Vol.3.Issue.4.2015 (Oct-Dec.)

http://www.bomsr.com

BULLETIN OF MATHEMATICS
AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

ISSN:2348-0580

RESEARCH ARTICLE

SOME COMMON FIXED POINT THEOREMS IN GENERALIZED INTUITIONISTIC FUZZY METRIC SPACES

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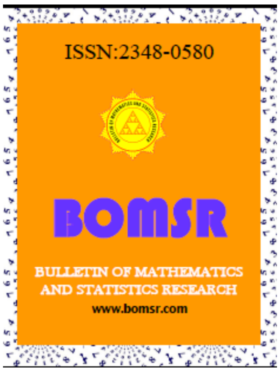

ABSTRACT

In this paper, we prove some common fixed point theorems for two nonlinear mappings in complete generalized intuitionistic fuzzy metric spaces. Our main results improved versions of several fixed point theorems in complete generalized intuitionistic fuzzy metric spaces.

Keywords: Common fixed point, Generalized intuitionistic fuzzy metric spaces, Property (C).

AMS Subject Classification : 47H10, 54H25.

1. INTRODUCTION

As a generalization of fuzzy sets, Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets. Park [8] using the idea of intuitionistic fuzzy sets defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous

t-conorm as a generalized of fuzzy metric spaces, George and Veeramani [5] showed that every metric induces an intuitionistic fuzzy metric, every fuzzy metric space in an intuitionsitic fuzzy metric space.

In 2006, Sedghi and Shobe [10] defined $\mathcal{M}$ -fuzzy metric spaces and proved a common fixed point theorem for four weakly compatible mappings in this spaces. In this paper, we prove some common fixed point theorems for two nonlinear mappings in complete generalized intuitionistic fuzzy metric spaces. Our main results improved versions of several fixed point theorems in complete generalized intuitionistic fuzzy metric spaces.

2. PRELIMINARIES

Definition 2.1:

A 5-tuple (X, $\mathcal{M}$, $\mathcal{N}$, *, ◊) is called an generalized intuitionistic fuzzy metric space if X is an arbitrary (non-empty) set, * is a continuous t-norm, ◊ a continuous t-conorm and $\mathcal{M}$, $\mathcal{N}$ are fuzzy sets on X^3 x (0, ∞), satisfying the following conditions:
for each x, y, z, a ∈ X and t, s > 0.

a) $\mathcal{M}$ (x, y, z, t) + $\mathcal{N}$ (x, y, z, t) ≤ 1,
b) $\mathcal{M}$ (x, y, z, t) > 0,
c) $\mathcal{M}$ (x, y, z, t) = 1 if and only if x = y= z,
d) $\mathcal{M}$ (x, y, z, t) = M (p{x, y, z}, t), where p is a permutation function,
e) $\mathcal{M}$ (x, y, z, a, t) * M (a, z, z, s) ≤ M (x, y, z, t + s),
f) $\mathcal{M}$ (x, y, z, .) : (0,∞) ⟶ [0,1] is continuous,
g) $\mathcal{N}$ (x, y, z, t) > 0,
h) $\mathcal{N}$ (x, y, z, t) = 0, if and only if x = y = z,
i) $\mathcal{N}$ (x, y, z, t) = $\mathcal{N}$ (p{x, y, z}, t) where p is a permutation function,
j) $\mathcal{N}$ (x, y, z, a, t) ◊ $\mathcal{N}$ (a, z, z, s) ≥ $\mathcal{N}$ (x, y, z, t + s),
k) $\mathcal{N}$ (x, y, z, .) : (0, ∞) ⟶ [0, 1] is continuous.

Then (M, $\mathcal{N}$) is called an generalized intuitionistic fuzzy metric on X.

