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RESEARCH ARTICLE

NUMERICAL SIMULATION OF BOUNDARY LAYER FLOW OF A NANOFLUID PAST AN EXPONENTIAL STRETCHING SHEET

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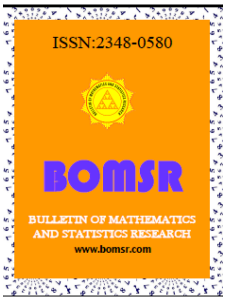

ABSTRACT

This paper considers the problem of steady laminar two dimensional boundary layer flow and heat transfer of nanofluids past an exponential stretching sheet. The governing boundary value problem, consisting of equations of motion and heat transfer for a nanofluid flow past an exponential stretching sheet, with their respective boundary conditions are considered for investigation. This boundary value problem consisting of nonlinear partial differential equations, are transformed into nonlinear ordinary differential equations using suitable similarity transformation and are solved numerically solved by using fourth order Runge-Kutta method, along with shooting technique. The solution mainly depends on Prandtl number Pr, Lewis number Le, Brownian motion parameter Nb and thermophoresis parameter Nt .The variation of the local Nusselt number and local Sherwood number with Nb and Nt for various values of Pr and Le is presented in tabular and graphical forms. It is found that the local Nusselt number is a decreasing function, while the local Sherwood number is an increasing function for each of the dimensionless parameters Pr,Le,Nb and Nt well thought-out.

Keywords:-Nanofluid, Stretching sheet, Brownian motion, Thermophoresis, Heat transfer, Similarity solution, boundary layer flow, Exponential stretching sheet.

Nomenclature

b,c	constant
c_f	skin friction coefficient
u_w	is the velocity of the stretching sheet
C_w	is nanoparticles volume fraction at the stretching surface
C_∞	ambient nanoparticles volume fraction
D_B	brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
$f(\eta)$	dimensionless stream function
κ	thermal conductivity
Pr	prandtl number
Le	lewis number
Nb	brownian motion parameter
Nt	thermophoresis parameter

Nu_x local Nusselt number
Sh_x local Sherwood number
Re_x local Reynolds number
T_W uniform temperature at the surface of the sheet
T_∞ ambient temperature or is the temperature far away from the sheet
T temperature of the fluid inside the boundary layer u,v velocity component along x and y-direction
p is the fluid pressure

Greek symbols

η dimensionless similarity variable
μ dynamic viscosity of the fluid
υ kinematic viscosity of the fluid
ϕ dimensionless concentration function
ρ_f density of the fluid
$(a\rho)_f$ heat capacity of the fluid
$(a\rho)_p$ effective heat capacity of a nano fluid
ψ stream function
α thermal diffusivity
θ dimensionless temperature
τ parameter defined by $(a\rho)_p/(a\rho)_f$

Subscripts

∞ condition at the free stream
w condition of the surface

1.INTRODUCTION

The theoretical studies of fluids flowing over a continuous stretching sheet has drawn attention of engineers and scientists for the last few decades due to the fact that this class of fluids corresponds to several industrially important fluids. Various applications of these fluids are present in the industry including cooling and drying of papers and textiles, extrusion of polymer fluids, artificial and natural gels, cooling of infinite metallic plate in cooling bath and spinning of fibers etc.

The heat transfer rate from the sheet into the fluid is very important, as in in such applications, it induces a direct impact on the quality of the products. However, the common conventional heat transfer fluids such as water,ethylene glycol, and engine oil have limited heat transfer capabilities owing to their low thermal conductivity, whereas metals have much higher thermal conductivities than these fluids. Therefore,dispersing high thermal conductive solid particles in a conventional heat transfer fluid may enhance the thermal conductivityof the resulting fluid.

After the pioneering work of Sakiadis [1], a huge amount of literature is accessible on boundary layer flow of Newtonian and non-Newtonian fluids over stretching surface [2-10].However, only a limited attention has been paid to the study of exponentially stretching surface .Further it is noticed that Magyari and Keller [11] were the first, who considered the boundary layer flow and heat transfer over an exponentially stretching sheet and also investigated the heat transfer aspects of the flow with varying wall temperature. Bidin and Nazar [12] , Ishak [13] and Nadeem et al.[14-,15] numerically examined the flow and heat transfer over an exponentially stretching surface with thermal radiation. Elbashbeshy[16] numerically examined the flow and heat transfer over an exponentially stretching surface considering wall mass suction. Sanjayanand and Khan[17] studied the viscous-elastic boundary layer flow and heat transfer due to and exponentially stretching sheet. Partha et al. [18] obtained similarity solution for mixed convection flow past an exponentially stretching surface by captivating into account the influence of viscous dissipation on the convective transport. Al-odat et al.[19] discussed the effect of magnetic field on thermal boundary layer flow on an exponentially stretching surface with an exponential temperature distribution. Sajid and Hayat [20] showed the influence of thermal radiation on the boundary layer flow and heat transfer of an incompressible viscous fluid due to an exponentially stretching sheet, and they reported series solutions for velocity and temperature using HAM.

Recently, Anuar Ishak[13] studied the MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. He solved it numerically by an implicit finite-difference method. V. Singh, Shweta Agarwal[21] explained the effects of heat transfer for two types of viscoelastic fluids over and exponentially stretching sheet with thermal conductivity and radiation in porous medium. He solved it by well known fourth order Runge-Kutta method with shooting technique. Krishnendu Bhattacharyya[22] presented the effect of steady boundary layer flow and reactive mass transfer past an exponentially stretching surface in an exponentially moving free stream. R.N.Jat and Gopi Chand [23] worked on MHD flow and heat transfer over an exponentially stretching sheet with viscous dissipation and radiation effects. Bikash Sahoo[24] analyzed the effects of flow and heat transfer of a third grade fluid past an exponentially sheet with partial slip boundary condition. Sohail Nadeen and Changhoon Lee[25] has discussed the boundary layer flow of a nanofluid over and exponentially stretching surface. He solved it analytically by HAM method. It is well known that Choi [30] was the first to introduce the term'nanofluid' that represents the fluid in which nano-scale particles are suspended in the base fluid with low thermal conductivity suchas water, ethylene glycol, oil etc. The use of chemical addition is a technique applied to enhance the heat transfer performance of base fluids.Nanofluids have been revealed to increase the thermal conductivity and convective heat transfer performance of base liquids [31]. There are numerous biomedical applications that involve nanofluids such as magnetic cell separation, drug delivery, hyperthermia and contrast enhancement in magnetic resonance imaging.

