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AMALGAMATION OF ODD HARMONIOUS GRAPHS WITH STAR GRAPHS

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ABSTRACT

A graph G (V, E) with ‘n’ vertices and ‘m’ edges is said to be odd harmonious graph if f is an injection from the vertices of G to the integers from 0 to (2q – 1) such that the induced mapping f* from the edges of G to{ 1,3,5, -------(2q - 1) } defined by f* (uv) = f (u) + f (v) is bijective.

Key words: path, ternary tree, complete graph, cycle graph, comb graph.

1. INTRODUCTION

Graph labeling were first introduced in 1960’s.Graph labeling serves as a frontier between number theory and structure of graphs. Labeling of graphs have various applications in network of transmitting station, X-ray, radar etc. Graham and Sloane introduced harmonious labeling in 1980.To begin with simple graph with ‘p’ vertices and ‘q’ edges. The definitions and other information which are used for the present investigation are given.

2. DEFINITIONS

Definition 2.1: Odd harmonious graph: A function f is said to be an odd harmonious labeling of graph G with q edges if f is an injection from the vertices of G to the integers of 0 to (2q – 1) such that the induced mapping f*(uv) = f (u) + f (v) from the edges of G to the odd integers between 1 to (2q-1) is a bijection.

Characteristics of labeling:

- The vertex labeling of G must be chosen from integer set 0 to (2q – 1) where ‘q’ is the number of edges in G.
- Label the vertices in randomly / clockwise / anticlockwise direction.
- It should satisfy f* (uv) = f (u) + f (v)

Definition 2.2: Ternary tree: A tree in which each node has out degree ≤ 3 is called ternary tree.

Definition 2.3: Comb graph: The graph obtained by joining a pendent edge at each vertex of path P_n is called a comb graph.

3. RESULTS:

Theorem 3.1: G is a path graph of order n and size m. The amalgamation of G with Star graph S_n of order 'n' and size (n-1) is odd harmonious graph.

Proof: G_{amal} = { V, E, f* } is a graph. Let the vertex set of G_{amal} (V) = { v_1, v_2, v_3,v_4 , v_5 -----v_n} and edge set of G_{amal} (E) = { e_1, e_2, e_3, e_4, e_5,-------e_n }.Attach central vertex of S_n to the any vertex of P_n and label the vertices of P_n and S_n by choosing numbers from 0 to 2q -1. Such that $G_{(amal)}$ = V x V → E is bijective and is defined as f*(uv) = f (u) + f (v) u ,v ϵ G_{amal} (V) and edge labels{ 1,3,5, ---- (2q-1) } are distinct .Hence G is odd harmonious graph.

Example:

The graph in (i) of Fig (1) is $G_{(amal)}$ of P_n with S_2 which is of order 6 and size 5.So G_{amal} (V)= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9 } then f* (uv) = f (u) + f (v) such that edge labels are { 1, 3, 5, 7, 9 } which admits odd harmonious labeling. Similarly, G_{amal} (P_3 ,S_3) in (ii) of Fig(1) is also odd harmonious graph. The graph on (iii) of Fig (1) is general case of the above graphs.

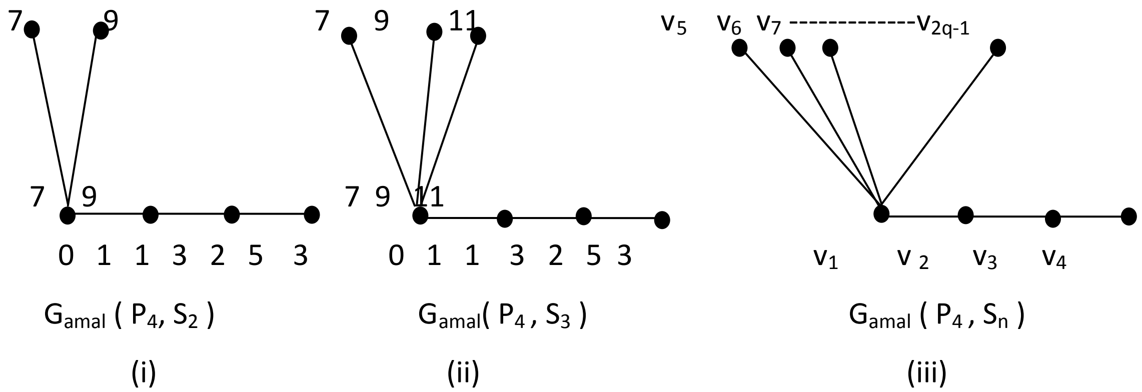


Fig (1)