Definition 2.2:

Let (X, $\mathcal{M},\mathcal{N}$, ∗, ◊) be an generalized intuitionistic fuzzy metric space. Then for any t > 0 $\mathcal{M}(x, x, y, t) = \mathcal{M}(x, y, y, t)$ and $\mathcal{N}(x, x, y, t) = \mathcal{N}(x, y, y, t)$. Let (X, $\mathcal{M},\mathcal{N}$, ∗, ◊) be an generalized intuitionistic fuzzy metric space. For any t > 0, the open ball $B_{\mathcal{M},\mathcal{N}}(x,r,t)$ with the center x ∈ X and radius 0 < r < 1 is defined by $B_{\mathcal{M}}(x,r,t) = \{y \in X: \mathcal{M}(x,y,y,t) > 1 - r\}$ and $B_{\mathcal{N}}(x,r,t) = \{y \in X: \mathcal{N}(x,y,y,t) < r\}$. A subset A of X is called an open set if, for all x ∈ A, there exist t > 0 and 0 < r < 1 such that $B_{\mathcal{M},\mathcal{N}}(x, r, t) \subseteq A$.

Example 2. 3 :

Let X is a nonempty set and D^*-metric on X. Denote a∗b = a.b and a ◊b = min {1, a + b} for all a, b ∈ [0,1]. For any t∈ [0, ∞), define $\mathcal{M}(x,y,z,t) = \frac{t}{t+D^*(x,y,z)}$, and
$\mathcal{N}(x,y,z,t) = \frac{D^*(x,y,z)}{t+D^*(x,y,z)}$ for all x, y, z ∈ X. It is easy to see that (X, $\mathcal{M},\mathcal{N}$, ∗, ◊) is a generalized intuitionistic fuzzy metric space.

Remark 2.4 :

Let (X, $\mathcal{M},\mathcal{N}$, ∗, ◊) is a generalized intuitionistic fuzzy metric space, If we define $\mathcal{M}$, $\mathcal{N}$: X^3×(0, ∞) →[0, 1] by $\mathcal{M}(x,y,z,t) = \mathcal{M}(x,y,t) * \mathcal{M}(y,z,t) * \mathcal{M}(z,x,t)$ and $\mathcal{N}(x,y,z,t) = \mathcal{N}(x,y,t) \lozenge \mathcal{N}(y,z,t) \lozenge \mathcal{N}(z,x,t)$, for all x, y, z ∈ X, then (X, $\mathcal{M},\mathcal{N}$, ∗, ◊) is a generalized intuitionistic fuzzy metric spaces.

Definition 2.5 :

Let (X, $\mathcal{M},\mathcal{N}$, ∗, ◊) be an generalized intuitionistic fuzzy metric space. $\mathcal{M}$ and $\mathcal{N}$ are said to be continuous function on X^3×(0,∞) if $\lim_{n\to\infty} \mathcal{M}(x_n,y_n,z_n,t_n) = \mathcal{M}(x,y,z,t)$ and
$\lim_{n\to\infty} \mathcal{N}(x_n,y_n,z_n,t_n) = \mathcal{N}(x,y,z,t)$ whenever a sequence {(x_n, y_n, z_n, t_n)} in X^3×(0,∞) converges to a point (x, y, z, t) ∈ X^3×(0,∞), that is, $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$, $\lim_{n\to\infty} z_n = z$,
$\lim_{n\to\infty} \mathcal{M}(x,y,z,t_n) = \mathcal{M}(x,y,z,t)$ and $\lim_{n\to\infty} \mathcal{N}(x,y,z,t_n) = \mathcal{N}(x,y,z,t)$.

Lemma 2.6 :

Let (X, $\mathcal{M}$, $\mathcal{N}$, $*$, $\Diamond$) be a generalized intuitionistic fuzzy metric space. Then, $\mathcal{M}$(x, y, z, t) and $\mathcal{N}$(x, y, z, t) are non-decreasing with respect to t, for all x, y, z in X.

Lemma 2.7 :

Let (X, $\mathcal{M}$, $\mathcal{N}$, $*$, $\Diamond$) be an generalized intuitionistic fuzzy metric space. Then, $\mathcal{M}$, $\mathcal{N}$ continuous function on $X^3 \times (0,\infty)$.

Lemma 2.8 :

Let (X, $\mathcal{M}$, $\mathcal{N}$ $*$, $\Diamond$) be an generalized intuitionistic fuzzy metric space. If there exists k >1 such that $\mathcal{M}(x_n, x_n, x_{n+1}, t) \geq \mathcal{M}(x_0, x_0, x_1, k^n t)$ and $\mathcal{N}(x_n, x_n, x_{n+1}, t) \leq \mathcal{N}(x_0, x_0, x_1, k^n t)$ for all $n > 1$, then $\{x_n\}$ is a Cauchy sequence in X.

