu_A(xy) = \nu_A(yx) \geq \nu_v(f(yx)) = \nu_v(f(y)f(x))$ which implies that $\nu_v(f(x)f(y)) = \nu_v(f(y)f(x))$ for all f(x) and f(y) in R^I. Hence V is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring R^I.

2.16 Theorem: Let (R, +, .) and (R^I, +, .) be any two nearrings. The homomorphic preimage of an (T, S)-intuitionistic fuzzy normal subnearring of R^I is an (T, S)-intuitionistic fuzzy normal subnearring of R.

Proof: Let $f : R \to R^I$ be a homomorphism. Let V = f(A) where V is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R^I. We have to prove that A is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R. Let x and y in R. Then clearly A is an (T, S)-intuitionistic fuzzy subnearring of a nearring R, since V is an (T, S)-intuitionistic fuzzy subnearring of a nearring R^I. Now $\mu_A(x+y) = \mu_v(f(x+y)) = \mu_v(f(x)+f(y)) = \mu_v(f(y)+f(x)) = \mu_v(f(y+x)) = \mu_A(y+x)$ which implies that $\mu_A(x+y) = \mu_A(y+x)$ for all x and y in R. And $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(x)f(y)) = \mu_v(f(y)f(x)) = \mu_v(f(yx)) = \mu_A(yx)$ which implies that $\mu_A(xy) = \mu_A(yx)$ for all x and y in R. Now $\nu_A(x+y) = \nu_v(f(x+y)) = \nu_v(f(x)+f(y)) = \nu_v(f(y)+f(x)) = \nu_v(f(y+x)) = \nu_A(y+x)$ which implies that $\nu_A(x+y) = \nu_A(y+x)$ for all x and y in R. And $\nu_A(xy) = \nu_v(f(xy)) = \nu_v(f(x)f(y)) = \nu_v(f(y)f(x)) = \nu_v(f(yx)) = \nu_A(yx)$ which implies that $\nu_A(xy) = \nu_A(yx)$ for all x and y in R. Hence A is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring R.

2.17 Theorem: Let (R, +, .) and (R^I, +, .) be any two nearrings. The anti-homomorphic image of an (T, S)-intuitionistic fuzzy normal subnearring of R is an (T, S)-intuitionistic fuzzy normal subnearring of R^I.

Proof: Let $f : R \to R^I$ be an anti-homomorphism. Then f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x) for all x and y in R. Let V = f(A) where A is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R. We have to prove that V is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R^I. Now for f(x) and f(y) in R^I, clearly V is an (T, S)-intuitionistic fuzzy subnearring of the nearring R^I, since A is an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Now $\mu_v(f(x)+f(y)) = \mu_v(f(y+x)) \geq \mu_A(y+x) = \mu_A(x+y) \leq \mu_v(f(x+y)) = \mu_v(f(y)+f(x))$ which implies that $\mu_v(f(x)+f(y)) = \mu_v(f(y)+f(x))$ for all f(x) and f(y) in R^I. And $\mu_v(f(x)f(y)) = \mu_v(f(yx)) \geq \mu_A(yx) = \mu_A(xy) \leq \mu_v(f(xy)) = \mu_v(f(y)f(x))$ which implies that $\mu_v(f(x)f(y)) = \mu_v(f(y)\, f(x))$ for all f(x) and f(y) in R^I. Also $\nu_v(f(x)+f(y)) = \nu_v(f(y+x)) \leq \nu_A(y+x) = \nu_A(x+y) \geq \nu_v(f(x+y)) = \nu_v(f(y)+f(x))$ which implies that $\nu_v(f(x)+f(y)) = \nu_v(f(y)+f(x))$ for all f(x) and f(y) in R^I. And $\nu_v(f(x)f(y)) = \nu_v(f(yx)) \leq \nu_A(yx) = \nu_A(xy) \geq \nu_v(f(xy)) = \nu_v(f(y)f(x))$ which implies that $\nu_v(f(x)f(y)) = \nu_v(f(y)f(x))$ for all f(x) and f(y) in R^I. Hence V is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R^I.

2.18 Theorem: Let (R, +, .) and (R^I, +, .) be any two nearrings. The anti-homomorphic preimage of an (T, S)-intuitionistic fuzzy normal subnearring of R^I is an (T, S)-intuitionistic fuzzy normal subnearring of R.