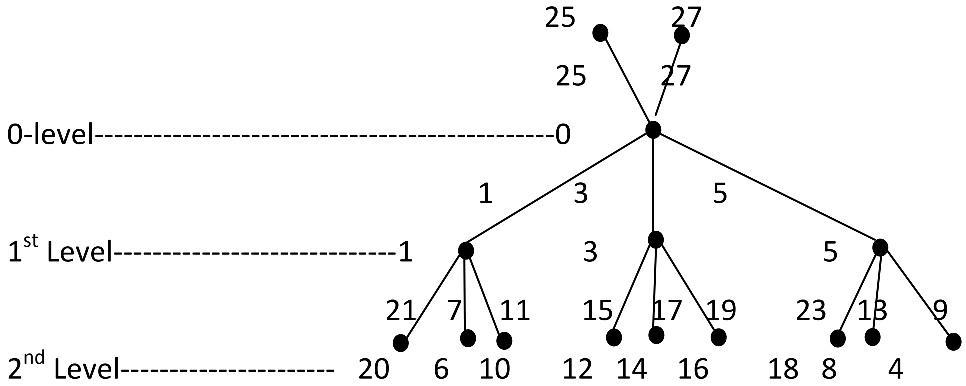
Note:

- If vertex labeling order is continued as { 0, 1, 2, 3, ------- (2q-1) }in clockwise / anticlockwise direction for the graph (G_{amal}) then we get the repeated edge labeling.
- Attach central vertex of S_n to the any vertex of P_n and label the vertices of P_n and S_n by choosing numbers from 0 to 2q -1

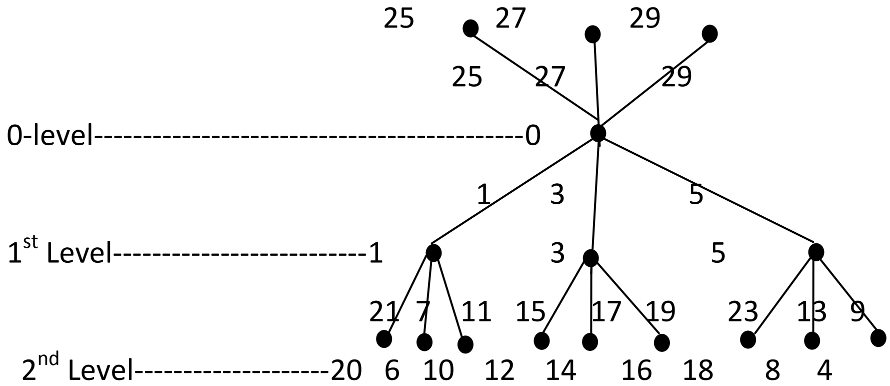
Theorem 3.2: G is a ternary graph of order n and size m .The amalgamation of G with star graph S_n of order 'n' and size (n-1) is odd harmonious graphs.

Proof: G_{amal} = { V, E, f* } is a graph. Let the vertex set of G_{amal} (V) = { v_1, v_2, v_3,v_4 v_5 -----v_n} and edge set G_{amal} (E) = { e_1, e_2, e_3, e_4,e_5,--------e_n }. Attach central vertex of S_n to the root vertex of a ternary tree and label that vertex as zero (0). Label the remaining vertices of a ternary tree by level wise in any direction. Such that $G_{(amal)}$ = V x V → E is bijective and is defined by. f* (uv) = f (u) + f (v) u ,v ϵ G_{amal} (V) and edge labels { 1, 3, 5, ------(2q-1) } are distinct .Hence G is odd harmonious graph.

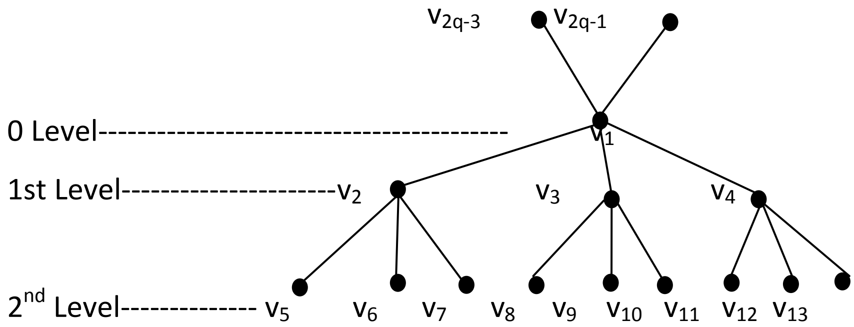
Example: The graph in (i) of Fig (2) is $G_{(amal)}$ of ternary tree (T) with S_2 which is of order 15 and size 14. S G_{amal} (V) = {0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16, 17,18,19,20,21, 22,23, 24, 25, 26, 27 } then f* (uv) = f (u) + f (v) such that edge labels are { 1,3,5,7,9,11,13, 15,17,19,21 ,23,25 ,27 } which admits odd harmonious labeling. Similarly, $G_{(amal)}$ (T, S_3) in (ii) of Fig (2) is also odd harmonious graph. The graph on (iii) of Fig (2) is general case of the above graphs.