Definition 2.9 :

We say that generalized intuitionistic fuzzy metric space (X, $\mathcal{M}$, $\mathcal{N}$, $*$, $\Diamond$) has the property (C) if it satisfies the following condition:

For some x, y, z ∈ X, $\mathcal{M}(x, y, z, t) = C \Rightarrow C=1$ and $\mathcal{N}(x, y, z, t) = C \Rightarrow C= 0$, for all t > 0.

3. The Main Results

Theorem 3.1 :

Let (X, $\mathcal{M}$, $\mathcal{N}$, $*$, $\Diamond$) be a complete generalized intuitionistic fuzzy metric space and S, T be two self-mappings of X satisfying the following conditions :

(i) There exists a constant k ∈ (0,1) such that

$$\mathcal{M}(Sx, TSx, Ty, kt) \geq \gamma\big(\mathcal{M}(x, Sx, y, t)\big) \quad (3.1.1)$$

(or)

$$\mathcal{M}(Ty, STy, Sx, kt) \geq \gamma\big(\mathcal{M}(y, Ty, x, t)\big), \text{for all } x, y \in X \quad (3.1.2)$$

$$\mathcal{N}(Sx, TSx, Ty, kt) \leq \varphi\big(\mathcal{N}(x, Sx, y, t)\big) \quad (3.1.3)$$

(or)

$$\mathcal{N}(Ty, STy, Sx, kt) \leq \varphi\big(\mathcal{N}(y, Ty, x, t)\big), \text{for all } x, y \in X. \quad (3.1.4)$$

where γ, φ : [0,1] → [0,1] is a function such that γ (a) ≥ a and φ(a) ≤ a for all a ∈ [0, 1],

(ii) ST = TS.

If (X, $\mathcal{M}$, $\mathcal{N}$, $*$, $\Diamond$) have the property (C), then S and T have a unique common fixed point in X.

Proof .

Let x_0 be an arbitrary point in X, define

$$\begin{cases} x_{2n+1} = T\,x_{2n,} \\ \quad x_{2n+2} = S\,x_{2n+1} \end{cases} \quad \text{for all } n \geq 0 \quad (3.1.5)$$

(1) Let $d_m(t) = \mathcal{M}(x_m, x_{m+1}, x_{m+1}, t)$ and $\delta_m(t) = \mathcal{N}(x_m, x_{m+1}, x_{m+1}, t)$ for any t > 0.

Then for any even m = 2n ∈ N, by (3.1.1), (3.1.3) and (3.1.5), we have

$$\begin{aligned} d_{2n}(kt) &= \mathcal{M}(x_{2n}, x_{2n+1}, x_{2n+1}, kt) \\ &= \mathcal{M}(Sx_{2n-1}, Tx_{2n}, Tx_{2n}, kt) \\ &= \mathcal{M}(Sx_{2n-1}, TSx_{2n-1}, Tx_{2n}, kt) \\ &\geq \gamma(\mathcal{M}(x_{2n-1}, Sx_{2n-1}, x_{2n}, t)) \\ &\geq \mathcal{M}(x_{2n-1}, x_{2n}, x_{2n}, t) \\ &= d_{2n-1}(t). \end{aligned}$$

$$\begin{aligned} \delta_{2n}(kt) &= \mathcal{N}(x_{2n}, x_{2n+1}, x_{2n+1}, kt) \\ &= \mathcal{N}(Sx_{2n-1}, Tx_{2n}, Tx_{2n}, kt) \\ &= \mathcal{N}(Sx_{2n-1}, TSx_{2n-1}, Tx_{2n}, kt) \\ &\leq \varphi(\mathcal{N}(x_{2n-1}, Sx_{2n-1}, x_{2n}, t)) \\ &\leq \mathcal{N}(x_{2n-1}, x_{2n}, x_{2n}, t) \\ &= \delta_{2n-1}(t). \end{aligned}$$

Thus $d_{2n}(kt) \geq d_{2n-1}(t)$ and $\delta_{2n}(kt) \leq \delta_{2n-1}(t)$ for all even m = 2n ∈ N and t > 0.

Similarly, for any odd m = 2n+1 ∈ N, we have also

$d_{2n+1}(kt) \geq d_{2n}(t)$ and $\delta_{2n+1}(kt) \leq \delta_{2n}(t)$.