Thus motivated by the above mentioned investigations and applications of exponential stretching sheet, we felt appropriate to discuss steady laminar two dimensional boundary layer flow and heat transfer of nanofluids past an exponential stretching sheet.

2. Mathematical formulation.

We consider a steady, incompressible, laminar, two dimensional boundary layer flow of a viscous nanofluid past a flat sheet coinciding with the plane y=0 and the flow being confined to y>0 .The flow is generated due to stretching of the sheet caused by the simultaneous application of two equal and opposite force along the x-axis. Keeping the origin fixed, the sheet is then stretched with a velocity $u = u_w(x) = U_0 \exp(x/L)$,where U_0 is the reference velocity, L is the reference length and x is the coordinate measured along the stretching surface varying exponentially with the distance from the slit as shown in Fig 1.

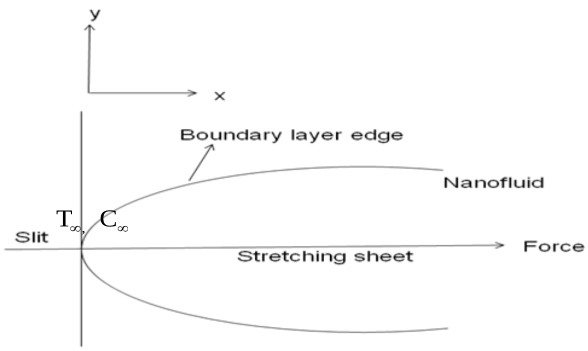

Fig 1. Physical model and coordinate system $u_w(x) = U_0 \exp(x/L)$

It is assumed that at the stretching surface, the temperature T and the nanoparticles fraction C take constant values T_w and C_w respectively. When y attends infinity, the ambient values of temperature T and nanoparticles fraction C are denoted by T_∞ and C_∞ respectively. The fluid is a water based nanofluid containing three types nanoparticles Cu, Al2O3 and TiO2. It is further assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium.

The basic steady conservation of mass, momentum, thermal energy and nanoparticles equations for nanofluids can be printed in Cartesian co-ordinates x and y as, see Kuznetsov and Nield

[26-27].The flow and heat transfer characteristics under the boundary layer approximations are governed by the following equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= \upsilon\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}= \alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left\{D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial y}\right)^2\right]\right\} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The boundary conditions are

$$v=0,\quad u=u_w(x),\quad T=T_w,\quad C=C_w,\quad \text{at} \quad y=0 \tag{5}$$

$$u=v=0,\quad T=T_\infty,\quad C=C_\infty \quad \text{as} \quad y\to\infty \tag{6}$$

Where u and v are the velocity components along x and y axis respectively, $\upsilon=\mu/\rho_f$ is the kinematic viscosity, $\alpha=\kappa/(c\rho)_f$ is the thermal diffusivity, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient and $\tau=(\rho a)_p/(\rho a)_f$ is the ratio between the effective heat capacity of the nanoparticles material and heat capacity of the nano fluid. T is the temperature inside the boundary layer, T_∞ is the temperature far away from the sheet. $u_w(x)=U_o\exp(x/L)$ is the stretching velocity of the sheet, $T_w=T_\infty+b\exp(x/2L)$ is the temperature of stretching surface and $C_w=C_\infty+c\exp(x/2L)$ is nanoparticles volume fraction at the stretching surface.

We are interested in similarity solution of the above boundary value problem therefore we introduce the following similarity transformations (dimensionless quantities).

$$\eta=y\sqrt{\frac{U_0}{2\nu L}}\exp(x/2L),\ \psi=\sqrt{2\nu LU_0}\exp(x/2L)f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

$$u=U_0\exp(x/L)f'(\eta),\quad v=-\sqrt{\frac{\nu U_0}{2L}}\exp(x/2L)\{f(\eta)+\eta f'(\eta)\} \tag{7}$$

In eqn(7) ,f denotes the non-dimensional stream function , the prime denotes differentiation with respect to η and the stream function ψ is defined in the usual way as $u=\partial\psi/\partial y,\ v=-\partial\psi/\partial x$. Making use of transformations(7) in (1), we can realize incompressibility condition (i.e. continuity equation) is identically satisfied and the governing eqns (2) - (4) takes the form of non-linear ordinary differential equations:

$$f'''+ff''-2f'^2=0 \tag{8}$$

$$\theta''+\Pr f\,\theta'+\Pr Nb\,\phi'\theta'+\Pr Nt\,\theta'^2=0 \tag{9}$$

$$\phi''+Le(f\,\phi'-f'\phi)+\frac{Nt}{Nb}\theta''=0 \tag{10}$$

The boundary conditions are

$$f(0)=0,\ f'(0)=1,\ \theta(0)=1,\ \phi(0)=1 \quad \text{at } \eta=0$$
$$f'(\infty)=0,\ \theta(\infty)=0,\ \phi(\infty)=0 \quad \text{as } \eta\to\infty \tag{11}$$

Where f, θ and ϕ are dimensionless velocity, temperature and nanoparticles concentration, respectively. η is the similarity variable, the prime denote differentiation with respect to η and the governing parameters appearing in eqs (8) to(10) are defined by

$$\left.\begin{aligned}
&Pr=\frac{\upsilon}{\alpha} && \to \text{Prandtl number}\\
&Le=\frac{\upsilon}{D_B} && \to \text{Lewis number}\\
&Nb=\frac{(\rho c)_p D_B (C_w-C_\infty)}{(\rho c)_f\,\upsilon} && \to \text{Brownian motion parameter}\\
&Nt=\frac{(\rho c)_p D_T (T_w-T_\infty)}{(\rho c)_f\,T_\infty \upsilon} && \to \text{Thermophoresis parameter}
\end{aligned}\right\} \tag{12}$$

It is important to note that this boundary value problem reduces to the classical problem of flow and heat and mass transfer due to a stretching surface in a viscous fluid when Nb and Nt are zero in eqs.(9)-(10) .

