Vol.3.Issue.3.2015 (Jul-Sept)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

ISSN:2348-0580

RESEARCH ARTICLE

A STUDY ON ANTI I-FUZZY SUBBIGROUP OF A BIGROUP

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of anti I-fuzzy (interval valued fuzzy) subbigroup of a bigroup.

2000 AMS Subject classification: 03F55, 06D72, 08A72.

KEY WORDS: Bigroup, fuzzy subset, I-fuzzy subset, fuzzy subbigroup, I-fuzzy subbigroup, anti I-fuzzy subbigroup.

INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh [9], after that several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld [2]. Interval-valued fuzzy sets were introduced independently by Zadeh [10], Grattan-Guiness [3], Jahn [4], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function. Jun.Y.B and Kin.K.H [5] defined an interval valued fuzzy R-subgroups of nearrings. M.G.Somasundra Moorthy & K.Arjunan [7, 8] defined interval valued fuzzy subrings of a ring. In this paper, we introduce the some theorems in anti interval valued fuzzy (denoted as anti I-fuzzy) subbigroup of a bigroup.

1.PRELIRMINARIES:

1.1 Definition: A set $(G, +, \bullet)$ with two binary operations + and $\bullet$ is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that (i) $G = G_1 \cup G_2$ (ii) $(G_1, +)$ is a group (iii) $(G_2, \bullet)$ is a group.

1.2 Definition: Let X be a non–empty set. A **fuzzy subset A** of X is a function $A: X \rightarrow [0, 1]$.

1.3 Definition: Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup. Then a fuzzy set A of G is said to be a fuzzy subbigroup of G if there exist two fuzzy subsets A_1 of G_1 and A_2 of G_2 such that (i) $A = A_1 \cup A_2$ (ii) A_1 is a fuzzy subgroup of $(G_1, +)$ (iii) A_2 is a fuzzy subgroup of $(G_2, \bullet)$.

1.4 Definition: Let X be any nonempty set. A mapping $[M] : X \rightarrow D[0, 1]$ is called an interval valued fuzzy subset (I-fuzzy subset) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq$

$M^{+}(x)$, for all x in X. Thus $M^{-}(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.

1.5 Definition: Let $[M] = \{\langle x, [M^{-}(x), M^{+}(x)] \rangle / x\in X \}$, $[N] = \{\langle x, [N^{-}(x), N^{+}(x)] \rangle / x\in X\}$ be any two interval valued fuzzy subsets of X. We define the following relations and operations:

(i) $[M] \subseteq [N]$ if and only if $M^{-}(x) \leq N^{-}(x)$ and $M^{+}(x) \leq N^{+}(x)$, for all x in X.

(ii) $[M] = [N]$ if and only if $M^{-}(x) = N^{-}(x)$ and $M^{+}(x) = N^{+}(x)$, for all x in X.

(iii) $[M] \cap [N] = \{ \langle x, [\min \{ M^{-}(x), N^{-}(x)\}, \min \{ M^{+}(x), N^{+}(x)\}] \rangle / x\in X \}$.

(iv) $[M] \cup [N] = \{ \langle x, [\max \{ M^{-}(x), N^{-}(x)\}, \max \{ M^{+}(x), N^{+}(x)\}] \rangle / x\in X \}$.

(v) $[M]^{C} = [1, 1] - [M] = \{ \langle x, [1- M^{+}(x), 1- M^{-}(x)] \rangle / x\in X \}$.

1.6 Definition: Let $G = (G_1\cup G_2, + , \bullet)$ be a bigroup. The I-fuzzy subset $[A]: G\to D[0, 1]$ of G is said to be a I-fuzzy subbigroup of G if there exist two I-fuzzy subsets $[A_1]: G_1\to D[0, 1]$ of G_1 and $[A_2]: G_2\to D[0, 1]$ of G_2 such that (i) $[A] = [A_1]\cup[A_2]$

(ii) $[A_1]$ is a I-fuzzy subgroup of $(G_1, +)$

(iii) $[A_2]$ is a I-fuzzy subgroup of $(G_2 , \bullet)$.

