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**WHAT IS THE APPROPRIATE ESTIMATOR TO EXCHANGE RATE MODEL OF SUDAN ECONOMY**

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**ABSTRACT**

Estimating economic relations through model system is valuable, hence this study tackled the system of exchange rate as a part of Sudan economy model in order to provide key tools to the policy makers which can help in managing macroeconomic objectives. The suggested system of exchange rate comprise from five equations which are well identified. Applying OLS to the target equation in the system (exchange rate) clarified good specification of the equation, then it estimated from the system by using 2SLS, 3SLS and SURE. After evaluating the simulated target equation by different estimators, the SURE estimator is detected as appropriate estimator to the model. The results affirmed the importance of GDP and price level in explaining the exchange rate in the presence of direct relationship with GDP and inverse relationship with general price level. In addition to existence of an inverse relationship between exchange rate and government budget balance, current account balance as well as capital account balance, hence lowering these balances deficit will improve the exchange rate position.

**Key Words:** Exchange Rate , 2SLS , 3SLS , SURE

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**INTRODUCTION**

The title of any economy is the exchange rate which reflects the real state of the economy. Managing exchange rate is important issue that faces policy makers. If stability is desired exchange rate must design to suit the running fiscal and monetary policies. There are many factors affecting exchange rate either directly or indirectly through transmission channels . Some factors effectively change exchange rate, while other touch it slightly. Exchange rate appreciation or depreciation depends upon relative strengthens of contradicted factors that working in economic atmosphere. If such atmosphere is properly controlled the resulted exchange rate could capable to serve the macroeconomic objectives.

As mentioned many variables interact in economic field to generate specific exchange rate , so there are different channels in economic world that produce given exchange rate. Hence in order to capture the interaction channels the formulation of system model frame is significantly needed in order to hold all variables which involve in the transmission operation upon which exchange rate is fed .

The system nature that underlying exchange rate interrelations necessitates specific manipulation in the process of estimation. The appropriate and proper estimator is one that count for simultaneous bias or the correlations of errors either with explanatory variables or across equations. Accordingly the study will examine 2SLS , 3SLS , SURE, FIML estimators along with OLS to detect the features of the target equation which is exchange rate.

### **Identification**

The key identification condition is that each explanatory variable is uncorrelated with the error term, this fundamental condition no longer holds, in general, for simultaneous equations models. However, if we have instrumental variables, we can still identify or consistently estimate the parameters in an simultaneous equations model, just as with omitted variables or measurement error.

It seems natural to ask, can we uniquely recover the structural parameters from the reduced form? If so, then we need only estimate the reduced form. If we can uniquely recover the parameters for a given structural equation from the reduced form, then that structural equation is said to be exactly identified. If we cannot, then the structural equation is said to be over identified. It is quite possible, and indeed common, to have a structural model in which some equations are exactly identified and some equations are over identified. If all structural equations are exactly identified, then estimation of the reduced form and algebraic recovery of the structural parameters, termed Indirect Least Squares, is consistent and efficient. To determine if an equation is exactly identified, we rely on the order condition, which is expressed for each equation as Order Condition: If the number of predetermined variables in the system equals the number of slope coefficients in the equation, then the parameters of the equation are (if the rank condition is satisfied) exactly identified. If the number of predetermined variables in the system exceeds the number of slope coefficients in the equation, then the parameters of the equation are over identified. (If there are fewer predetermined variables than slope coefficients, the parameters in the equation are not identified.). The order condition is trivial to check once all equations have been specified. The rank condition requires more: at least one of the exogenous variables excluded from one equation must have a nonzero population coefficient in other equation. This ensures that at least one of the exogenous variables omitted from one equation appears in the reduced form of other equation.

### **RESET (Specification Error Test)**

Pesaran and Taylor (1999) propose the RESET general specification error test for appropriate functional form and/or omitted variables. The test has good relative power (compared to other tests in their paper) and is robust to heteroscedasticity. To conduct the test proceed as follows:

1. Square the 2SLS forecasts from the estimated model :  $\tilde{y}_t^2$
2. Run a 2SLS/IV regression of  $y$  against the original RHS variables and  $\tilde{y}_t^2$ , use as instruments the original instruments and  $\tilde{y}_t^2$ .
3. The t-statistic on the  $\tilde{y}_t^2$  variable is the test statistic.
4. If the test statistic is significant then there is a specification error.