The important physical quantities of interest in this problem are local Skin friction coefficient C_f , the local Nusselt number Nu_x and the local Sherwood number Sh_x are defined as:

$$C_f=-\frac{\tau_w}{\rho u_w^2},\quad Nu_x=\frac{xq_w}{k(T_w-T_\infty)},\quad Sh_x=\frac{xq_m}{D_B(C_w-C_\infty)} \tag{13}$$

Where wall shear stress τ_w , wall heat flux q_w , mass flux q_m are given by:

$$\tau_w=\rho\nu\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0},\quad q_m=-D_B\left(\frac{\partial \phi}{\partial y}\right)_{y=0} \tag{14}$$

Where C_f, $Nu_x(Nur)$, $Sh_x(Shr)$, Re_x are the skin friction, local Nusselt number, local Sherwood number and local Reynolds number respectively.

By solving eqs.(13) using eqs.(7),(14).We get

$$C_f\sqrt{2Re_x}=-f''(0),\qquad \sqrt{\frac{2}{X}}\left(\frac{Nu_x}{\sqrt{Re_x}}\right)=-\theta'(0)=Nur,\qquad \sqrt{\frac{2}{X}}\left(\frac{Sh_x}{\sqrt{Re_x}}\right)=-\phi'(0)=Shr \tag{15}$$

Where X=x/L is dimensionless coordinate along the sheet, L is the length of the sheet, C_f, $Nu_x(Nur)$, $Sh_x(Shr)$, Re_x are the skin friction, local Nusselt number , local Sherwood number and local Reynolds number respectively.

3. Numerical solution.

An efficient fourth order Runge-Kutta method along with shooting technique has been engaged to study the flow model of the above coupled non-linear ordinary differential equations (8)-(10) for different values of governing parameters viz. Prandtl number Pr, Lewis parameter Le, Brownian motion parameter Nb and thermophoresis parameter Nt. The non-linear differential equations are first decomposed into a system of first order differential equations. The coupled ordinary differential eqs.(8)-(10) are third order in f and second order in θ and ϕ which have been reduced to a system of seven to a system of seven simultaneous equations for seven unknowns. In order to numerically solve this system of equations using Runge-Kutta method, the solution requires seven initial conditions but two initial conditions in f one initial condition in each of θ and ϕ are known. However, the values of f, θ and ϕ are known at $\eta \to \infty$.These end conditions are utilized to produce unknown initial conditions at $\eta = 0$ by Shooting technique. The most important step of this scheme is to choose the appropriate finite value of η_∞.Thus to estimate the value of η_∞, we start with some initial guess value and solve the boundary value problem consisting of Eqs. (8)-(10) to obtain $f''(0), \theta'(0) \text{ and } \phi'(0)$.The solution process is repeated with another larger value of η_∞ until two successive values of $f''(0), \theta'(0) \text{ and } \phi'(0)$ differ only after desired significant digit. The last value η_∞ is taken as the finite value of the limit η_∞ for the particular set of physical parameters for determining velocity, temperature, and concentration, respectively are $f(0), \theta(0) \text{ and } \phi((0)$ in the boundary layer. After attaining all the initial conditions, we solve this system of simultaneous equations using fourth order Runge-Kutta integration scheme. The value of η_∞ is selected to vary from 5 to 20 depending on the physical parameters governing the flow so that no numerical oscillation would occur. Thus, the coupled boundary value problem of third-order in f, second order in θ and ϕ has been reduced to a system of seven simultaneous equations of first-order for seven unknowns as mentioned below,

The eqs.(8)-(10) can be expressed as:

$$
\begin{aligned}
f''' &= -f f'' + 2 f'^2 \\
\theta'' &= -Pr[f\theta' + Nb\,\phi'\theta' + Nt\,\theta'^2] \\
\phi'' &= -Le(f\phi' - f'\phi) + Pr\left(\frac{Nt}{Nb}\right)[f\theta' + Nb\,\phi'\theta' + Nt\theta'^2]
\end{aligned}
\tag{16}
$$

Now we can define new variable variables by the equations:

$$f_1 = f,\ f_2 = f',\ f_3 = f'',\ f_4 = \theta,\ f_5 = \theta',\ f_6 = \phi,\ f_7 = \phi' \tag{17}$$

The coupled higher order non-linear differential equations (8)-(10)) with the boundary conditions (11) may be transformed to seven equivalent first order differential equations and boundary conditions as given below:

$$
\begin{aligned}
f_1' &= f_2, \\
f_2' &= f_3, \\
f_3' &= -f_1 f_3 + 2f_2^2, \\
f_4' &= f_{5,} \\
f_5' &= -Pr[f_1 f_5 + Nb\, f_5 f_7 + Nt f_5^{\ 2}], \\
f_6' &= f_7, \\
f_7' &= -Le(f_1 f_7 - f_2 f_6) + Pr\left(\frac{Nt}{Nb}\right)[f_1 f_5 + Nb f_5 f_7 + Nt f_5^2]
\end{aligned}
\tag{18}
$$

A prime denotes the differentiation with respect of η and the boundary conditions are:

$$f_1(0) = 0,\ f_2(0) = 1,\ f_4(0) = 1,\ f_6(0) = 1,\ f_2(\infty) = 0,\ f_4(\infty) = 0,\ f_6(\infty) = 0 \tag{19}$$

In this study, the boundary value problem is first converted into an initial value problem (IVP).Then the IVP is solved by appropriately guessing the missing initial value using the shooting method for several sets of parameters. The step size h=0.1 is used for the computational purpose The error tolerance of 10^{-6} is also being used. The results obtained are presented through tables and graphs, and the main features of the problems are discussed and analyzed.