1.7 Definition: Let $G = (G_1\cup G_2, + , \bullet)$ be a bigroup. The I-fuzzy subset $[A]: G\to D[0, 1]$ of G is said to be an anti I-fuzzy subbigroup of G if there exist two I-fuzzy subsets

$[A_1]: G_1\to D[0, 1]$ of G_1 and $[A_2]: G_2\to D[0, 1]$ of G_2 such that (i) $[A] = [A_1]\cup[A_2]$

(ii) $[A_1]$ is an anti I-fuzzy subgroup of $(G_1, +)$

(iii) $[A_2]$ is an anti I-fuzzy subgroup of $(G_2 , \bullet)$.

2. PROPERTIES:

2.1 Theorem: If $[A] = [M]\cup[N]$ is an anti I-fuzzy subbigroup of a bigroup $G = E\cup F$, then $\mu_{[M]}(-x) = \mu_{[M]}(x)$, $\mu_{[M]}(x) \geq \mu_{[M]}(e)$, $\mu_{[N]}(x^{-1}) = \mu_{[N]}(x)$, $\mu_{[N]}(x) \geq \mu_{[N]}(e')$ for all x, e in E and x, e' in F.

Proof: Let x, e in E and x, e' in F. Now $\mu_{[M]}(x) = \mu_{[M]}((-(-x))) \leq \mu_{[M]}(-x) \leq \mu_{[M]}(x)$. Therefore $\mu_{[M]}(-x) = \mu_{[M]}(x)$ for all x in E. And $\mu_{[M]}(e) = \mu_{[M]}(x-x) \leq \text{rmax} \{ \mu_{[M]}(x), \mu_{[M]}(x) \}= \mu_{[M]}(x)$. Therefore $\mu_{[M]}(e) \leq \mu_{[M]}(x)$ for all x, e in E. Also $\mu_{[N]}(x) = \mu_{[N]}((x^{-1})^{-1}) \leq \mu_{[N]}(x^{-1}) \leq \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x^{-1}) = \mu_{[N]}(x)$ for all x in F. And $\mu_{[N]}(e') = \mu_{[N]}(xx^{-1}) \leq \text{rmax}\{ \mu_{[N]}(x), \mu_{[N]}(x^{-1}) \}= \mu_{[N]}(x)$. Therefore $\mu_{[N]}(e') \leq \mu_{[N]}(x)$ for all x, e' in F.

2.2 Theorem: If $[A] = [M]\cup[N]$ is an anti I-fuzzy subbigroup of a bigroup $G = E\cup F$, then (i) $\mu_{[M]}(x-y) = \mu_{[M]}(e)$ gives $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x, y and e in E

(ii) $\mu_{[N]}(xy^{-1}) = \mu_{[N]}(e')$ gives $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x, y and e' in F.

Proof: (i) Let x, y and e in E. Then $\mu_{[M]}(x) = \mu_{[M]}(x-y+y) \leq \text{rmax} \{ \mu_{[M]}(x-y), \mu_{[M]}(y) \} = \text{rmax} \{ \mu_{[M]}(e), \mu_{[M]}(y) \}= \mu_{[M]}(y) = \mu_{[M]}(y-x+x) \leq \text{rmax} \{ \mu_{[M]}(y-x), \mu_{[M]}(x) \}= \text{rmax} \{\mu_{[M]}(e), \mu_{[M]}(x) \}= \mu_{[M]}(x)$. Therefore $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x and y in E.

(ii) Let x, y and e' in F. Then $\mu_{[N]}(x) = \mu_{[N]}(xy^{-1}y) \leq \text{rmax} \{ \mu_{[N]}(xy^{-1}), \mu_{[N]}(y) \} = \text{rmax} \{ \mu_{[N]}(e'), \mu_{[N]}(y) \}= \mu_{[N]}(y) = \mu_{[N]}(yx^{-1}x) \leq \text{rmax} \{ \mu_{[N]}(yx^{-1}), \mu_{[N]}(x) \}= \text{rmax} \{ \mu_{[N]}(e'), \mu_{[N]}(x) \}= \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x and y in F.

2.3 Theorem: If $[A] = [M]\cup[N]$ is an anti I-fuzzy subbigroup of a bigroup $G = E\cup F$,

then (i) $H_1= \{ x / x\in E \text{ and } \mu_{[M]}(x) = [0, 0] \}$ is either empty or a subgroup of E.

(ii) $H_2 = \{ x / x\in F \text{ and } \mu_{[N]}(x) = [0, 0] \}$ is either empty or a subgroup of F.