The logic underlying this test suggests, in a correctly specified model the predictions from the model should not have any explanatory power in the original model. Effectively, the predictions

from the model should not be able to explain any variation in the residuals in a correctly specified model.

Consider the model

$$\hat{y} = E\{y|x\} = \beta x.$$

The Ramsey test then tests whether  $(\beta x)^2, (\beta x)^3, \dots, (\beta x)^k$  has any power in explaining y. This is executed by estimating the following linear regression

$$y = \alpha x + \gamma_1 \hat{y}^2 + \dots + \gamma_{k-1} \hat{y}^k + \epsilon$$

and then testing, by a means of a F-test whether  $\gamma_1$  through  $\gamma_{k-1}$  are zero. If the null-hypothesis that all  $\gamma$  coefficients are zero is rejected, then the model suffers from mis-specification

### **Advantage of 2SLS**

The advantages of using 2SLS over the more conventional maximum likelihood (ML) method for SEM include:

- It does not require any distributional assumptions for RHS independent variables, they can be non-normal, binary, etc.
- In the context of a multi-equation non-recursive SEM it isolates specification errors to single equations,
- It is computationally simple and does not require the use of numerical optimization algorithms.
- It easily caters for non-linear and interactions effects
- It permits the routine use of often ignored diagnostic testing procedures for problems such as heteroscedasticity and specification error
- Simulation evidence from econometrics suggests that 2SLS may perform better in small samples than ML
- There are however some disadvantages in using 2SLS compared to ML, these include:
- The ML estimator is more efficient than 2SLS given its simultaneous estimation of all relationships, hence ML will dominate 2SLS always in sufficiently large samples if all assumptions are valid and the model specification is correct. Effectively ML is more efficient (if the model is valid) as it uses much more information than 2SLS.
- Unlike the ML method, the 2SLS estimator depends upon the choice of reference variable. The implication being that different 2SLS estimates result given different scaling variables.
- Programs with diagram facilities such as EQS do not exist for 2SLS. One needs to logically work through the structure of the model to specify individual equations for all the relationships for the 2SLS estimator.

### **Three Stage**

3SLS obtains three stage least squares estimates of a set of nonlinear equations. Three stage least squares estimates are consistent and asymptotically normal, and, under some conditions, asymptotically more efficient than single equation estimates. 3SLS is asymptotically less efficient than FIML. Three stage least squares is a combination of multivariate regression (SUR estimation) and two stage least squares. It obtains instrumental variable estimates, taking into account the covariances across equation disturbances as well. The objective function for three stage least squares is the sum of squared transformed fitted residuals. Three stage least squares estimates are obtained by estimating a set of nonlinear (or linear) equations with cross-equation constraints imposed, but with a diagonal covariance matrix of the disturbances across equations. This is the constrained two stage least squares estimator. The parameter estimates thus obtained are used to form a consistent estimate of the covariance matrix of the disturbances, which is then used as a

weighting matrix when the model is reestimated to obtain new values of the parameters. The actual method of parameter estimation is the Gauss-Newton method for nonlinear least squares described under LSQ. If the model is linear in the parameters and endogenous variables, only two iterations will be required, one to obtain the 3SLS covariance matrix estimate, and one to obtain parameter estimates.

A number of special cases are readily derived from this in which 2SLS and 3SIs are coincided. If the errors of all equations are uncorrelated, 2SLS is identical to 3SLS. If all equations are just identified, and hence  $X'Z_i$  is square and non-singular for each  $i$  (so that the  $X'Z_i$  span the  $A$ -dimensional space for all  $i$ ), 2SLS equals 3SLS. A third example would be that the first  $p$  equations are just identified with freely covarying errors, whereas the last  $M - p$  equations may be over identified but their errors would have to be mutually uncorrelated and uncorrelated with the errors of the first  $p$  equations. As noticed by Srivastava and Tiwari, a seemingly unrelated regressions system is a special case of a simultaneous system, with only purely exogenous variables as explanatory variables. For that special case the proposition implies that GLS will be identical to OLS applied equation by equation if and only if for all equations  $i, j$  with non-zero error covariance of the matrices of explanatory variables  $X_i$  and  $X_j$  span the same column space.

