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RESEARCH ARTICLE

BIPOLAR INTERVAL VALUED FUZZY SUBGROUPS OF A GROUP

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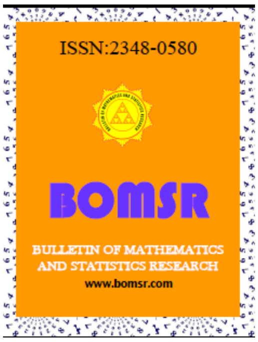

ABSTRACT

In this paper, we study some of the properties of bipolar interval valued fuzzy subgroup of a group and prove some results on these.

KEY WORDS: Bipolar valued fuzzy set, bipolar interval valued fuzzy set, bipolar interval valued fuzzy subgroup.

INTRODUCTION

In 1965, Zadeh [14] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [6]. Lee [8] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [8, 9]. Somasundara Moorthy.M.G & K.Arjunan [12] introduced the interval valued fuzzy subrings of a ring under homomorphism. In this paper we introduce the concept of bipolar interval valued fuzzy subgroup and established some results.

1.PRELIMINARIES

1.1 Definition: A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = { < x, $A^+(x)$, $A^-(x)$ >/ $x \in X$}, where $A^+ : X \to [0, 1]$ and $A^- : X \to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A. If $A^+(x) \neq 0$ and $A^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for A and if $A^+(x) = 0$ and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.2 Example: A = { < a, 0.5, –0.3 >, < b, 0.1, –0.7 >, < c, 0.5, –0.4 >} is a bipolar valued fuzzy subset of X= {a, b, c }.

1.3 Definition: A bipolar interval valued fuzzy set (BIVFS) [A] in X is defined as an object of the form $[A] = \{ < x, [A]^+(x), [A]^-(x) > / x \in X\}$, where $[A]^+ : X \to D[0, 1]$ and $[A]^- : X \to D[-1, 0]$. The positive membership degree $[A]^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar interval valued fuzzy set [A] and the negative membership degree $[A]^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar interval valued fuzzy set [A]. If $[A]^+(x) \neq [0, 0]$ and $[A]^-(x) = [0, 0]$, it is the situation that x is regarded as having only positive satisfaction for [A] and if $[A]^+(x) = [0, 0]$ and $[A]^-(x) \neq [0, 0]$, it is the situation that x does not satisfy the property of [A], but somewhat satisfies the counter property of [A]. It is possible for an element x to be such that $[A]^+(x) \neq [0, 0]$ and $[A]^-(x) \neq [0, 0]$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.4 Example: [A] = { < a, [0.5, 0.6], [–0.6, –0.4] >, < b, [0.1, 0.4], [–0.7, –0.5] >, < c, [0.5, 0.6], [–0.6, –0.4] >} is a bipolar interval valued fuzzy subset of X = {a, b, c }.

1.5 Definition: Let G be a group. A bipolar interval valued fuzzy subset [A] of G is said to be a bipolar interval valued fuzzy subgroup of G if the following conditions are satisfied

(i) $[A]^+(xy) \geq \text{rmin} \{ [A]^+(x), [A]^+(y) \}$

(ii) $[A]^+(x^{-1}) \geq [A]^+(x)$

(iii) $[A]^-(xy) \leq \text{rmax} \{ [A]^-(x), [A]^-(y) \}$

(iv) $[A]^-(x^{-1}) \leq [A]^-(x)$ for all x and y in G.

1.6 Example: Let G = { 1, –1, i, –i } be a group with respect to the ordinary multiplication. Then [A] = { < 1, [0.5, 0.5], [–0.6, – 0.6] >, < –1, [0.4, 0.4], [–0.5, – 0.5] >, < i, [0.2, 0.2], [–0.4, –0.4] >, < –i, [0.2, 0.2], [–0.4, –0.4] > } is a bipolar interval valued fuzzy subgroup of G.

1.7 Definition: Let G be a group. A bipolar interval valued fuzzy subgroup [A] of G is said to be a bipolar interval valued fuzzy normal subgroup of G if

(i) $[A]^+(xy) = [A]^+(yx)$

(ii) $[A]^-(xy) = [A]^-(yx)$ for all x and y in G.

1.8 Definition: Let [A] be a bipolar interval valued fuzzy subgroup of a group G. For any $a \in G$, a[A] defined by $(a[A]^+)(x) = [A]^+(a^{-1}x)$ and $(a[A]^-)(x) = [A]^-(a^{-1}x)$, for every $x \in G$ is called the bipolar interval valued fuzzy coset of the group G.