(i) G_{amal} (T, S_2)



(ii) G_{amal}(T , S_3)



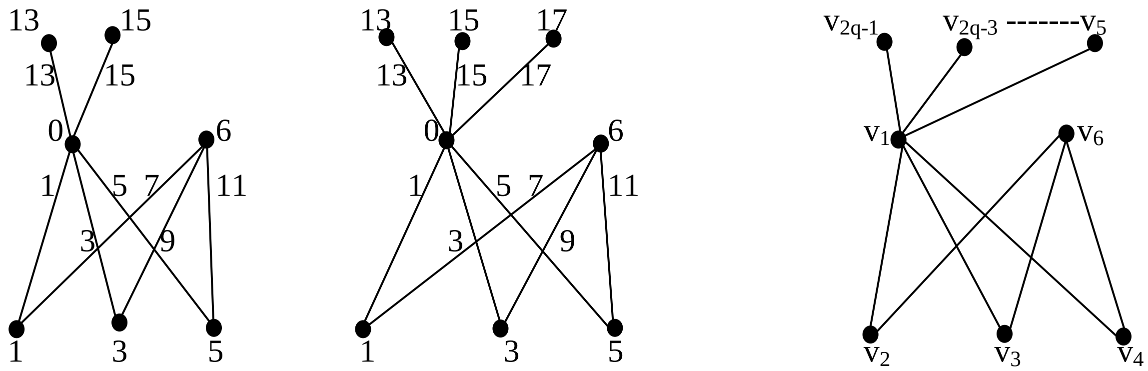
(iii) G_{amal} (T, S_n)

Fig (2)

Theorem 3.3: G is a $K_{n,m}$ graph , (for all values of n, m)the amalgamation of G with star graph S_n of order 'n' and size (n -1) is odd harmonious graphs.

Proof: G_{amal} = { V, E, f* } is a graph. Let the vertex set of G_{amal} (V) = { v_1, v_2, v_3,v_4,v_5 -----v_n} and edge set G_{amal} (E) = { e_1, e_2, e_3, e_4,e_5,--------e_n }. Attach central vertex of a S_n to the top of a right or left side of the $K_{n,m}$ graph and label it as zero (0),label the odd integers to the bottom vertices, such that $G_{(amal)}$ = V x V → E is bijective defined as f* (uv) = f (u) + f (v) u, v є G_{amal} (V) and edge labels { 1,3,5,---- (2q-1) } are distinct. Hence G is odd harmonious graph.

Example: The graph in (i) of Fig (3) is G_{amal} of $K_{n,m}$ with S_2 which is of order 7 and size 8. So G_{amal} (V) = {0, 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 } then f* (uv) = f (u) + f (v) such that edge labels are { 1, 3, 5,7,9,11,13,15, } which admits odd harmonious labeling similarly, $G_{(amal)}$ ($K_{2,3}$, S_3) in (ii) of Fig(3) is also odd harmonious graph. The graph on (iii) of Fig (3) is general case of the above graphs.



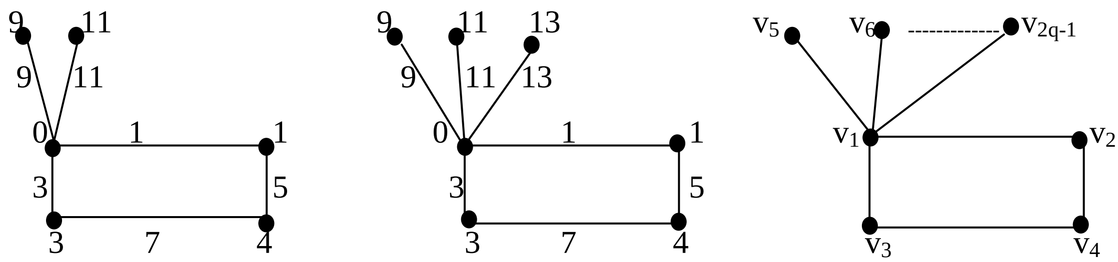
(i) $G_{(amal)}$ ($K_{2,3}$,S_2) (ii) $G_{(amal)}$ ($K_{2,3}$,S_3) (iii) $G_{(amal)}$ ($K_{2,3}$,S_n)

Fig (3)

Theorem 3.4: G is a Cycle graph (C_n) where n ≡ 0 (mod 4) which is of order n and size m. The amalgamation of G with Star graph S_n of order n and size (n-1) is odd harmonious graph.