#### **FIML:**

Full information maximum likelihood is almost universally abbreviated FIML, and it is often pronounced like "fimble" if "fimble" was an English word. FIML is often the ideal tool to use when your data contains missing values because FIML uses the raw data as input and hence can use all the available information in the data. This is opposed to other methods which use the observed covariance matrix which necessarily contains less information than the raw data. An observed covariance matrix contains less information than the raw data because one data set will always produce the same observed covariance matrix, but one covariance matrix could be generated by many different raw data sets. Mathematically, the mapping from a data set to a covariance matrix is not one-to-one (i.e. the function is non-injective), but rather many-to-one. Although there is a loss of information between a raw data set and an observed covariance matrix, in structural equation modeling we are often only modeling the observed covariance matrix and the observed means. We want to adjust the model parameters to make the observed covariance and means matrices as close as possible to the model-implied covariance and means matrices. Therefore, we are usually not concerned with the loss of information from raw data to observed covariance matrix. However, when some raw data is missing, the standard maximum likelihood method for determining how close the observed covariance and means matrices are to the model-expected covariance and means matrices fails to use all of the information available in the raw data. This failure of maximum likelihood (ML) estimation, as opposed to FIML, is due to ML exploiting for the sake of computational efficiency some mathematical properties of matrices that do not hold true in the presence of missing data.

#### **SURE**

The seemingly unrelated regressions (SUR) model, proposed by Zellner, can be viewed as a special case of the generalized regression model  $E(y) = X\beta$ ,  $V(y) = \sigma^2\Omega$ ; however, it does not share all of the features or problems of other leading special cases. While, like those models, the matrix  $\Omega$  generally involves unknown parameters which must be estimated, the usual estimators for the covariance matrix of the least squares estimator  $\hat{\beta}_{LS}$  are valid, so that the usual inference procedures based on normal theory are valid if the dependent variable  $y$  is multinormal or if the sample size  $N$  is large and suitable limit theorems are applicable. Also, unlike those other models, there is little reason to test the null hypothesis  $H_0 : \Omega = I$ ; the form of  $\Omega$  is straightforward and its

parameters are easy to estimate consistently, so a feasible version of Aitken's GLS estimator is an attractive alternative to the asymptotically-inefficient LS estimator.

A seemingly unrelated regression (SUR) system comprises several individual relationships that are linked by the fact that their disturbances are correlated. Such models have found many applications. For example, demand functions can be estimated for different households (or household types) for a given commodity. The correlation among the equation disturbances could come from several sources such as correlated shocks to household income. alternatively, one could model the demand of a household for different commodities, but adding-up constraints leads to restrictions on the parameters of different equations in this case. On the other hand, equations explaining some phenomenon in different cities, states, countries, firms or industries provide a natural application as these various entities are likely to be subject to spillovers from economy-wide or worldwide shocks. There are two main motivations for use of SUR. The first one is to gain efficiency in estimation by combining information on different equations. The second motivation is to impose and/or test restrictions that involve parameters in different equations

### **Model Evaluation**

In order to evaluate an economic model it is valuable to investigate the transmission channels, testing exogeneity and invariance as well as model performance. The concept weak exogeneity and parameter invariance refer to different questions concerning valid conditioning in the context of estimation and valid policy analysis respectively. Weak exogeneity of the conditional variables for the parameters of the model implies that these parameters are free to vary with respect to the parameters of marginal models of national income. Valid policy analysis involves as a necessary condition that the coefficients of the model are invariant to the interaction occurring in the marginal models. Finally in evaluating the model the performance of estimated model gives insight about feasibility of future model implementations. Here the tracking performance under simulation is the best technique in comparing actual quantities versus estimated ones. Many suggestion concerning estimation and simulation are available like full information maximum likelihood. According to the mode performance checks the forecast power of the model apparently becomes judgable.

### **Empirical Model**

The system model of exchange rate contains five equations, the main equation is exchange rate equation, while other equations are GDP equation, current account equation, capital account equation and price level equation respectively. As far as identification principle the model was satisfied the necessary and sufficient conditions by applying order and rank methods in testing the identification of equations. Accordingly all equation in the model were over identified so we can apply different simultaneous equations estimators to detect the best one. As a matter of fact the study is interested and concerned with the exchange rate equation. Here we shall review different estimators to this equation within the overall model and neglect the estimates of remaining equations. Such limitation in estimates is due to the objective of the study which conveys in the title.