Hence we have

$d_n(kt) \geq d_{n-1}(t)$ and $\delta_n(kt) \leq \delta_{n-1}(t)$. for all n ≥ 1 (3.1.6)

Thus, we have

$\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t) \geq \mathcal{M}(x_{n-1}, x_n, x_n, \frac{1}{k}t) \geq \ldots \geq \mathcal{M}(x_0, x_1, x_1, \frac{1}{k^n}t)$ and

$\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t) \leq \mathcal{N}(x_{n-1}, x_n, x_n, \frac{1}{k}t) \leq \ldots \leq \mathcal{N}(x_0, x_1, x_1, \frac{1}{k^n}t)$.

Therefore, by lemma (2.8), $\{x_n\}$ is a Cauchy sequence in X and by the completeness of X, $\{x_n\}$ converges to a point x in X and so

$$\lim_{n\to\infty} x_{2n+1} = \lim_{n\to\infty} Tx_{2n} = \lim_{n\to\infty} Sx_{2n+1} = \lim_{n\to\infty} x_{2n+2} = x.$$

Now, we prove that Tx = x.

Put x = x_{2n-1} and y = x, in (i), we obtain

$\mathcal{M}(Sx_{2n-1}, TSx_{2n-1}, Tx, kt) \geq \gamma(\mathcal{M}(x_{2n-1}, Sx_{2n-1}, x, t))$,

$\mathcal{M}(x_{2n}, x_{2n+1}, Tx, kt) \geq \gamma(\mathcal{M}(x_{2n-1}, x_{2n}, x, t))$

$\geq \mathcal{M}(x_{2n-1}, x_{2n}, x, t)$ (3.1.7)

$\mathcal{N}(Sx_{2n-1}, TSx_{2n-1}, Tx, kt) \leq \varphi(\mathcal{N}(x_{2n-1}, Sx_{2n-1}, x, t))$,

$\mathcal{N}(x_{2n}, x_{2n+1}, Tx, kt) \leq \varphi(\mathcal{N}(x_{2n-1}, x_{2n}, x, t))$

$\leq \mathcal{N}(x_{2n-1}, x_{2n}, x, t)$ (3.1.8)

Letting n→∞ in (3.1.7) and (3.1.8), we have

$\mathcal{M}(x, x, Tx, kt) \geq \mathcal{M}(x, x, x, t) = 1$ and $\mathcal{N}(x, x, Tx, kt) \leq \mathcal{N}(x, x, x, t) = 0$

which implies that Tx = x, that is, x is a fixed point of T.

Next, we prove that Sx = x.

Put x = x and y = x_{2n} in (3.1.1) and (3.1.3),we obtain

$\mathcal{M}(Sx, TSx, Tx_{2n}, kt) \geq \gamma(\mathcal{M}(x, Sx, x_{2n}, t) \geq \mathcal{M}(x, Sx, x_{2n}, t)$ and

$\mathcal{N}(Sx, TSx, Tx_{2n}, kt) \leq \varphi(\mathcal{N}(x, Sx, x_{2n}, t) \leq \mathcal{N}(x, Sx, x_{2n}, t)$.

By (ii), since TS = ST, we get

$\mathcal{M}(Sx, Sx, Tx_{2n}, kt) \geq \gamma(\mathcal{M}(x, Sx, x_{2n}, t) \geq \mathcal{M}(x, Sx, x_{2n}, t)$ and

$\mathcal{N}(Sx, Sx, Tx_{2n}, kt) \leq \varphi(\mathcal{N}(x, Sx, x_{2n}, t) \leq \mathcal{N}(x, Sx, x_{2n}, t)$. (3.1.9)

Letting n→∞ in (3.1.9), we have

$\mathcal{M}(Sx, Sx, x, kt) \geq \mathcal{M}(x, Sx, x, t)$, $\mathcal{N}(Sx, Sx, x, kt) \leq \mathcal{N}(x, Sx, x, t)$ and hence

$\mathcal{M}(x, Sx, x, t) \geq \mathcal{M}(x, Sx, x, \frac{1}{k}t) \geq \mathcal{M}(x, Sx, x, \frac{1}{k^2}t) \ldots \geq \mathcal{M}(x, Sx, x, \frac{1}{k^n}t)$ and

$\mathcal{N}(x, Sx, x, t) \leq \mathcal{N}\left(x, Sx, x, \frac{1}{k}t\right) \leq \mathcal{N}(x, Sx, x, \frac{1}{k^2}t) \ldots \leq \mathcal{N}(x, Sx, x, \frac{1}{k^n}t)$.

