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RESEARCH ARTICLE

CRITERION FOR A PRIME BELOW A GIVEN INTEGER TO BE RELATIVELY PRIME NUMBER

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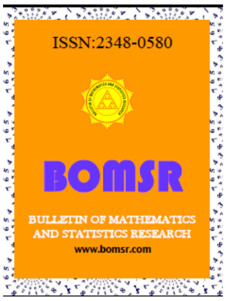

ABSTRACT

When N is a composite integer then N has prime factors less than or equal to $\sqrt{N}$. The prime factors of N that are less than or equal to $\sqrt{N}$ are not relatively prime to N. Also some prime numbers less than or equal to $\frac{N}{2}$ are also not relatively prime to N. In this paper we proved a criterion under which a prime number less than N becomes relatively prime to N.

Keywords: condition, criterion for which a prime number of a composite number is relatively prime.

INTRODUCTION

Let us consider a composite number 28. The prime factors 2,7 of 28 are not relatively prime to 28. The prime numbers greater than 7 and less than 28 are 11,13,17,19 and 23. We find these prime numbers to be relatively prime to 28. Thus, if a composite number $N \neq 2q, 3q$ where q is a prime number, then the prime numbers greater than $\frac{N}{4}$ are relatively prime to N is discussed in this paper.

Similarly, when $N = 3q$ where q is an odd prime or $N = 2q$ where q is a prime number then the primes greater than $\frac{N}{3}$ or $\frac{N}{2}$ are relatively prime to N.

Theorem 1. Let $N \neq 2q, 3q$ where q is a prime number, be a composite integer. The prime numbers greater than $\frac{N}{4}$ are relatively prime to N.

Proof: Let $N \neq 2q, 3q$ where q is a prime number, be a composite integer and $\frac{N}{4} < p < N$ be a prime number.

We prove that $gcd(p, N) = 1$.

Let $gcd(p, N) = d$.

Then $d|p$ and $d|N$.

Since p is a prime number and hence $d = 1$ or $d = p$.

If $d = 1$ then the theorem is proved.
If $d = p$ then $gcd(p, N) = p$.
$p|N$ implies $N = kp$ for some integer k, $1 < k < N$ and $\frac{\boldsymbol{N}}{\boldsymbol{4}} < p < N$.
$k = \frac{N}{p} < 4$ (since $\frac{\boldsymbol{N}}{\boldsymbol{4}} < p$ implies $\frac{\boldsymbol{N}}{\boldsymbol{p}} < 4$).
Therefore, $k < 4$ implies $k = 1,2,3$.
If $k = 1$ then $N = p$ is a prime.
This contradicts our hypothesis as N is a composite number.
Therefore, $k \neq 1$
When $k = 2$ then $N = 2p$.
This contradicts our hypothesis as $N \neq 2p$, where p is a prime number .
Therefore, $k \neq 2$
When $k = 3$ then $N = 3p$.
This contradicts our hypothesis as $N \neq 3p$, where p is a prime number .
Therefore $k \neq 3$
Thus we proved $gcd(p, N) = 1$.
Hence all prime numbers greater than $\frac{N}{4}$ and less than N are relatively prime to N.

Theorem 2. Let $N = 2q$ where q is a prime number, be a composite integer. The prime numbers greater than $\frac{N}{2}$ are relatively prime to N.

Proof. There is nothing to prove here because we know that all primes greater than $\frac{N}{2}$ are relatively prime to N.

Theorem 3. Let $N = 3q$ where q is an odd prime number, be a composite integer. The prime numbers greater than $\frac{N}{3}$ are relatively prime to N.

Proof. The proof is same as Theorem 1.

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Reference

Elementary Number Theory, David M. Burton, University of New Hampshire.