1.9 Definition: Let [A] be a bipolar interval valued fuzzy subgroup of a group G and $H = \{x \in G / [A]^+(x) = [A]^+(e) \text{ and } [A]^-(x) = [A]^-(e) \}$, then o([A]), order of [A] is defined as o([A]) = o(H).

1.10 Definition: Let [A] and [B] be two bipolar interval valued fuzzy subgroups of a group G. Then [A] and [B] are said to be conjugate bipolar interval valued fuzzy subgroup of G if for some $g \in G$, $[A]^+(x) = [B]^+(g^{-1}xg)$ and $[A]^-(x) = [B]^-(g^{-1}xg)$, for every $x \in G$.

1.11 Definition: Let [A] be a bipolar interval valued fuzzy subgroup of a group G. Then for any a and b in G, a bipolar interval valued fuzzy middle coset a[A]b of G is defined by $(a[A]^+b)(x) = [A]^+(a^{-1}xb^{-1})$ and $(a[A]^-b)(x) = [A]^-(a^{-1}x b^{-1})$ for every $x \in G$.

2. PROPERTIES:

2.1 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. If $[A]^+(x) < [A]^+(y)$ and $[A]^-(x) > [A]^-(y)$ for some x and y in G, then $[A]^+(xy) = [A]^+(x) = [A]^+(yx)$ and $[A]^-(xy) = [A]^-(x) = [A]^-(yx)$.

Proof: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. Let $[A]^+(x) < [A]^+(y)$ and $[A]^-(x) > [A]^-(y)$ for some x and y in G. Now $[A]^+(xy) \geq \text{rmin}\{[A]^+(x), [A]^+(y)\}= [A]^+(x)$; and $[A]^+(x) = [A]^+(xyy^{-1}) \geq \text{rmin}\{[A]^+(xy), [A]^+(y)\} = [A]^+(xy)$. Also $[A]^+(yx) \geq \text{rmin}\{[A]^+(y), [A]^+(x)\} = [A]^+(x)$; and $[A]^+(x) = [A]^+(y^{-1}yx) \geq \text{rmin}\{[A]^+(y), [A]^+(yx)\} = [A]^+(yx)$. Therefore $[A]^+(xy) = [A]^+(x) = [A]^+(yx)$. Now $[A]^-(xy) \leq \text{rmax}\{[A]^-(x), [A]^-(y)\}= [A]^-(x)$; and $[A]^-(x) = [A]^-(xyy^{-1}) \leq \text{rmax}\{[A]^-(xy), [A]^-(y)\}= [A]^-(xy)$. Also $[A]^-(yx) \leq \text{rmax}\{[A]^-(y), [A]^-(x)\}= [A]^-(x)$; and $[A]^-(x) = [A]^-(y^{-1}yx) \leq \text{rmax}\{[A]^-(y), [A]^-(yx)\}= [A]^-(yx)$. Therefore $[A]^-(xy) = [A]^-(x) = [A]^-(yx)$.

2.2 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. If $[A]^+(x) < [A]^+(y)$ and $[A]^-(x) < [A]^-(y)$ for some x and y in G, then $[A]^+(xy) = [A]^+(x) = [A]^+(yx)$ and $[A]^-(xy) = [A]^-(y) = [A]^-(yx)$.

Proof: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. Let $[A]^+(x) < [A]^+(y)$ and $[A]^-(x) < [A]^-(y)$ for some x and y in G. Now $[A]^+(xy) \geq \text{rmin}\{[A]^+(x), [A]^+(y)\}= [A]^+(x)$; and $[A]^+(x) = [A]^+(xyy^{-1}) \geq \text{rmin}\{[A]^+(xy), [A]^+(y)\}= [A]^+(xy)$. And $[A]^+(yx) \geq \text{rmin}\{[A]^+(y), [A]^+(x)\}= [A]^+(x)$; and $[A]^+(x) = [A]^+(y^{-1}yx) \geq \text{rmin}\{[A]^+(y), [A]^+(yx)\}= [A]^+(yx)$. Therefore $[A]^+(xy) = [A]^+(x) = [A]^+(yx)$. Now $[A]^-(xy) \leq \text{rmax}\{[A]^-(x), [A]^-(y)\}= [A]^-(y)$; and $[A]^-(y) = [A]^-(x^{-1}xy) \leq \text{rmax}\{[A]^-(x), [A]^-(xy)\}= [A]^-(xy)$. And $[A]^-(yx) \leq \text{rmax}\{[A]^-(y), [A]^-(x)\}= [A]^-(y)$; and
$[A]^-(y) = [A]^-(yxx^{-1}) \leq \text{rmax}\{[A]^-(yx), [A]^-(x)\}= [A]^-(yx)$. Therefore $[A]^-(xy) = [A]^-(y) = [A]^-(yx)$.