On the other hand, that

$\mathcal{M}(x, Sx, x, k^n t) \geq \mathcal{M}(x, Sx, x, t)$ and $\mathcal{N}(x, Sx, x, k^n t) \leq \mathcal{N}(x, Sx, x, t)$.

Hence $\mathcal{M}(x, Sx, x, t) = C$ and for all t > 0. Since $(X, \mathcal{M}, \mathcal{N} *, \lozenge)$ has the property (C), it follows that C = 1 and so Sx = x, $\mathcal{N}(x, Sx, x, t) = C \Rightarrow C = 0$ and so Sx = x, that is, x is a fixed point of S. Therefore, x is a common fixed point of the self-mappings S and T.

(2) By using (3.1.2), (3.1.4) and (3.1.5). Let $d_m(t) = \mathcal{M}(x_{m+1}, x_m, x_m, t)$ and $\delta_m(t) = \mathcal{N}(x_{m+1}, x_m, x_m, t)$ for any t > 0. Then, for any even m = 2n ∈ N, we have

$$\begin{aligned} d_{2n}(kt) &= \mathcal{M}(x_{2n+1}, x_{2n}, x_{2n}, kt) \\ &= \mathcal{M}(Tx_{2n}, Sx_{2n-1}, Sx_{2n-1}, kt) \\ &= \mathcal{M}(Tx_{2n}, STx_{2n-2}, Sx_{2n-1}, kt) \\ &\geq \gamma(\mathcal{M}(x_{2n}, Tx_{2n-2}, x_{2n-1}, t)) \end{aligned}$$

$\geq \mathcal{M}(x_{2n}, Tx_{2n-2}, x_{2n-1}, t)$
$\geq \mathcal{M}(x_{2n}, x_{2n-1}, x_{2n-1}, t) = d_{2n-1}(t)$.

$\delta_{2n}(kt) = \mathcal{N}(x_{2n+1}, x_{2n}, x_{2n}, kt)$
$= \mathcal{N}(Tx_{2n}, Sx_{2n-1}, Sx_{2n-1}, kt)$
$= \mathcal{N}(Tx_{2n}, STx_{2n-2}, Sx_{2n-1}, kt)$
$\leq \phi(\mathcal{N}(x_{2n}, Tx_{2n-2}, x_{2n-1}, t))$
$\leq \mathcal{N}(x_{2n}, Tx_{2n-2}, x_{2n-1}, t)$
$\leq \mathcal{N}(x_{2n}, x_{2n-1}, x_{2n-1}, t)$
$= \delta_{2n-1}(t)$.

Thus $d_{2n}(kt) \geq d_{2n-1}(t)$ and $\delta_{2n}(kt) \leq \delta_{2n-1}(t)$ for all even m = 2n ∈ N and t > 0.
Similarly, for any odd m = 2n +1 ∈ N,
we have also $d_{2n+1}(kt) \geq d_{2n}(t)$ and $\delta_{2n+1}(kt) \leq \delta_{2n}(t)$ for all n ≥ 1.
The remains of the proof are almost same to the case of (3.1.1) and (3.1.3).