(iii) $K = H_1 \cup H_2$ is either empty or a subbigroup of G.

Proof: If no element satisfies this condition, then H_1 and H_2 are empty. Also $K = H_1\cup H_2$ is empty. (i) If x and y in H_1, then $\mu_{[M]}(x-y) \leq \text{rmax} \{ \mu_{[M]}(x), \mu_{[M]}(y) \}\leq \text{rmax} \{ [0, 0], [0, 0] \} = [0, 0]$. Therefore $\mu_{[M]}(x-y) = [0, 0]$. We get $x-y$ in H_1. Hence H_1 is a subgroup of G_1. (ii) If x and y in H_2, then $\mu_{[N]}(xy^{-1}) \leq$ rmax

{ $\mu_{[N]}(x)$, $\mu_{[N]}(y)$} = rmax {[0, 0], [0, 0]} = [0, 0]. Therefore $\mu_{[N]}(xy^{-1})$ = [0, 0]. We get xy^{-1} in H_2. Hence H_2 is a subgroup of G_2. (iii) From (i) and (ii) we get K = $H_1 \cup H_2$ is a subbigroup of G.

2.4 Theorem: If [A] = [M]$\cup$[N] is an anti I-fuzzy subbigroup of a bigroup G = E$\cup$F, then (i) H_1= { x / x$\in$E and $\mu_{[M]}(x) = \mu_{[M]}(e)$ } is a subgroup of E

(ii) H_2 = { x / x$\in$F and $\mu_{[N]}(x) = \mu_{[N]}(e')$ } is a subgroup of F

(iii) K = $H_1 \cup H_2$ is a subbigroup of G.

Proof: (i) Clearly e in H_1 so H_1 is a non empty. Let x and y be in H_1. Then $\mu_{[M]}(x–y) \le$ rmax { $\mu_{[M]}(x)$, $\mu_{[M]}(y)$ }= rmax { $\mu_{[M]}(e)$, $\mu_{[M]}(e)$ }= $\mu_{[M]}(e)$. Therefore $\mu_{[M]}(x–y) \le \mu_{[M]}(e)$ for all x and y in H_1. We get $\mu_{[M]}(x–y) = \mu_{[M]}(e)$ for all x and y in H_1. Therefore x–y in H_1. Hence H_1 is a subgroup of E.

(ii) Clearly e' in H_2 so H_2 is a non empty. Let x and y be in H_2. Then $\mu_{[N]}(xy^{-1}) \le$ rmax { $\mu_{[N]}(x)$, $\mu_{[N]}(y)$ }= rmax {$\mu_{[N]}(e')$, $\mu_{[N]}(e')$ } = $\mu_{[N]}(e')$. Therefore $\mu_{[N]}(xy^{-1}) \le \mu_{[N]}(e')$ for all x and y in H_2. We get $\mu_{[N]}(xy^{-1}) = \mu_{[N]}(e')$ for all x and y in H_2. Therefore xy^{-1} in H_2. Hence H_2 is a subgroup of F.

(iii) From (i) and (ii) we get K = $H_1 \cup H_2$ is a subbigroup of G.

2.5 Theorem: Let [A]= [M]$\cup$[N] be an anti I-fuzzy subbigroup of a bigroup G = E$\cup$F.

(i) If $\mu_{[M]}(x–y)$ = [0, 0], then $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x and y in E.

(ii) If $\mu_{[N]}(xy^{-1})$ = [0, 0], then $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x and y in F.

Proof: (i) Let x and y belongs to E. Then $\mu_{[M]}(x) = \mu_{[M]}(x–y+y) \le$ rmax {$\mu_{[M]}(x–y)$, $\mu_{[M]}(y)$ } = rmax { [0, 0], $\mu_{[M]}(y)$ } = $\mu_{[M]}(y) = \mu_{[M]}(–y) = \mu_{[M]}(–x+x–y) \le$ rmax { $\mu_{[M]}(–x)$, $\mu_{[M]}(x–y)$ } = rmax { $\mu_{[M]}(–x)$, [0, 0] } = $\mu_{[M]}(–x) = \mu_{[M]}(x)$. Therefore $\mu_{[M]}(x) = \mu_{[M]}(y)$ for all x and y in E.

