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RESEARCH ARTICLE

HOMOMORPHISM AND ANTI-HOMOMORPHISM OF BIPOLAR-VALUED MULTI FUZZY SUBSEMIRINGS OF A SEMIRING

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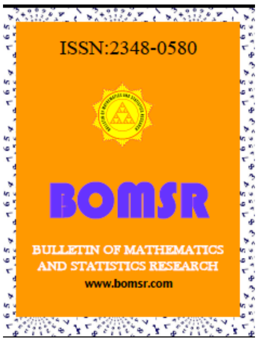

ABSTRACT

In this paper, we made an attempt to study the algebraic nature of bipolar-valued multi fuzzy subsemirings under homomorphism and anti-homomorphism and prove some results on these.

KEY WORDS: Bipolar-valued fuzzy set, bipolar-valued multi fuzzy set, bipolar-valued multi fuzzy subsemiring, bipolar-valued multi fuzzy normal subsemiring.

INTRODUCTION

In 1965, Zadeh [13] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [6]. Lee [8] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [–1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [–1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [8, 9]. Anitha.M.S., Muruganantha Prasad & K.Arjunan[1] defined as Bipolar-valued fuzzy subgroups of a group. We introduce the concept of bipolar-valued multi fuzzy subsemiring under homomorphism, antihomomorphism and established some results.

1.PRELIMINARIES:

1.1 Definition: A bipolar-valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) > / x \in X\}$, where $A^+ : X \rightarrow [0, 1]$ and $A^- : X \rightarrow [-1, 0]$. The positive membership degree $A^+(x)$

denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued fuzzy set A. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.2 Example: A = { < a, 0.5, –0.3 >, < b, 0.1, –0.7 >, < c, 0.5, –0.4 >} is a bipolar-valued fuzzy subset of X= {a, b, c }.

1.3 Definition: A bipolar-valued multi fuzzy set (BVMFS) A in X is defined as an object of the form A = {<x, $A_i^+(x)$, $A_i^-(x)$ >/ $x\in X$}, where $A_i^+ : X\to [0, 1]$ and $A_i^- : X\to [-1, 0]$. The positive membership degrees $A_i^+(x)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar-valued multi fuzzy set A and the negative membership degrees $A_i^-(x)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued multi fuzzy set A. If $A_i^+(x) \neq 0$ and $A_i^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A_i^+(x) = 0$ and $A_i^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A_i^+(x) \neq 0$ and $A_i^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X, where i = 1 to n.

1.4 Example: A = { < a, 0.5, 0,6, 0.3, –0.3, –0.6, –0.5 >, < b, 0.1, 0.4, 0.7, –0.7, –0.3, –0.6 >, < c, 0.5, 0.3, 0.8, –0.4, –0.5, –0.3 >} is a bipolar-valued multi fuzzy subset of X = { a, b, c }.

1.5 Definition: Let R be a semiring. A bipolar-valued multi fuzzy subset A of R is said to be a bipolar-valued multi fuzzy subsemiring of R (BVMFSSR) if the following conditions are satisfied,

(i) $A_i^+(x+y) \geq \min\{ A_i^+(x), A_i^+(y) \}$

(ii) $A_i^+(xy) \geq \min\{ A_i^+(x), A_i^+(y) \}$

(iii) $A_i^-(x+y) \leq \max\{ A_i^-(x), A_i^-(y) \}$

(iv) $A_i^-(xy) \leq \max\{ A_i^-(x), A_i^-(y) \}$ for all x and y in R.

1.6 Example: Let R = Z_3 = { 0, 1, 2 } be a semiring with respect to the ordinary addition and multiplication. Then A = {< 0, 0.5, 0.8, 0.6, –0.6, –0.5, –0.7>, <1, 0.4, 0.7, 0.5, –0.5, –0.4, –0.6 >, < 2, 0.4, 0.7, 0.5, –0.5, –0.4, –0.6 > } is a bipolar-valued multi fuzzy subsemiring of R.