2.3 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. If $[A]^+(x) > [A]^+(y)$ and $[A]^-(x) > [A]^-(y)$ for some x and y in G, then $[A]^+(xy) = [A]^+(y) = [A]^+(yx)$ and $[A]^-(xy) = [A]^-(x) = [A]^-(yx)$.

Proof: It is trivial.

2.4 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a group G. If $[A]^+(x) > [A]^+(y)$ and $[A]^-(x) < [A]^-(y)$ for some x and y in G, then $[A]^+(xy) = [A]^+(y) = [A]^+(yx)$ and $[A]^-(xy) = [A]^-(y) = [A]^-(yx)$.

Proof: It is trivial.

2.5 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy subgroup of a finite group G, then o([A]) divides o(G).

Proof: Let [A] be a bipolar interval valued fuzzy subgroup of a finite group G with e as its identity element. Clearly $H = \{ x \in G \,/\, [A]^+(x) = [A]^+(e) \text{ and } [A]^-(x) = [A]^-(e) \}$ is a subgroup of the group G. By Lagranges theorem $o(H) \mid o(G)$.

Hence by the definition of the order of the bipolar interval valued fuzzy subgroup of the group G, we have $o([A]) \mid o(G)$.

2.6 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ and $[B] = \langle [B]^+, [B]^- \rangle$ be two bipolar interval valued fuzzy subsets of a abelian group G. Then A and B are conjugate bipolar interval valued fuzzy subsets of the group G if and only if A = B.

Proof: Let A and B be conjugate bipolar interval valued fuzzy subsets of group G, then for some $y \in G$, we have $[A]^+(x) = [B]^+(y^{-1}xy) = [B]^+(y^{-1}yx) = [B]^+(ex) = [B]^+(x)$. Therefore $[A]^+(x) = [B]^+(x)$. And $[A]^-(x) = [B]^-(y^{-1}xy) = [B]^-(y^{-1}yx) = [B]^-(ex) = [B]^-(x)$. Therefore $[A]^-(x) = [B]^-(x)$. Hence [A] = [B]. Conversely if [A] = [B] then for the identity element e of group G, we have $[A]^+(x) = [B]^+(e^{-1}xe)$ and $[A]^-(x) = [B]^-(e^{-1}xe)$ for every $x \in G$. Hence [A] and [B] are conjugate bipolar interval valued fuzzy subsets of the group G.

2.7 Theorem: If $[A] = \langle [A]^+, [A]^- \rangle$ and $[B] = \langle [B]^+, [B]^- \rangle$ are conjugate bipolar interval valued fuzzy subgroups of the group G, then o([A]) = o([B]).

Proof: Let [A] and [B] are conjugate bipolar interval valued fuzzy subgroups of the group G.

Now o([A]) = order of $\{ x \in G \,/\, [A]^+(x) = [A]^+(e)$ and $[A]^-(x) = [A]^-(e) \}$

= order of $\{ x \in G \,/\, [B]^+(y^{-1}xy) = [B]^+(y^{-1}ey)$ and $[B]^-(y^{-1}xy) = [B]^-(y^{-1}ey) \}$

= order of $\{ x \in G \,/\, [B]^+(x) = [B]^+(e)$ and $[B]^-(x) = [B]^-(e) \}$

= o([B]).

Hence o([A]) = o([B]).

2.8 Theorem: Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar interval valued fuzzy normal subgroup of a group G. Then for any y in G we have $[A]^+(yxy^{-1}) = [A]^+(y^{-1}xy)$ and $[A]^-(yxy^{-1}) = [A]^-(y^{-1}xy)$ for every $x \in G$.

Proof: Let [A] be a bipolar interval valued fuzzy normal subgroup of a group G.

For any y in G. Then we have $[A]^+(yxy^{-1}) = [A]^+(x) = [A]^+(xyy^{-1}) = [A]^+(y^{-1}xy)$.

Therefore $[A]^+(yxy^{-1}) = [A]^+(y^{-1}xy)$.

And $[A]^-(yxy^{-1}) = [A]^-(x) = [A]^-(xyy^{-1}) = [A]^-(y^{-1}xy)$.

Therefore $[A]^-(yxy^{-1}) = [A]^-(y^{-1}xy)$.

2.9 Theorem: A bipolar interval valued fuzzy subgroup $[A] = \langle [A]^+, [A]^- \rangle$ of a group G is normalized if and only if $[A]^+(e) = [1, 1]$ and $[A]^-(e) = [0, 0]$ where e is the identity element of the group G.