4. Results and discussion.

The numerical solutions are obtained for velocity, temperature and concentration profiles for different values of governing parameters. The obtained results are displayed through graphs from,Figs.2-13 for velocity,temperature and concentration profiles respectively.

Figs 2, 3 and 4. shows the effects of Nt and Nb parameters for the selected values of Pr and Le numbers . As expected, the boundary layer profiles for the temperature function $\theta(\eta)$ are essentially the same form as in the case of a regular fluid. It is observed that the temperature increases as the parameters (fig 2,Nt,Nb=0.1,0.3,0.5) ,(fig3,Nb=0.1,0.2,0.3) and (fig 4,Nt=0.1,0.3,0.5) increases, which results in thickening of thermal boundary layer of the fluid . An increase in Nb corresponds to the effective motion of nanoparticles within the flow. The intensity of this chaotic motion increases the kinetic energy of the nanoparticles and as a consequence the nanofluid's temperature rises. In nanofluids the Brownian motion takes place due to the size of nanoparticles which is of nanometer scale and at this level, the particle motion and its effect on the fluid have a pivotal role in the heat transfer. . From the definition of thermophoretic parameter Nt, it is obvious

that larger values of Nt correspond to the larger temperature difference and shear gradient. Thus increase in Nt leads to the larger temperature inside the boundary layer as depicted in Fig 4.

Figs. 5 and 6 shows the effects of Pr and Le numbers on the temperature profiles for the selected values of Nb and Nt parameters .It is observed that the temperature decrease,as the parameters in (fig5, Pr,Le=1,10) and (fig6, Pr=10,15,20) increases, which results in thinning of thermal boundary layer thickness of the fluid .

Figs. 7, 8 and 9 shows the effects of Nb ,Le and Nt parameters on the concentration profiles for the selected values of other parameters .It is observed that the concentration decrease as the parameters (fig7, Nb=0.1, 0.3, 0.5), (fig8 ,Le=10,20,30) increases, while concentration increases as the parameter(fig9,Nt=0.1,0.2,0.3) increases. In fig 7and 8 which results in thinning of concentration boundary layer thickness of the fluid. Whereas in fig 9,it is noticed, the thickening of concentration boundary layer thickness of the nano fluid

From fig 2. and fig 7, we can say that the temperature profiles converge quickly than the concentration profiles. The thickness of the boundary layer for the concentration profiles $\phi(\eta)$ is found to be lesser than the thermal boundary layer thickness when Le>1.It decreases with the increase in Nb and this decrease diminishes when Nb>5.

Figs. 10(a) and 10(b) shows , the variation in dimensionless heat transfer rates(i.e. Nusselt number) vs Nt for Pr=1and Pr=10 respectively. These figures illustrate the effects of Pr and Nb on the dimensionless heat transfer rates for the same combination of Le. It is noticed that the local Nusselt number decreases with the increase in Nb and Nt , but increase with increase in Pr, with higher Prandtl number has a relatively lower thermal conductivity, which results in reduction of the thermal boundary layer thickness.

Figs. 11(a) and 11(b) shows for both the cases of LSS and ESS, the variation in dimensionless heat transfer rates vs Nt for Le=5 and Le= 25 respectively. These figures show the effects of Le and Nb on the dimensionless heat transfer rates for the same combination of Prandtl numbers. It is noticed that the local Nusselt number decreases with the increase in Nb and Nt but decrease with increase in Le. The alteration in the dimensionless heat transfer rates is found to be higher for smaller values of Nb and this change decreases with the increase of Nt.

Figs. 12(a) and 12(b) depicts the variation in dimensionless mass transfer rates (i.e. local Sherwood number) vs Nt parameter for Pr=1 and Pr=10 respectively. These figures show the effects of Pr and Nb on the dimensionless mass transfer rates for the same combination of Le .Further it is noticed from figs12 (a) and 12(b),that the local Sherwood number increases with the increase in Nb and Nt, but increases with increase in Pr.

Figs. 13(a) and 13(b) shows the variation in dimensionless concentration rate $-\phi'(0)$ vs Nt for Le=5 and Le= 25 respectively. These figures show the effects of Le and Nb on the dimensionless heat transfer rates for the same combination of Prandtl numbers. It is noticed that in fig 13(a) the local Sherwood number increases with the increase in Nb and Nt for Le=5 and in fig 13(b) local Sherwood number decreases with the increase in Nb and Nt for Le=25,but however local Sherwood number increase with increase in Le.

Finally, a comparison with published work available in the literature has been performed in order to check the accuracy of the present results.

From table 1, it shows a test of accuracy of the solution, the values of local Nusselt number $-\theta'(0)$ for different values of Prandtl number are compared with solutions reported by Magyari and Keller[11](1999), El-Aziz[28](2009), Bidin and Nazar[12](2009), Anur Ishak[13](2011) and Swati Mukhopadhyay[29](2012). The table shows the numerical solution obtained by the present fourth order Runge-Kutta method along with Shooting technique is in very good agreement. Therefore, we are confident that results obtained by us are very much accurate to analyze the flow problem.

Tables 2 and 3 shows the variation of the local Nusselt number and local Sherwood number respectively for different values of Nb, Nt for Pr=10,Le=10.It is noticed that local Sherwood number is a decreasing function, while it is is an increasing function for(Nb=0.1 to Nb=0.5 keeping Nt =0.1,0.2,0.3,0.4,0.5) and initially decreasing function for (Nt=0.1 to Nt=0.5 for Nb=0.1,0.2) later appears to be an increasing function for (Nt=0.1 to 0.5 for Nb=0.3 ,0.4,0.5).