1.7 Definition: Let R be a semiring. A bipolar-valued multi fuzzy subsemiring A of R is said to be a bipolar-valued multi fuzzy normal subsemiring of R if $A_i^+(x+y) = A_i^+(y+x)$, $A_i^+(xy) = A_i^+(yx)$, $A_i^-(x+y) = A_i^-(y+x)$ and $A_i^-(xy) = A_i^-(yx)$ for all x and y in R.

1.8 Definition: Let R and R^I be any two semirings. Then the function f: R → R^I is said to be an antihomomorphism if f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x) for all x and y in R.

1.9 Definition: Let X and X^I be any two sets. Let f : X→ X^I be any function and let A be a bipolar-valued multi fuzzy subset in X, V be a bipolar-valued multi fuzzy subset in f(X) = X^I, defined by $V_i^+(y) = \sup_{x\in f^{-1}(y)} A_i^+(x)$ and $V_i^-(y) = \inf_{x\in f^{-1}(y)} A_i^-(x)$, for all x in X and y in X^I. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2. SOME PROPERTIES:

2.1 Theorem: Let R and R^I be any two semirings. The homomorphic image of a bipolar-valued multi fuzzy subsemiring of R is a bipolar-valued multi fuzzy subsemiring of R^I.

Proof: Let f : R $\rightarrow$ R^{I} be a homomorphism. Let V = f(A) where A is a bipolar-valued multi fuzzy subsemiring of R. We have to prove that V is a bipolar-valued multi fuzzy subsemiring of R^{I}. Now for f(x), f(y) in R^{I}, V_i^+(f(x)+f(y)) = V_i^+(f(x+y)) ≥ A_i^+(x+y) ≥ min{ A_i^+(x), A_i^+(y)} = min {V_i^+(f(x)), V_i^+(f(y)) } which implies that V_i^+(f(x)+f(y)) ≥ min{ V_i^+(f(x)), V_i^+(f(y)) }. And V_i^+(f(x)f(y)) = V_i^+(f(xy)) ≥ A_i^+(xy) ≥ min{ A_i^+(x), A_i^+(y)} = min { V_i^+(f(x)), V_i^+(f(y)) } which implies that V_i^+(f(x)f(y)) ≥ min{ V_i^+(f(x)), V_i^+(f(y)) }. Also V_i^-(f(x)+f(y)) = V_i^-(f(x+y)) ≤ A_i^-(x+y) ≤ max{ A_i^-(x), A_i^-(y) } = max { V_i^-(f(x)), V_i^-(f(y))} which implies that V_i^-(f(x)+f(y)) ≤ max{V_i^-(f(x)), V_i^-(f(y)) }. And V_i^-(f(x)f(y)) = V_i^-(f(xy)) ≤ A_i^-(xy) ≤ max{ A_i^-(x), A_i^-(y) } = max{ V_i^-(f(x)), V_i^-(f(y))} which implies that V_i^-(f(x)f(y)) ≤ max{ V_i^-(f(x)), V_i^-(f(y)) }. Hence V is a bipolar-valued multi fuzzy subsemiring of R^{I}.

2.2 Theorem: Let R and R^{I} be any two semirings. The homomorphic preimage of a bipolar-valued multi fuzzy subsemiring of R^{I} is a bipolar-valued multi fuzzy subsemiring of R.