$$\text{Exch} = a - b*Y + c*GP - d *CAP + f*KAP + h*P \quad (1)$$

$$Y = v + t*G - o*T + n*M^s \quad (2)$$

$$GP = l + s*T + j*G \quad (3)$$

$$CAP = w + k*exch + q*Y + z \quad (4)$$

$$KAP = q + u* R \quad (5)$$

Symbol	Variable	Type
Exch	Exchange Rate	Endogenous
GP	Budget Position	Endogenous
CAP	Current Account Position	Endogenous
KAP	Capital Account Position	Endogenous
Y	GDP	Endogenous
P	Price Level	Exogenous
R	Interest Rate	Exogenous
G	Government Expenditure	Exogenous
T	Indirect Tax	Exogenous
M <sup>s</sup>	Money Supply	Exogenous

### The Findings

The study chosen the period between 2000 to 2014 as target period. The data were gathered from official sources. In the first step we run OLS to cover the overall features of the target equation (exchange rate), next we compare the estimates of 2SLS, 3SLS and SURE, where as the full information maxima likelihood estimator was not incorporated in the analysis because the result revealed near singular matrix.

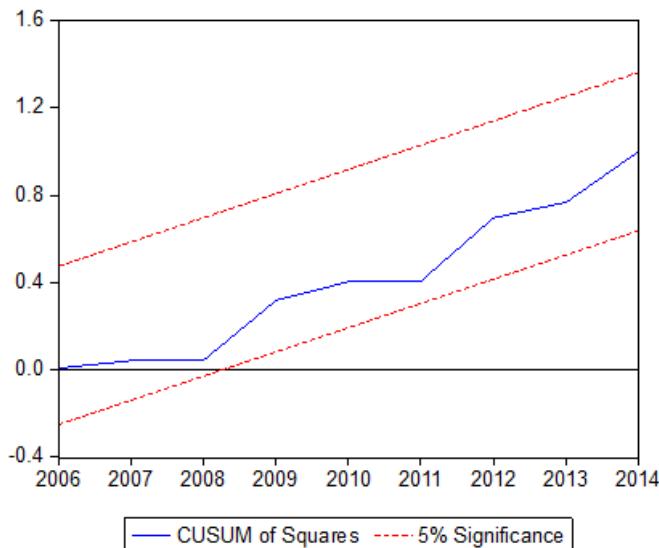
From the results (exhibited below) of OLS estimation Ramsy reset test indicates the absence of specification error or none of either redundant nor omitted significant variables from the target equation. Hence the model is well specified, more over the test indicates no relation between the error term and the explanatory variables, beside lineal shape of the model. R square is comparatively high with high forecast power as confirmed by Thiel inequality . Finally the parameters are stable as shown by the stability graph (cumulative sum of squares at 5% significance).

From the coefficients of the equation there exist direct relationship between exchange rate and GDP while there exist inverse relationship between exchange rate and budget position, current account position , capital account position and price level respectively. Exchange rate is less sensitive to these above variables except the price level to some degree. The constant term is positive which indicates automatic enhancement in exchange rate position.

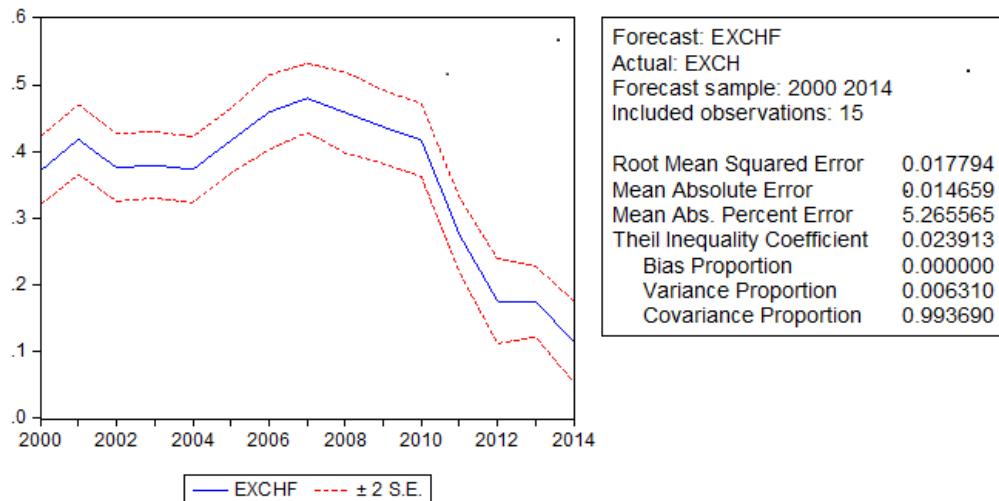
### OLS (results)