Table 1: Comparison of results for the local Nusselt number $-\theta'(0)$ for Nt =Nb=Le=0.

Pr	Magyari and Keller[11] (1999)	El-Aziz[28] (2009)	Bidin & Nazar[12] (2009)	Anur Ishak[13] (2011)	S.Mukho-adhyay &Gorla[29] (2012)	Present Results
1.0	0.954782	0.954785	0.9548	0.9548	0.9547	0.951556
2.0	--------	------	1.4714	1.4715	1.4714	1.465304
3.0	1.869075	1.869074	1.8691	1.8691	1.8691	1.859997
5.0	2.500135	2.500132	-------	2.5001	2.5001	2.485222
10.0	3.660379	3.660372	-------	3.6604	3.6603	3.630831

Table2 :Variation of Nu_x with Nb and Nt for Pr=10,Le=10.

ESS	Nb=0.1	Nb=0.2	Nb=0.3	Nb=0.4	Nb=0.5
Nt	Nu_x	Nu_x	Nu_x	Nu_x	Nu_x
0.1	1.996804	1.398096	1.010997	0.762334	0.600227
0.2	1.680884	1.214909	0.906994	0.703189	0.565804
0.3	1.463489	1.087530	0.833110	0.659919	0.539755
0.4	1.309018	0.995461	0.778371	0.626898	0.519242
0.5	1.195238	0.926248	0.736165	0.605640	0.502535

Table3: Variation of Sh_x with Nb and Nt for Pr=10,Le=10.

ESS	Nb=0.1	Nb=0.2	Nb=0.3	Nb=0.4	Nb=0.5
Nt	Sh_x	Sh_x	Sh_x	Sh_x	Sh_x
0.1	2.977544	3.536209	3.661603	3.696171	3.703874
0.2	2.608756	3.493767	3.676965	3.727476	3.738362
0.3	2.533735	3.475994	3.690462	3.750683	3.764038
0.4	2.438263	3.460115	3.697129	3.765798	3.782225
0.5	2.351303	3.438623	3.696437	3.775423	3.792153

5. Conclusions:

In this paper, effects of Prandtl number(Pr),Lewis number(Le),Brownian motion parameter(Nb),thermophoresis parameter(Nt) on temperature profiles, concentration profiles, local Nusselt number and local Sherwood number , on the boundary layer flow and heat transfer of nanofluids past an exponential stretching sheet is investigated. The variation of the local Nusselt number and local Sherwood numbers with Nb and Nt for various values of Pr and Le is presented in tabular and graphical forms. The numerical results obtained are in excellent agreement with the previously published data in limiting condition and for some particular cases of the present study. The following conclusions have been drawn from the present study:

The effects of Pr,Le and are inversely proportional to temperature where as the reverse effect is seen in case of Nt,Nb.

The effects of N is directly proportional to concentration (mass fraction) where as the reverse effect is seen in case of Nb and Le .

In the case of LSS, it is found that the local Nusselt number is a decreasing function, while the local Sherwood number is an increasing function for each of the dimensionless parameters Pr,Le,Nb and Nt considered .

The local Sherwood number is an increasing function of higher Pr and a decreasing function of lower Pr , while local Nusselt number is a decreasing function for lower Pr and increasing function for higher Pr for each of Le, Nb and Nt.

The local Nusselt number is a decreasing function of higher Le and an increasing function for lower Le, while local Sherwood number is an increasing function of higher Le and decreasing function for lower Le for each Pr,Nb,and Nt.