Proof: Let $f : R \to R^I$ be an anti-homomorphism. Let V = f(A) where V is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R^I. We have to prove that A is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R. Let x and y in R. Then clearly A is an (T, S)-intuitionistic fuzzy subnearring of the nearring R, since V is an (T, S)-intuitionistic fuzzy subnearring of a nearring R^I. Now $\mu_A(x+y) = \mu_v(f(x+y)) = \mu_v(f(y)+f(x)) = \mu_v(f(x)+f(y)) = \mu_v(f(y+x)) = \mu_A(y+x)$ which implies that $\mu_A(x+y) = \mu_A(y+x)$ for all x and y in R. And $\mu_A(xy) = \mu_v(f(xy)) = \mu_v(f(y)f(x)) = \mu_v(f(x)f(y)) = \mu_v(f(yx)) = \mu_A(yx)$ which implies that $\mu_A(xy) = \mu_A(yx)$ for all x and y in R. Also $\nu_A(x+y) = \nu_v(f(x+y)) = \nu_v(f(y)+f(x)) = \nu_v(f(x)+f(y)) = \nu_v(f(y+x)) = \nu_A(y+x)$ which implies that $\nu_A(x+y) = \nu_A(y+x)$ for all x and y in R. And $\nu_A(xy) = \nu_v(f(xy)) = \nu_v(f(y)f(x)) = \nu_v(f(x)f(y)) = \nu_v(f(yx)) = \nu_A(yx)$ which implies that $\nu_A(xy) = \nu_A(yx)$ for all x and y in R. Hence A is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring R.

In the following Theorem $\circ$ is the composition operation of functions:

2.19 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H and f is an isomorphism from a nearring R onto H. If A is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring H, then $A \circ f$ is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring R.

Proof: Let x and y in R and A be an (T, S)-intuitionistic fuzzy normal subnearring of a nearring H. Then clearly $A \circ f$ is an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Now $(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(x)+f(y)) = \mu_A(f(y)+f(x)) = \mu_A(f(y+x)) = (\mu_A \circ f)(y+x)$ which implies that $(\mu_A \circ f)(x+y) = (\mu_A \circ f)(y+x)$ for all x and y in R. And $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(x)f(y)) = \mu_A(f(y)f(x)) = \mu_A(f(yx)) =$

$(\mu_A \circ f)(yx)$ which implies that $(\mu_A \circ f)(xy) = (\mu_A \circ f)(yx)$ for all x and y in R. Also $(\nu_A \circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(x)+f(y)) = \nu_A(f(y)+f(x)) = \nu_A(f(y+x)) = (\nu_A \circ f)(y+x)$ which implies that $(\nu_A \circ f)(x+y) = (\nu_A \circ f)(y+x)$ for all x and y in R. And $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(x)f(y)) = \nu_A(f(y)f(x)) = \nu_A(f(yx)) = (\nu_A \circ f)(yx)$ which implies that $(\nu_A \circ f)(xy) = (\nu_A \circ f)(yx)$ for all x and y in R. Hence A∘f is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R.

2.20 Theorem: Let A be an (T, S)-intuitionistic fuzzy subnearring of a nearring H and f is an anti-isomorphism from a nearring R onto H. If A is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring H, then A∘f is an (T, S)-intuitionistic fuzzy normal subnearring of the nearring R.

Proof: Let x and y in R and A be an (T, S)-intuitionistic fuzzy normal subnearring of a nearring H. Then clearly A∘f is an (T, S)-intuitionistic fuzzy subnearring of a nearring R. Now $(\mu_A \circ f)(x+y) = \mu_A(f(x+y)) = \mu_A(f(y)+f(x)) = \mu_A(f(x)+f(y)) = \mu_A(f(y+x)) = (\mu_A \circ f)(y+x)$ which implies that $(\mu_A \circ f)(x+y) = (\mu_A \circ f)(y+x)$ for all x and y in R. And $(\mu_A \circ f)(xy) = \mu_A(f(xy)) = \mu_A(f(y)f(x)) = \mu_A(f(x)f(y)) = \mu_A(f(yx)) = (\mu_A \circ f)(yx)$ which implies that $(\mu_A \circ f)(xy) = (\mu_A \circ f)(yx)$ for all x and y in R. Also $(\nu_A \circ f)(x+y) = \nu_A(f(x+y)) = \nu_A(f(y)+f(x)) = \nu_A(f(x)+f(y)) = \nu_A(f(y+x)) = (\nu_A \circ f)(y+x)$ which implies that $(\nu_A \circ f)(x+y) = (\nu_A \circ f)(y+x)$ for all x and y in R. And $(\nu_A \circ f)(xy) = \nu_A(f(xy)) = \nu_A(f(y)f(x)) = \nu_A(f(x)f(y)) = \nu_A(f(yx)) = (\nu_A \circ f)(yx)$ which implies that $(\nu_A \circ f)(xy) = (\nu_A \circ f)(yx)$ for all x and y in R. Hence A∘f is an (T, S)-intuitionistic fuzzy normal subnearring of a nearring R.

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