	Coefficient	t	Sig
Constant	0.367627	15.95644	0.000
GDP	5.96E-06	6.563977	0.000
GP	-7.90E0-06	-4.773085	0.001
CAP	-1.86E-05	-2.441461	0.0373
KAP	-1.11E-05	-1.642866	0.1348
P	-0.805110	-4.925947	0.0008
Adjusted R <sup>2</sup>	0.975077		
D W	2.399081		
Serial autocorrelation (f)	1.273658		0.2918
Heteroscadasticity (f)	0.904867		<b>0.5179</b>
Ramsey RESET test (f)	0.990528		0.3488
Thiel Inequality	0.023913		
Bias Proportion	0.000		
Variance Proportion	0.006310		
Covariance Proportion	0.993690		

### Coefficients stability (OLS)



### Forecast Test



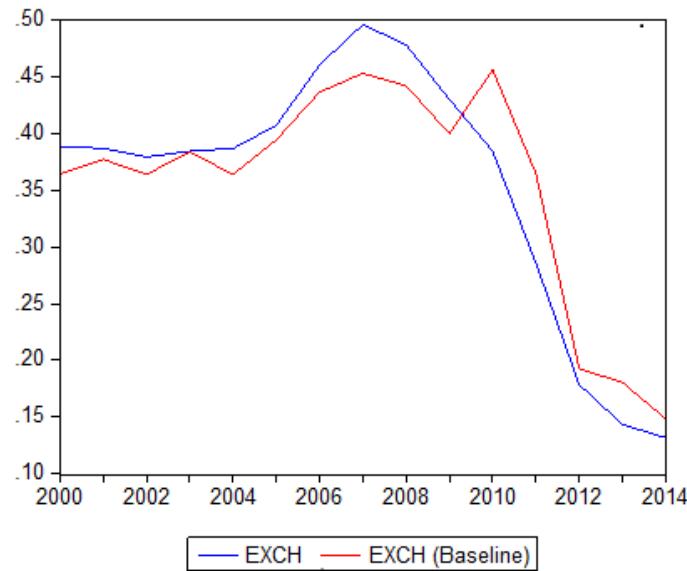
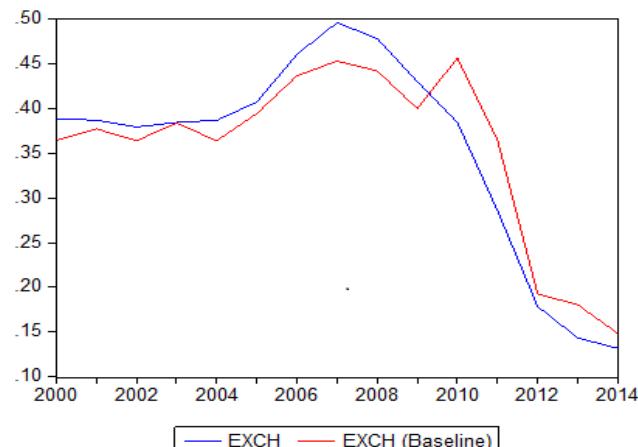
The results of 2SLS, 3SLS and SURE (exhibited below) reveal that R square is comparatively high (more than 90%) to all three estimators with Durbin Watson values closed to two. There is direct relationship between exchange rate and GDP, while inverse relationship between exchange rate and budget position, current account position, capital account position and general price level respectively. Exchange rate is less sensitive to these above variables except the price level to some degree. The constant term is positive which indicates automatic enhancement in exchange rate position.