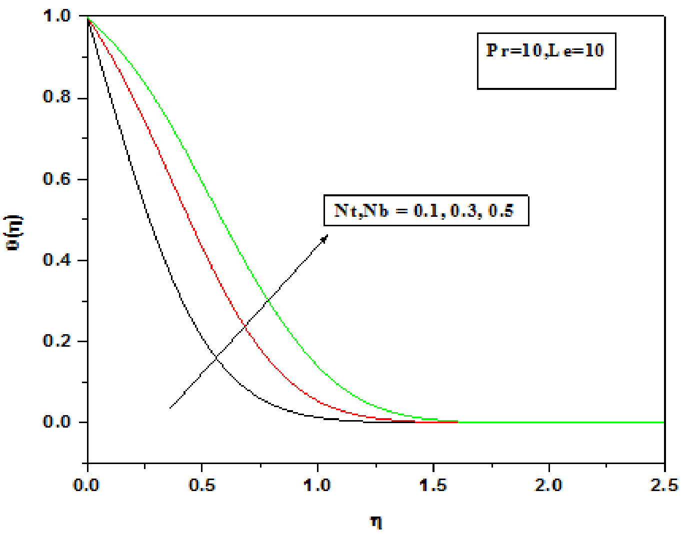

Fig 2. Effects of Nt and Nb on temperature profiles

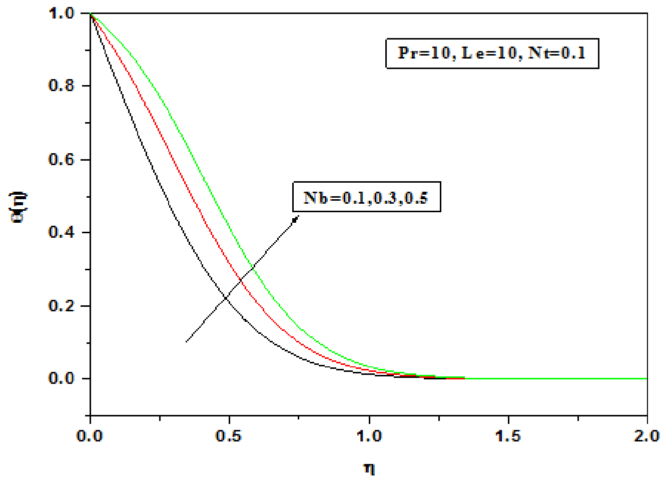

Fig 3. Effects of Nb on temperature profiles

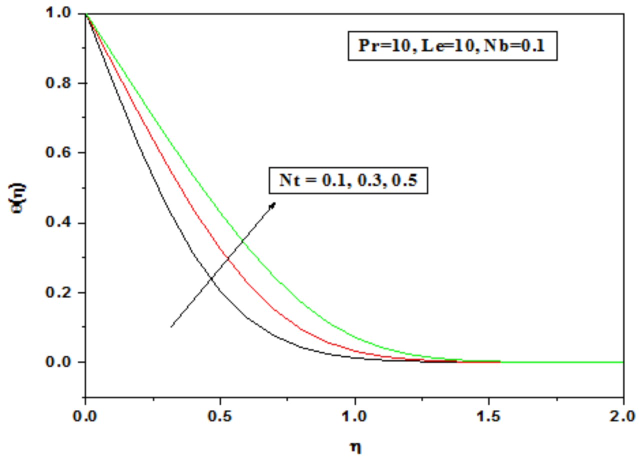

Fig 4. Effects of Nt on temperature profiles

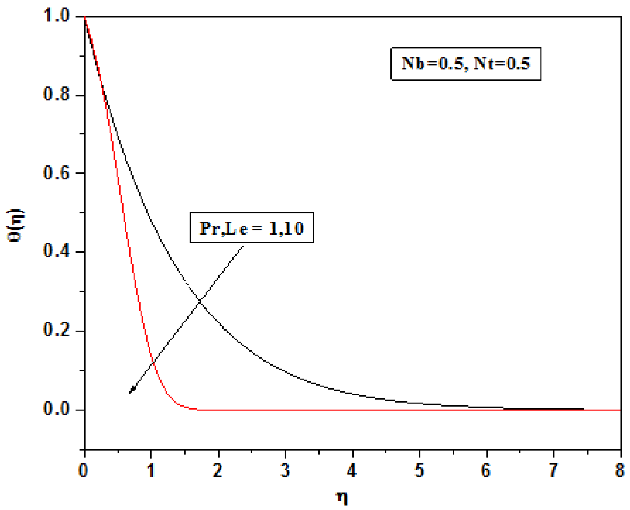

Fig 5. Effects of Pr and Le on temperature profiles

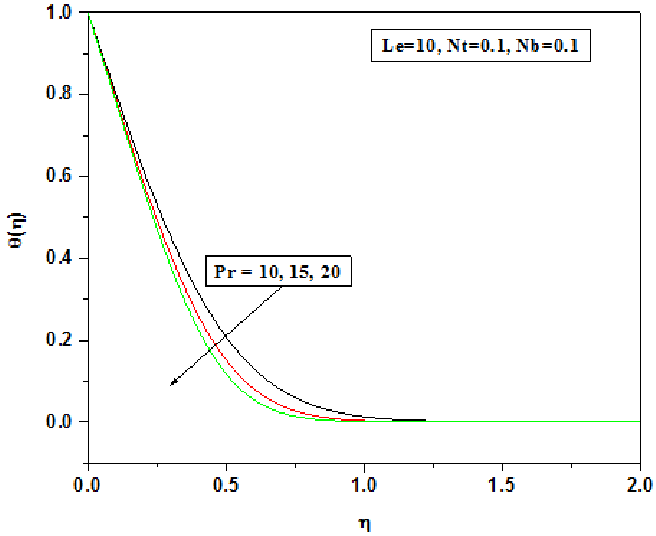

Fig 6. Effects of Pr number on temperature profiles

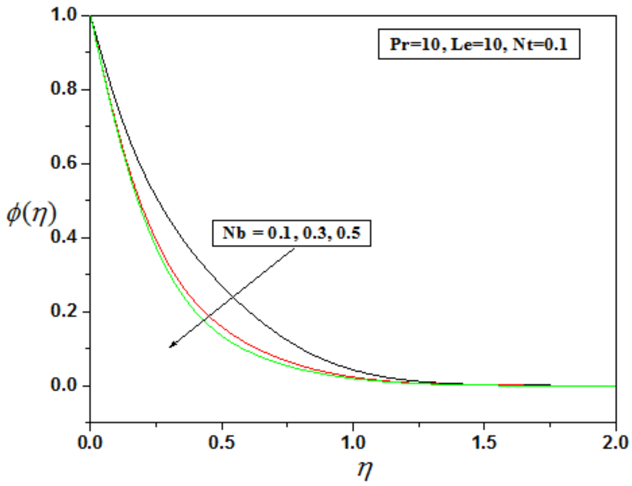