Proof: If [A] is normalized then there exists $x \in G$ such that $[A]^+(x) = [1, 1]$ and $[A]^-(x) = [0, 0]$, but by properties of a bipolar interval valued fuzzy subgroup [A] of the group G, $[A]^+(x) \leq [A]^+(e)$ and $[A]^-(x) \geq [A]^-(e)$ for every $x \in G$.

since $[A]^+(x) = [1, 1]$ and $[A]^-(x) = [0, 0]$ and $[A]^+(x) \leq [A]^+(e)$ and $[A]^-(x) \geq [A]^-(e)$.

Therefore $[1, 1] \leq [A]^+(e)$ and $[0, 0] \geq [A]^-(e)$. But $[1, 1] \geq [A]^+(e)$ and $[0, 0] \leq [A]^-(e)$.

Hence $[A]^+(e) = [1, 1]$ and $[A]^-(e) = [0, 0]$.

Conversely if $[A]^+(e) = [1, 1]$ and $[A]^-(e) = [0, 0]$, then by the definition of normalized bipolar interval valued fuzzy subset [A] is normalized.

2.10 Theorem: If $[A] = \langle [A]^+, [A]^- \rangle$ is a bipolar interval valued fuzzy subgroup of a group G, then for any a in G the bipolar interval valued fuzzy middle coset $a[A]a^{-1}$ of G is also a bipolar interval valued fuzzy subgroup of a group G.

Proof: Let [A] is a bipolar interval valued fuzzy subgroup of a group G and a in G. To prove $a[A]a^{-1} = (x, a[A]^+a^{-1}, a[A]^-a^{-1})$ is a bipolar interval valued fuzzy subgroup of G. Let x and y in G.

Then $(a\,[A]^+a^{-1})(xy^{-1}) = [A]^+(a^{-1}xy^{-1}a)$

$= [A]^+(a^{-1}xaa^{-1}y^{-1}a)$

$= [A]^+(a^{-1}xa(a^{-1}ya)^{-1})$

$\geq \text{rmin}\{ [A]^+(a^{-1}xa), [A]^+(a^{-1}ya) \}$

$= \text{rmin}\{ (a\,[A]^+a^{-1})(x), (a\,[A]^+a^{-1})(y) \}$.

Therefore $(a\,[A]^+a^{-1})(xy^{-1}) \geq \text{rmin}\{ (a\,[A]^+a^{-1})(x), (a\,[A]^+a^{-1})(y) \}$.

And $(a\,[A]^-a^{-1})(xy^{-1}) = [A]^-(a^{-1}xy^{-1}a)$

$= [A]^-(a^{-1}xaa^{-1}y^{-1}a)$

$= [A]^-(a^{-1}xa(a^{-1}ya)^{-1})$

$\leq \text{rmax}\{ [A]^-(a^{-1}xa), [A]^-(a^{-1}ya) \}$

$= \text{rmax}\{ (a\,[A]^-a^{-1})(x), (a\,[A]^-a^{-1})(y) \}$.

Therefore $(a\,[A]^-a^{-1})(xy^{-1}) \leq \text{rmax}\{ (a\,[A]^-a^{-1})(x), (a\,[A]^-a^{-1})(y) \}$.

Hence $a[A]a^{-1}$ is a bipolar interval valued fuzzy subgroup of a group G.

2.11 Theorem: Let [A] = 〈 $[A]^+$, $[A]^-$ 〉 be a bipolar interval valued fuzzy subgroup of a group G and $a[A]a^{-1}$ be a bipolar interval valued fuzzy middle coset of the group G, then $o(a[A]a^{-1}) = o([A])$ for any $a\in G$.

Proof: Let [A] be a bipolar interval valued fuzzy subgroup of a group G and $a\in G$. By Theorem 2.10, the bipolar interval valued fuzzy middle coset $a[A]a^{-1}$ is a bipolar interval valued fuzzy subgroup of a group G. Further by the definition of a bipolar interval valued fuzzy middle coset of the group G we have $(a\,[A]^+a^{-1})(x) = [A]^+(a^{-1}xa)$ and $(a\,[A]^-a^{-1})(x) = [A]^-(a^{-1}xa)$ for every x in G.

Hence for any a in G, [A] and $a[A]a^{-1}$ are conjugate bipolar interval valued fuzzy subgroup of the group G as there exists $a\in G$ such that $(a\,[A]^+a^{-1})(x) = [A]^+(a^{-1}xa)$ and $(a\,[A]^-a^{-1})(x) = [A]^-(a^{-1}xa)$ for every x in G. By Theorem 2.6, $o(a[A]a^{-1}) = o([A])$ for any a in G.