(ii) Let x and y belongs to F. Then $\mu_{[N]}(x) = \mu_{[N]}(xy^{-1}y) \le$ rmax {$\mu_{[N]}(xy^{-1})$, $\mu_{[N]}(y)$ } = rmax{ [0, 0], $\mu_{[N]}(y)$ } = $\mu_{[N]}(y) = \mu_{[N]}(y^{-1}) = \mu_{[N]}(x^{-1}xy^{-1}) \le$ rmax {$\mu_{[N]}(x^{-1})$, $\mu_{[N]}(xy^{-1})$} = rmax { $\mu_{[N]}(x^{-1})$, [0, 0]} = $\mu_{[N]}(x^{-1}) = \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x) = \mu_{[N]}(y)$ for all x and y in F.

2.6 Theorem: If [A] = [M]$\cup$[N] is an anti I-fuzzy subbigroup of a bigroup G = E$\cup$F, then (i) $\mu_{[M]}(x+y)$ = rmax{ $\mu_{[M]}(x)$, $\mu_{[M]}(y)$ } for each x and y in E with $\mu_{[M]}(x) \neq \mu_{[M]}(y)$

(ii) $\mu_{[N]}(xy)$ = rmax{ $\mu_{[N]}(x)$, $\mu_{[N]}(y)$ } for each x and y in F with $\mu_{[N](}x) \neq \mu_{[N](}y)$.

Proof: (i) Let x and y belongs to E. Assume that $\mu_{[M]}(x) < \mu_{[M]}(y)$, then $\mu_{[M]}(y) = \mu_{[M]}(–x+x+y) \le$ rmax{ $\mu_{[M]}(–x)$, $\mu_{[M]}(x+y)$ } $\le$ rmax{ $\mu_{[M]}(x)$, $\mu_{[M]}(x+y)$} = $\mu_{[M]}(x+y) \le$ rmax{ $\mu_{[M]}(x)$, $\mu_{[M]}(y)$}= $\mu_{[M]}(y)$. Therefore $\mu_{[M]}(x+y) = \mu_{[M]}(y)$ = rmax{ $\mu_{[M]}(x)$, $\mu_{[M]}(y)$ } for x and y in E.

(ii) Let x and y belongs to F. Assume that $\mu_{[N]}(x) < \mu_{[N]}(y)$, then $\mu_{[N]}(y) = \mu_{[N]}(x^{-1}xy) \le$ rmax{ $\mu_{[N]}(x^{-1})$, $\mu_{[N]}(xy)$ } $\le$ rmax{ $\mu_{[N]}(x)$, $\mu_{[N]}(xy)$ } = $\mu_{[N]}(xy) \le$ rmax {$\mu_{[N]}(x)$, $\mu_{[N]}(y)$ }= $\mu_{[N\,]\,=}\,y)$.

Therefore $\mu_{[N]}(xy) = \mu_{[N]}(y)$ = rmax{ $\mu_{[N]}(x)$, $\mu_{[N]}(y)$ } for x and y in F.

2.7 Theorem: If [A] = [M]$\cup$[N] and [B] = [O]$\cup$[P] are two anti I-fuzzy subbigroups of a bigroup G = E$\cup$F, then their union [A]$\cup$[B] is an anti I-fuzzy subbigroup of G.