Proof: Let f : R $\rightarrow$ R^{I} be a homomorphism. Let V = f(A) where V is a bipolar-valued multi fuzzy subsemiring of R^{I}. We have to prove that A is a bipolar-valued multi fuzzy subsemiring of R. Let x and y in R. Now A_i^+(x+y) = V_i^+(f(x+y)) = V_i^+(f(x)+f(y)) ≥ min{ V_i^+(f(x)), V_i^+(f(y)) }= min{ A_i^+(x), A_i^+(y)} which implies that A_i^+(x+y) ≥ min{ A_i^+(x), A_i^+(y) }. And A_i^+(xy) = V_i^+(f(xy)) = V_i^+(f(x)f(y)) ≥ min{ V_i^+(f(x)), V_i^+(f(y)) }= min { A_i^+(x), A_i^+(y)} which implies that A_i^+(xy) ≥ min{ A_i^+(x), A_i^+(y) }. Also A_i^-(x+y) = V_i^-(f(x+y)) = V_i^-(f(x)+f(y)) ≤ max{ V_i^-(f(x)), V_i^-(f(y))} = max{ A_i^-(x), A_i^-(y) } which implies that A_i^-(x+y) ≤ max{ A_i^-(x), A_i^-(y) }. And A_i^-(xy) = V_i^-(f(xy)) = V_i^-(f(x)f(y)) ≤ max{ V_i^-(f(x)), V_i^-(f(y))} = max{ A_i^-(x), A_i^-(y)} which implies that A_i^-(xy) ≤ max{A_i^-(x), A_i^-(y)}. Hence A is a bipolar-valued multi fuzzy subsemiring of R.

2.3 Theorem: Let R and R^{I} be any two semirings. The antihomomorphic image of a bipolar valued multi fuzzy subsemiring of R is a bipolar-valued multi fuzzy subsemiring of R^{I}.

Proof: Let f : R $\rightarrow$ R^{I} be an antihomomorphism. Let V = f(A) where A is a bipolar-valued multi fuzzy subsemiring of R. We have to prove that V is a bipolar-valued multi fuzzy subsemiring of R^{I}. Now for f(x), f(y) in R^{I}, V_i^+(f(x)+f(y)) = V_i^+(f(y+x)) ≥ A_i^+(y+x) ≥ min { A_i^+(x), A_i^+(y)} = min { V_i^+(f(x)), V_i^+(f(y)) } which implies that V_i^+(f(x)+f(y)) ≥ min { V_i^+(f(x)), V_i^+(f(y)) }. And V_i^+(f(x)f(y)) = V_i^+(f(yx)) ≥ A_i^+(yx) ≥ min { A_i^+(x), A_i^+(y)} = min { V_i^+(f(x)), V_i^+(f(y)) } which implies that V_i^+(f(x)f(y)) ≥ min { V_i^+(f(x)), V_i^+(f(y)) }. Also V_i^-(f(x)+f(y)) = V_i^-(f(y+x)) ≤ A_i^-(y+x) ≤ max{ A_i^-(x), A_i^-(y)} = max{ V_i^-(f(x)), V_i^-(f(y))} which implies that V_i^-(f(x)+f(y)) ≤ max { V_i^-(f(x)), V_i^-(f(y))}. And V_i^-(f(x)f(y)) = V_i^-(f(yx)) ≤ A_i^-(yx) ≤ max{ A_i^-(x), A_i^-(y)} = max{ V_i^-(f(x)), V_i^-(f(y)) } which implies that V_i^-(f(x)f(y)) ≤ max { V_i^-(f(x)), V_i^-(f(y))}. Hence V is a bipolar-valued multi fuzzy subsemiring of R^{I}.

2.4 Theorem: Let R and R^{I} be any two semirings. The antihomomorphic preimage of a bipolar-valued multi fuzzy subsemiring of R^{I} is a bipolar-valued multi fuzzy subsemiring of R.