Fig 7. Effects of Nb on concentration profiles

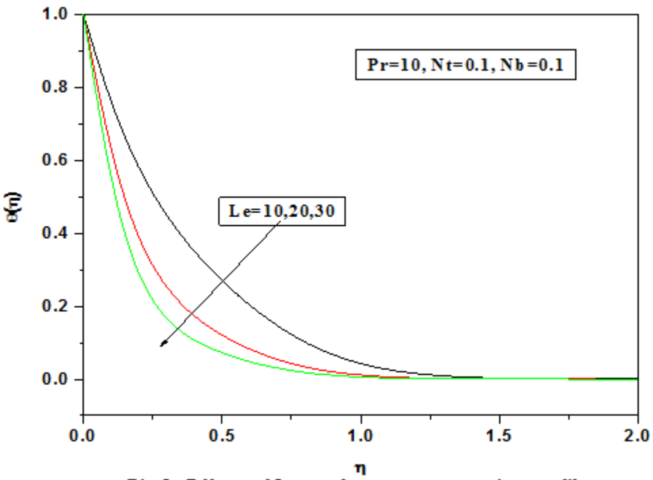

Fig 8. Effects of Le number on concentration profiles

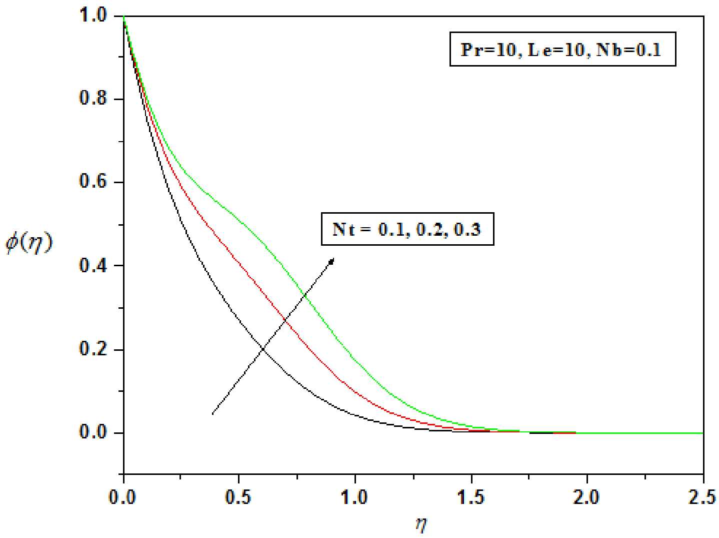

Fig 9. Effects of Nt on concentration profies

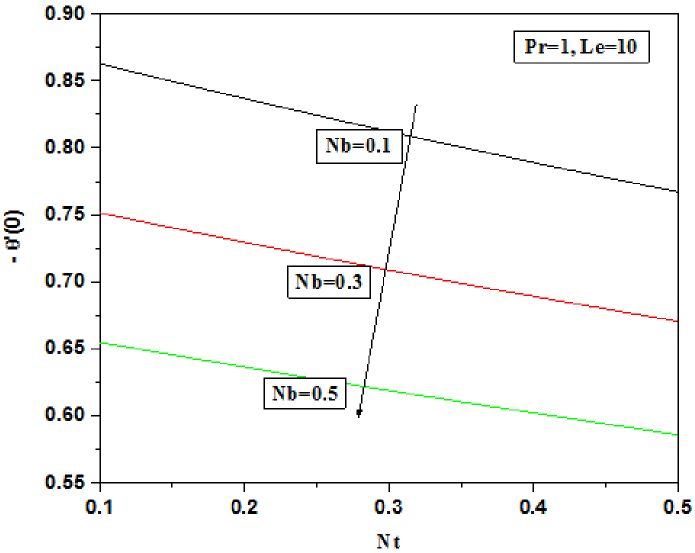

Fig 10(a)Effects of Nb on dimensionless heat transfer rate

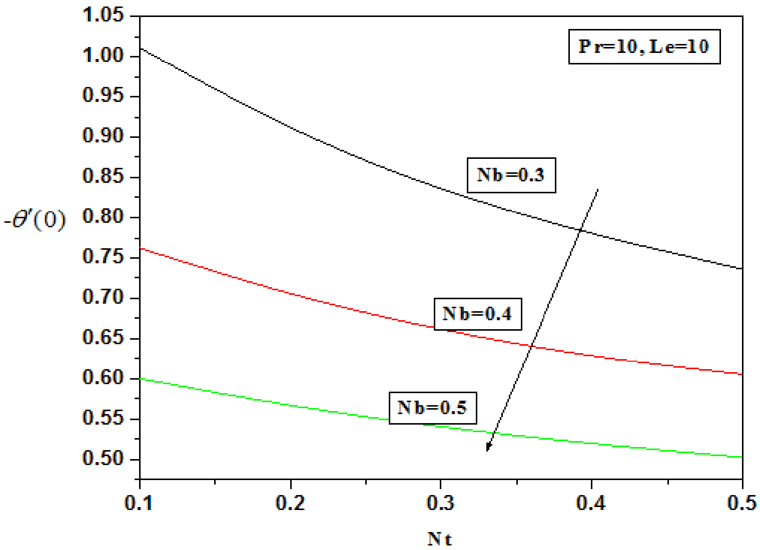

Fig 10(b) Effects of Nb on dimensionless heat transfer rates

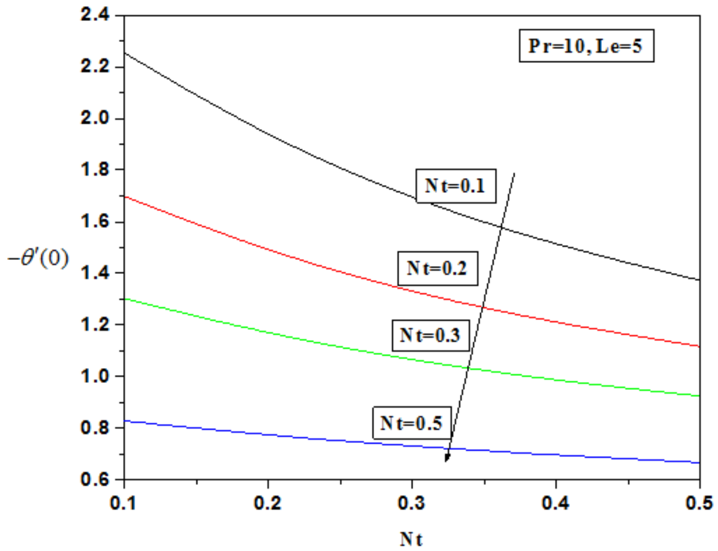