2.12 Theorem: Let [A] = 〈 $[A]^+$, $[A]^-$ 〉 be a bipolar interval valued fuzzy subgroup of a group G and [B] = 〈 $[B]^+$, $[B]^-$ 〉 be a bipolar interval valued fuzzy subset of a group G. If [A] and [B] are conjugate bipolar interval valued fuzzy subsets of the group G then [B] is a bipolar interval valued fuzzy subgroup of a group G.

Proof: Let [A] be a bipolar interval valued fuzzy subgroup of a group G and [B] be a bipolar interval valued fuzzy subset of a group G. And let [A] and [B] are conjugate bipolar interval valued fuzzy subsets of the group G. To prove [B] is a bipolar interval valued fuzzy subgroup of the group G.

Let x and y in G. Then xy^{-1} in G.

Now, $[B]^+(xy^{-1}) = [A]^+(g^{-1}xy^{-1}g) = [A]^+(g^{-1}xgg^{-1}y^{-1}g) = [A]^+(g^{-1}xg(g^{-1}yg)^{-1}) \geq \text{rmin}\,\{[A]^+(g^{-1}xg), [A]^+(g^{-1}yg)\} = \text{rmin}\{[B]^+(x), [B]^+(y)\}$. Therefore $[B]^+(xy^{-1}) \geq \text{rmin}\,\{[B]^+(x), [B]^+(y)\}$. And $[B]^-(xy^{-1}) = [A]^-(g^{-1}xy^{-1}g) = [A]^-(g^{-1}xgg^{-1}y^{-1}g) = [A]^-(g^{-1}xg(g^{-1}yg)^{-1}) \leq \text{rmax}\,\{[A]^-(g^{-1}xg), [A]^-(g^{-1}yg)\} = \text{rmax}\{[B]^-(x), [B]^-(y)\}$. Therefore $[B]^-(xy^{-1}) \leq \text{rmax}\,\{[B]^-(x), [B]^-(y)\}$.

Hence [B] is a bipolar interval valued fuzzy subgroup of the group G.

2.13 Theorem: Let a bipolar interval valued fuzzy subgroup [A] = 〈 $[A]^+$, $[A]^-$ 〉 of a group G be conjugate to a bipolar interval valued fuzzy subgroup [M] = 〈 $[M]^+$, $[M]^-$ 〉 of G and a bipolar interval valued fuzzy subgroup [B] = 〈 $[B]^+$, $[B]^-$ 〉 of a group H be conjugate to a bipolar interval valued fuzzy subgroup [N] = 〈 $[N]^+$, $[N]^-$ 〉 of H. Then a bipolar interval valued fuzzy subgroup [A]×[B] = 〈 $([A]\times[B])^+$, $([A]\times[B])^-$ 〉 of a group G×H is conjugate to a bipolar interval valued fuzzy subgroup [M]×[N] = 〈$([M]\times[N])^+$, $([M]\times[N])^-$〉 of G×H.

Proof: Let [A] and [B] be bipolar interval valued fuzzy subgroups of the groups G and H. Let x, x^{-1} and f be in G and y, y^{-1} and g be in H. Then (x, y), (x^{-1}, y^{-1}) and (f, g) are in G×H. Now, $([A]\times[B])^+(f, g) = \text{rmin}\,\{[A]^+(f), [B]^+(g)\} = \text{rmin}\{[M]^+(xfx^{-1}), [N]^+(ygy^{-1})\}$

$=([M]\times[N])^+(xfx^{-1}, ygy^{-1}) = ([M]\times[N])^+[(x,y)(f, g)(x^{-1},y^{-1})] = ([M]\times[N])^+[(x,y)(f,g)(x,y)^{-1}]$.

Therefore $([A]\times[B])^+(f, g) = ([M]\times[N])^+[(x, y)(f, g)(x, y)^{-1}]$. And $([A]\times[B])^-(f, g) = \text{rmax}\,\{[A]^-(f), [B]^-(g)\} = \text{rmax}\{[M]^-(xf\,x^{-1}), [N]^-(yg\,y^{-1})\} = ([M]\times[N])^-(xf\,x^{-1}, yg\,y^{-1}) = ([M]\times[N])^-[(x, y)(f, g)(x^{-1}, y^{-1})] = ([M]\times[N])^-[(x, y)(f, g)(x, y)^{-1}]$. Therefore $([A]\times[B])^-(f, g) = ([M]\times[N])^-[(x, y)(f, g)(x, y)^{-1}]$. Hence a bipolar interval valued fuzzy subgroup [A]×[B] of a group G×H is conjugate to a bipolar interval valued fuzzy subgroup [M]×[N] of G×H.

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