Proof: Let f : R $\rightarrow$ R^{I} be an antihomomorphism. Let V = f(A) where V is a bipolar-valued multi fuzzy subsemiring of R^{I}. We have to prove that A is a bipolar-valued multi fuzzy subsemiring of R. Let x and y in R. Now A_i^+(x+y) = V_i^+(f(x+y)) = V_i^+(f(y)+f(x)) ≥ min { V_i^+(f(x)), V_i^+(f(y))}= min{ A_i^+(x), A_i^+(y)} which implies that A_i^+(x+y) ≥ min{ A_i^+(x), A_i^+(y)}. And A_i^+(xy) = V_i^+(f(xy)) = V_i^+(f(y)f(x)) ≥ min { V_i^+(f(x)), V_i^+(f(y))}= min{ A_i^+(x), A_i^+(y)} which implies that A_i^+(xy) ≥ min{ A_i^+(x), A_i^+(y)}. Also A_i^-(x+y) = V_i^-(f(x+y)) = V_i^-(f(y)+f(x)) ≤ max{ V_i^-(f(x)), V_i^-(f(y))} = max{ A_i^-(x), A_i^-(y)} which implies that A_i^-(x+y) ≤ max{ A_i^-(x), A_i^-(y) }. And A_i^-(xy) = V_i^-(f(xy)) = V_i^-(f(y)f(x)) ≤ max{ V_i^-(f(x)), V_i^-(f(y))} = max{ A_i^-(x), A_i^-(y)} which implies that A_i^-(xy) ≤ max { A_i^-(x), A_i^-(y) }. Hence A is a bipolar-valued multi fuzzy subsemiring of R.

2.5 Theorem: Let R and R^{I} be any two semirings. The homomorphic image of a bipolar-valued multi fuzzy normal subsemiring of R is a bipolar-valued multi fuzzy normal subsemiring of R^{I}.

Proof: Let f : R $\rightarrow$ R^{I} be a homomorphism. Let V = f(A) where A is a bipolar-valued multi fuzzy normal subsemiring of R. We have to prove that V is a bipolar-valued multi fuzzy normal subsemiring of R^{I}.

Now for f(x), f(y) in R^I, V_i^+(f(x)+f(y)) = V_i^+(f(x+y)) ≥ A_i^+(x+y) = A_i^+(y+x) ≤ V_i^+(f(y+x)) = V_i^+(f(y)+f(x)) which implies that V_i^+(f(x)+f(y)) = V_i^+(f(y)+f(x)). And V_i^+(f(x)f(y)) = V_i^+(f(xy)) ≥ A_i^+(xy) = A_i^+(yx) ≤ V_i^+(f(yx)) = V_i^+(f(y)f(x)) which implies that V_i^+(f(x)f(y)) = V_i^+(f(y)f(x)). Also V_i^-(f(x)+f(y)) = V_i^-(f(x+y)) ≥ A_i^-(x+y) = A_i^-(y+x) ≤ V_i^-(f(y+x)) = V_i^-(f(y)+f(x)) which implies that V_i^-(f(x)+f(y)) = V_i^-(f(y)+f(x)). And V_i^-(f(x)f(y)) = V_i^-(f(xy)) ≥ A_i^-(xy) = A_i^-(yx) ≤ V_i^-(f(yx)) = V_i^-(f(y)f(x)) which implies that V_i^-(f(x)f(y)) = V_i^-(f(y)f(x)). Hence V is a bipolar-valued multi fuzzy normal subsemiring of R^I.

2.6 Theorem: Let R and R^I be any two semirings. The homomorphic preimage of a bipolar-valued multi fuzzy normal subsemiring of R^I is a bipolar-valued multi fuzzy normal subsemiring of R.

Proof: Let f : R → R^I be a homomorphism. Let V = f(A) where V is a bipolar-valued multi fuzzy normal subsemiring of R^I. We have to prove that A is a bipolar-valued multi fuzzy normal subsemiring of R. Let x and y in R. Now A_i^+(x+y) = V_i^+(f(x+y)) = V_i^+(f(x)+f(y)) = V_i^+(f(y)+f(x)) = V_i^+(f(y+x)) = A_i^+(y+x) which implies that A_i^+(x+y) = A_i^+(y+x). And A_i^+(xy) = V_i^+(f(xy)) = V_i^+(f(x)f(y)) = V_i^+(f(y)f(x)) = V_i^+(f(yx)) = A_i^+(yx) which implies that A_i^+(xy) = A_i^+(yx). Also A_i^-(x+y) = V_i^-(f(x+y)) = V_i^-(f(x)+f(y)) = V_i^-(f(y)+f(x)) = V_i^-(f(y+x)) = A_i^-(y+x) which implies that A_i^-(x+y) = A_i^-(y+x). And A_i^-(xy) = V_i^-(f(xy)) = V_i^-(f(x)f(y)) = V_i^-(f(y)f(x)) = V_i^-(f(yx)) = A_i^-(yx) which implies that A_i^-(xy)= A_i^-(yx). Hence A is a bipolar-valued multi fuzzy normal subsemiring of R.