Fig 11(a) Effect of Nt on dimensionless heat transfer rates

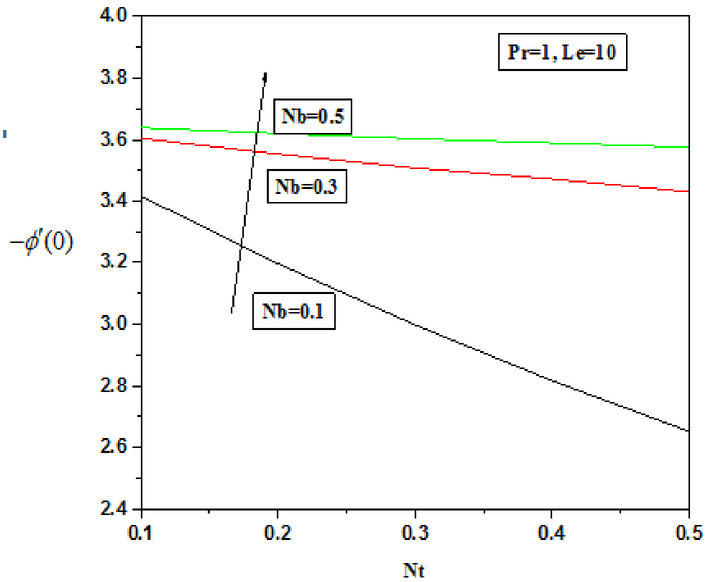

Fig 12(a) Effect of Nb on dimensionless concentration rate

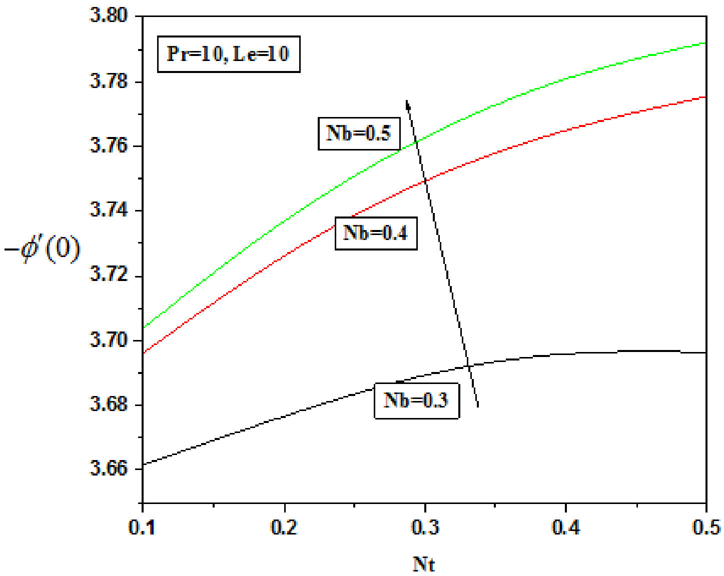

Fig 12(b) Effect of Nb on dimensionless concentration rate

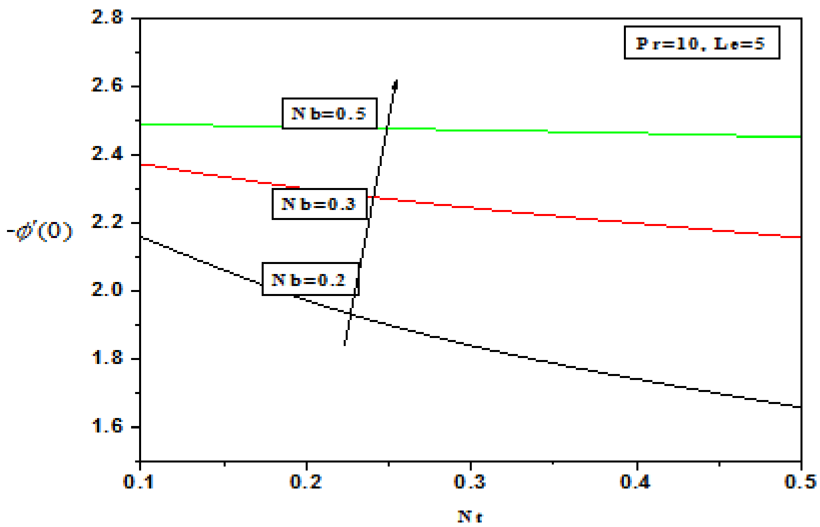

Fig 13(a) Effect of Nb on dimensionless concentration rate

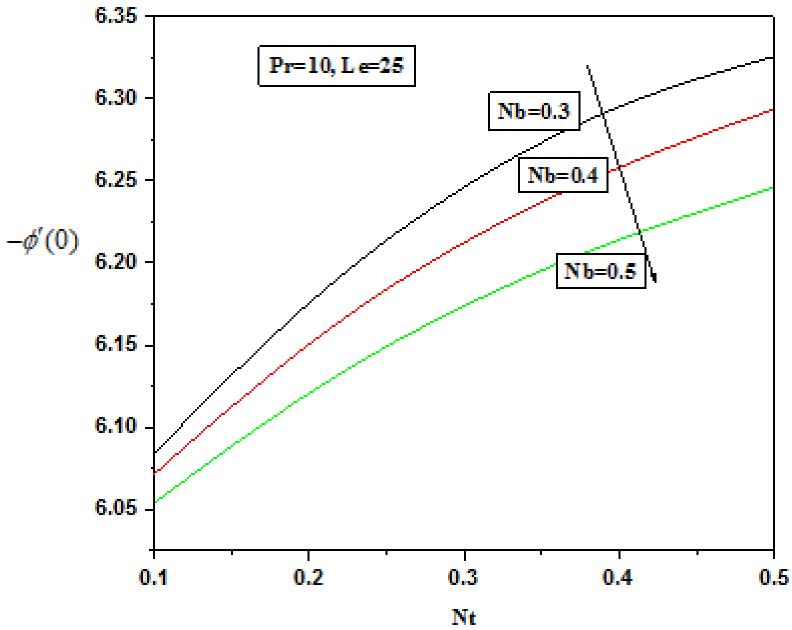

Fig 13(b) Effect of Nb on dimensionless concentration rate

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