Proof: G_{amal} = { V, E, f* } is a graph. Let the vertex set of G_{amal} (V) = { v_1, v_2, v_3,v_4,v_5 -----v_n} and edge set G_{amal} (E) = { e_1, e_2, e_3, e_4,e_5,--------e_n }. Attach central vertex of S_n to the any vertex of a C_n label that vertex as zero (0) ,and continuing the labeling to the remaining vertices in any direction, such that $G_{(amal)}$ = V x V → E is bijective defined as f* (uv) = f (u) + f (v) u, v є G_{amal} (V) and edge labels { 1, 3, 5, ---- (2q-1) } are distinct. Hence G is odd harmonious graph.

Example: The graph in (i) of Fig (4) is G_{amal} of C_4 with S_2 which is of order 6 and size 6.So G_{amal} (V) = {0, 1,2,3,4,5,6,7,8,9,10,11 } then f* (uv) = f (u) + f (v) such that edge labels are {1, 3, 5, 7, 9, 11 } which admits odd harmonious labeling similarly, G_{amal} (C_4 , S_2) in (ii) of Fig (4) is also odd harmonious graph. The graph on (iii) of Fig (4) is general case of the above graphs.



G_{amal} (C_4 , S_2) (i) G_{amal} (C_4, S_3) (ii) G_{amal} (C_4, S_n) (iii)

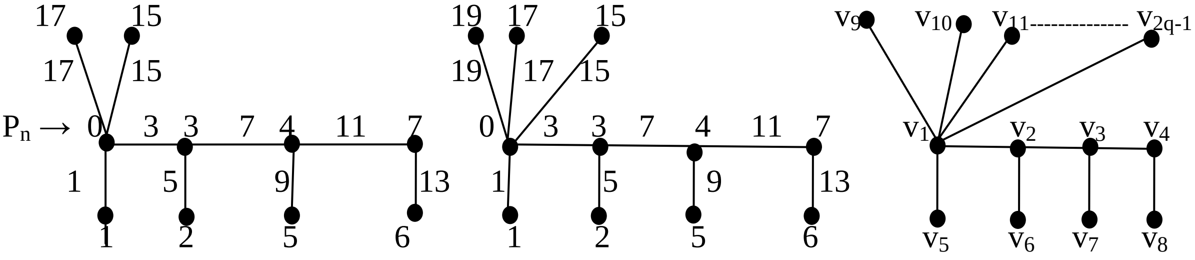
Fig (4)

Theorem 3.5: G is a Comb graph (P_n● K_1) which is of order n and size m. The amalgamation of G with Star graph S_n of order n and size (n-1) is odd harmonious graph.

Proof: G_{amal} = { V, E, f* } is a graph. Let the vertex set of G_{amal} (V) = { v_1, v_2, v_3,v_4 v_5 -----v_n} and edge set G_{amal} (E) = { e_1, e_2, e_3, e_4 ,e_5,--------e_n } Attach central vertex of S_n to any vertex of a P_n and label that vertex as zero(0) . Label the remaining vertices of a graph G_{amal} , such that $G_{(amal)}$ = V x V → E is bijective defined as f* (uv) = f (u) + f (v) u, v є G_{amal} (V) and edge labels { 1, 3, 5, ---- (2q-1) } are distinct .Hence G is odd harmonious graph.

Example: The graph in (i) of Fig (5) is G_{amal} of (P_n● K_1) with S_2 which is of order 10 and size 9.So G_{amal} (V) = { 0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17 } then f* (uv) = f (u) + f (v) such that edge labels are { 1,3,5,7,9,11,13,15,17 } which admits odd harmonious labeling similarly, G_{amal} [(P_n ● K_1)

,S_2] in (ii) of Fig (5) is also odd harmonious graph. The graph on (iii) of Fig (5) is general case of the above graphs.



G_{amal} [($P_4 \bullet K_1$) , S_2] (i) G_{amal} [($P_4 \bullet K_1$) , S_3] (ii) G_{amal} [($P_4 \bullet K_1$), S_n] (iii)

Fig (5)

Conclusion: In this paper we have observed that odd harmonious graphs most of the graph obtained by amalgamation are odd harmonious in future the same process will be analyzed for some graphs.

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