Proof: Let [A] = [M]$\cup$[N] = { $\langle$ x, $\mu_{[A]}(x)$ $\rangle$ / x$\in$G } where [M] = { $\langle$ x, $\mu_{[M]}(x)$ $\rangle$ / x$\in$E } and [N] = { $\langle$ x, $\mu_{[N]}(x)$ $\rangle$ / x$\in$F } and [B] = [O]$\cup$[P] = { $\langle$ x, $\mu_{[B]}(x)$ $\rangle$ / x$\in$G } where [O] = { $\langle$ x, $\mu_{[O]}(x)$ $\rangle$ / x$\in$E } and [P] = { $\langle$ x, $\mu_{[P]}(x)$ $\rangle$ / x$\in$F }. Let [C] = [A]$\cup$[B] = [R]$\cup$[S] where [C] = { $\langle$ x, $\mu_{C]}(x)$ $\rangle$ / x$\in$G }, [R] = [M]$\cup$ [O] = { $\langle$ x, $\mu_{R]}(x)$ $\rangle$ / x$\in$E } and [S] = [N]$\cup$[P] = { $\langle$ x, $\mu_{[S]}(x)$ $\rangle$ / x$\in$F }. Let x and y belong to E. Then $\mu_{[R]}(x–y)$ = rmax{$\mu_{[M]}(x–y)$, $\mu_{[O]}(x–y)$ }$\le$ rmax{rmax{ $\mu_{[M]}(x)$, $\mu_{[M]}(y)$ }, rmax { $\mu_{[O]}(x)$, $\mu_{[O]}(y)$ }} $\le$ rmaxn { rmax { $\mu_{[M]}(x)$, $\mu_{[O]}(x)$ }, rmax { $\mu_{[M]}(y)$, $\mu_{[O]}(y)$ }} = rmax { $\mu_{[R]}(x)$, $\mu_{[R]}(y)$ }. Therefore $\mu_{[R]}(x–y) \le$ rmax{ $\mu_{[R]}(x)$, $\mu_{[R]}(y)$ } for all x and y in E. Let x and y belong to F. Then $\mu_{[S]}(xy^{-1})$ = rmax{ $\mu_{[N]}(xy^{-1})$, $\mu_{[P]}(xy^{-1})$ }$\ge$ rmax{rmax{$\mu_{[N]}(x)$, $\mu_{[N]}(y)$}, rmax{$\mu_{[P]}(x)$, $\mu_{[P]}(y)$}}$\le$ rmax{ rmax{ $\mu_{[N]}(x)$, $\mu_{[P]}(x)$}, rmax{ $\mu_{[N]}(y)$, $\mu_{[P]}(y)$ }} = rmax $\mu_{[S]}(x)$, $\mu_{[S]}(y)$}. Therefore $\mu_{[S]}(xy^{-1}) \le$ rmax{ $\mu_{[S]}(x)$, $\mu_{[S]}(y)$ } for all x and y in F. Hence [A]$\cup$[B] is an anti I-fuzzy subbigroup of G.

2.8 Theorem: The union of a family of anti I-fuzzy subbigroups of a bigroup G is an anti

I-fuzzy subbigroup of G.

Proof: It is trivial.

2.9 Theorem: If [A] = [M]$\cup$[N] is an anti I-fuzzy subbigroup of a bigroup G = E$\cup$F, then (i) $\mu_{[M]}(x+y) = \mu_{[M]}(y+x)$ if and only if $\mu_{[M]}(x) = \mu_{[M]}(-y+x+y)$ for all x and y in E

(ii) $\mu_{[N]}(xy) = \mu_{[N]}(yx)$ if and only if $\mu_{[N]}(x) = \mu_{[N]}(y^{-1}xy)$ for all x and y in F.

Proof: (i) Let x and y be in E. Assume that $\mu_{[M]}(x+y) = \mu_{[M]}(y+x)$, then $\mu_{[M]}(-y+x+y) = \mu_{[M]}(-y+y+x) = \mu_{[M]}(e_1+x) = \mu_{[M]}(x)$. Therefore $\mu_{[M]}(x) = \mu_{[M]}(-y+x+y)$ for all x and y in E. Conversely, assume that $\mu_{[M]}(x) = \mu_{[M]}(-y+x+y)$, then $\mu_{[M]}(x+y) = \mu_{[M]}(x+y-x+x) = \mu_{[M]}(y+x)$. Therefore $\mu_{[M]}(x+y) = \mu_{[M]}(y+x)$ for all x and y in E.

(ii) Let x and y be in F. Assume that $\mu_{[N]}(x+y) = \mu_{[N]}(y+x)$, then $\mu_{[N]}(y^{-1}xy) = \mu_{[N]}(y^{-1}yx) = \mu_{[N]}(e_2x) = \mu_{[N]}(x)$. Therefore $\mu_{[N]}(x) = \mu_{[N]}(y^{-1}xy)$ for all x and y in F. Conversely, assume that $\mu_{[N]}(x) = \mu_{[N]}(y^{-1}xy)$, then $\mu_{[N]}(xy) = \mu_{[N]}(xyxx^{-1}) = \mu_{[N]}(yx)$. Therefore $\mu_{[N]}(xy) = \mu_{[N]}(yx)$ for all x and y in F.

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