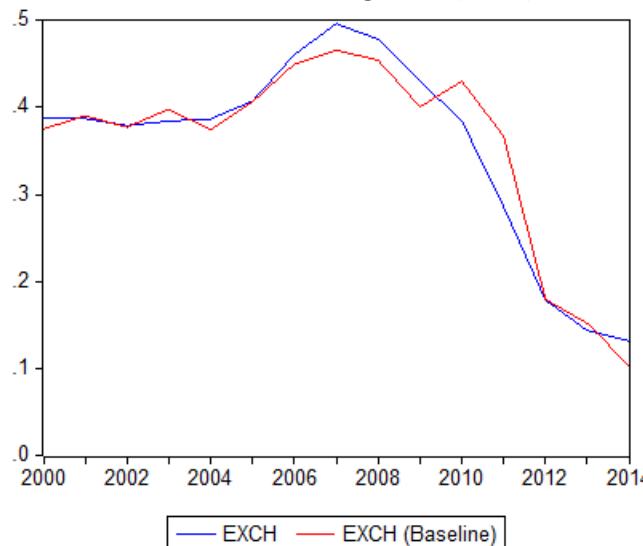
The coefficients in 2SLS coincided with those in 3SLS which might indicate no bias in the target equation or no correlation between the error term and the explanatory variables, while the coefficients are different from those in OLS which affirmed the existence of simultaneous equation bias. Reference to SURE estimator coefficients which were different from those in the rest estimators which assures the existence of the bias across equations or error terms are correlated across equations.

As far as model evaluation the historical simulation of the models exhibited in below graphs. The model were simulated by using base line method. From the graphs 2SLS and 3SLS experienced same

historical simulation while the SURE estimator model of exchange rate has different simulation from the former estimators. By quick glance to the forecast graphs it is clear that the SURE model is best one compare to other ones because most inflection points of all most actual and simulated exchange rate are move in the same direction, more over the actual and simulated graphs are closed to each as compare to ones in 2SLS and 3SLS.

	2SLS			3SLS			SURE		
	coef	T	Sig	coef	t	Sig	Coef	t	sig
Constant	0.367627	15.95644	0.000	0.367627	20.5997	0.00	0.358086	21.6984	0.000
GDP	5.96E-06	6.563977	0.000	5.96E-06	8.47406	0.00	6.32E-06	9.85193	0.000
GP	-7.90E-06	-4.773085	0.000	-7.90E-06	-6.16203	0.000	-8.51E-06	-7.5199	0.000
CAP	-1.86E-05	-2.441461	0.017	-1.86E-05	-3.15191	0.003	-1.64E-05	-3.2339	0.002
KAP	-1.11E-05	-1.642866	0.106	-1.11E-05	-2.12093	0.04	-7.67E-05	-1.7162	0.091
P	- 0.805110	-4.925947 0.805110	0.000	- 0.805110	-6.35937 0.000	- 0.805110	-0.76706 -6.9379	- 0.000	- 0.000
Adj R <sup>2</sup>	0.975			0.975			0.973962		
D W	2.399081			2.399081			2.328656		

**Simulation of exchange rate (2SLS)****Simulation of exchange rate (3SLS)**

**Simulation of exchange rate (SURE)**

### The Results

- 1) Exchange rate equation is well specified so there are no either redundant variables in the equation as indicated by Rasmy test.
- 2) Exchange rate model is over identified for all equations so it can be examined by 2SLS , 3SLS and SURE.
- 3) Examination of the model by (FIML) estimator revealed near singular matrix.
- 4) The results of different estimators assured the existence of simultaneous equations bias between the error terms across equations.
- 5) The historical simulation of different estimators was favour to SURE estimator, so the study relied on its results .
- 6) Exchange rate position is automatically enhanced as confirmed by the constant term.
- 7) Exchange rate is less sensitive to all explanatory variables except the general price level to some degree.
- 8) There exist direct relationship between exchange rate and GDP, so increases in GDP will improve the exchange rate position.
- 9) There exist an inverse relationship between exchange rate and government budget position, hence narrow difference in the budget will improve the exchange rate position.
- 10) (10) There exist an inverse relationship between exchange rate and current account position, hence improvement in current account deficit will improve the exchange rate position.
- 11) There exist an inverse relationship between exchange rate and capital account position, hence improvement in capital account deficit will make the exchange rate position better off.
- 12) There exist an inverse relationship between exchange rate and general price level, hence lowering inflation rate will improve the exchange rate position.

### Conclusion

The simultaneous equations model of exchange rate is appropriately and properly estimated by seemingly unrelated equations estimator (SURE). The results confirmed the importance of GDP and general price level in determining the exchange rate which combat with the economic theory if we bear in mind the resulted directions of relations of the two variables with

exchange rate as shown by the signs of coefficients. Other worthy remark is that managing balanced position in government budget, current account balance as well as capital account balance will significantly could make theposition of exchange rate better off.

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