2.7 Theorem: Let R and R^I be any two semirings. The antihomomorphic image of a bipolar-valued multi fuzzy normal subsemiring of R is a bipolar-valued multi fuzzy normal subsemiring of R^I.

Proof: Let f : R → R^I be an antihomomorphism. Let V = f(A) where A is a bipolar-valued multi fuzzy normal subsemiring of R. We have to prove that V is a bipolar-valued multi fuzzy normal subsemiring of R^I. Now for f(x), f(y) in G^I, V_i^+(f(x)+f(y)) = V_i^+(f(y+x)) ≥ A_i^+(y+x) = A_i^+(x+y) ≤ V_i^+(f(x+y)) = V_i^+(f(y)+f(x)) which implies that V_i^+(f(x)+f(y)) = V_i^+(f(y)+f(x)). And V_i^+(f(x)f(y)) = V_i^+(f(yx)) ≥ A_i^+(yx) = A_i^+(xy) ≤ V_i^+(f(xy)) = V_i^+(f(y)f(x)) which implies that V_i^+(f(x)f(y)) = V_i^+(f(y)f(x)). Also V_i^-(f(x)+f(y)) = V_i^-(f(y+x)) ≤ A_i^-(y+x) = A_i^-(x+y) ≥ V_i^-(f(x+y)) = V_i^-(f(y)+f(x)) which implies that V_i^-(f(x)+f(y)) = V_i^-(f(y)+f(x)). And V_i^-(f(x)f(y)) = V_i^-(f(yx)) ≤ A_i^-(yx) = A_i^-(xy) ≥ V_i^-(f(xy)) = V_i^-(f(y)f(x)) which implies that V_i^-(f(x)f(y)) = V_i^-(f(y)f(x)). Hence V is a bipolar-valued multi fuzzy normal subsemiring of R^I.

2.8 Theorem: Let R and R^I be any two semirings. The antihomomorphic preimage of a bipolar-valued multi fuzzy normal subsemiring of R^I is a bipolar-valued multi fuzzy normal subsemiring of R.

Proof: Let f : R → R^I be an antihomomorphism. Let V = f(A) where V is a bipolar-valued multi fuzzy normal subsemiring of R^I. We have to prove that A is a bipolar-valued multi fuzzy normal subsemiring of R. Let x and y in R. Now A_i^+(x+y) = V_i^+(f(x+y)) = V_i^+(f(y)+f(x)) = V_i^+(f(x)+f(y)) = V_i^+(f(y+x)) = A_i^+(y+x) which implies that A_i^+(x+y) = A_i^+(y+x). And A_i^+(xy) = V_i^+(f(xy)) = V_i^+(f(y)f(x)) = V_i^+(f(x)f(y)) = V_i^+(f(yx)) = A_i^+(yx) which implies that A_i^+(xy) = A_i^+(yx). Also A_i^-(x+y) = V^-(f(x+y)) = V_i^-(f(y)+f(x)) = V_i^-(f(x)+f(y)) = V_i^-(f(y+x)) = A_i^-(y+x) which implies that A_i^-(x+y) = A_i^-(y+x). And A_i^-(xy) = V^-(f(xy)) = V_i^-(f(y)f(x)) = V_i^-(f(x)f(y)) = V_i^-(f(yx)) = A_i^-(yx) which implies that A_i^-(xy) = A_i^-(yx). Hence A is a bipolar-valued multi fuzzy normal subsemiring of R.

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