Uniqueness: Let x' be another common fixed point of S and T. Then we have
$\mathcal{M}(x, x, x', kt) = \mathcal{M}(Sx, TSx, Tx', kt) \geq \gamma(\mathcal{M}(x, Sx, x', t)) \geq \mathcal{M}(x, x, x', t)$ and
$\mathcal{N}(x, x, x', kt) = \mathcal{N}(Sx, TSx, Tx', kt) \leq \phi\,(\mathcal{N}(x, Sx, x', t)) \leq \mathcal{N}(x, x, x', t)$,
which implies that
$\mathcal{M}(x, x, x', t) \geq \mathcal{M}(x, x, x', \frac{1}{k}t) \geq \mathcal{M}(x, x, x', \frac{1}{k^2}t) \ldots \geq \mathcal{M}\left(x, x, x', \frac{1}{k^n}t\right)$ and
$\mathcal{N}(x, x, x', t) \leq \mathcal{N}(x, x, x', \frac{1}{k}t) \leq \mathcal{N}(x, x, x', \frac{1}{k^2}t) \ldots \leq \mathcal{N}(x, x, x', \frac{1}{k^n}t)$
On the other hand, it follows from lemma (2.6) that
$\mathcal{M}(x, x, x', t) \geq \mathcal{M}(x, x, x', \frac{1}{k^n}t)$ and $\mathcal{N}(x, x, x', t) \leq \mathcal{N}(x, x, x', \frac{1}{k^n}t)$.
Since (X, $\mathcal{M}, \mathcal{N}, *, \lozenge$) has the property (C),
$\mathcal{M}(x, x, x', t) = C \Rightarrow C = 1$ and $\mathcal{N}(x, x, x', t) = C \Rightarrow C = 0$, that is $x = x'$.
Therefore, x is a unique common fixed point of S and T.

Corollary 3.2 :

Let (X, $\mathcal{M}, \mathcal{N}, *, \lozenge$) be a complete generalized intuitionistic fuzzy metric space. Let T be a mapping from X into itself such that there exists a constant k ∈ (0, 1) such that
$\mathcal{M}(Tx, T^2x, Ty, kt) \geq \mathcal{M}(x, Tx, y, t)$ and $\mathcal{N}(Tx, T^2x, Ty, kt) \leq \mathcal{N}(x, Tx, y, t)$, for all x, y ∈ X. If (X, $\mathcal{M}, \mathcal{N}, *, \lozenge$) have the property (C), then T have a unique fixed point in X.

Proof :

By Theorem (3.1), if we set $\gamma(a) = a$, $\varphi(a) = a$ and S = T, then the conclusion follows

Corollary 3.3 :

Let (X, $\mathcal{M}, \mathcal{N}, *, \lozenge$) be a complete generalized intuitionistic fuzzy metric space. Let T be a mapping from X into itself such that there exists a constant k ∈ (0, 1) such that $\mathcal{M}(T^n x, T^{2n}x, T^n y, kt) \geq \mathcal{M}(x, T^n x, y, t)$ and $\mathcal{N}(T^n x, T^{2n}x, T^n y, kt) \leq \mathcal{N}(x, T^n x, y, t)$ for all x, y ∈ X and n ≥ 2. If (X, $\mathcal{M}, \mathcal{N}, *, \lozenge$) has the property (C), then T have a unique fixed point in X.

Proof :

By corollary (3.2), T^n have a unique fixed point in X. Thus there exists x ∈ X such that $T^n x = x$. Since $T^{n+1}x = T^n(Tx) = T(T^n x) = Tx$, We have Tx = x.
Next, by using Lemma (2.8) and the property (C), we can prove the main results in this paper.

Theorem 3.4 :

Let (X, $\mathcal{M}, \mathcal{N}, *, \lozenge$) be a complete generalized intuitionistic fuzzy metric space with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq 1-t$ for all t ∈ [0,1]. Let S and T be mappings from X into itself such that there exists a constant k ∈ (0, 1) such that

$$\mathcal{M}(\mathrm{Sx},\mathrm{Ty},\mathrm{Ty},\mathrm{kt}) \geq \left\{\begin{array}{c} \mathrm{a(t)}\mathcal{M}(\mathrm{x},\mathrm{Sx},\mathrm{Sx},\mathrm{t}) + \mathrm{b(t)}\mathcal{M}(\mathrm{y},\mathrm{Ty},\mathrm{Ty},\mathrm{t}) + \\ \mathrm{c(t)}\mathcal{M}(\mathrm{x},\mathrm{Ty},\mathrm{Ty},\alpha\mathrm{t}) + \mathrm{d(t)}\mathcal{M}(\mathrm{y},\mathrm{Sx},\mathrm{Sx},(2-\alpha)\mathrm{t}) + \\ \mathrm{e(t)}\,\mathcal{M}(x,y,y,t) \end{array}\right\}$$

$$\mathcal{N}(\mathrm{Sx},\mathrm{Ty},\mathrm{Ty},\mathrm{kt}) \leq \left\{\begin{array}{c} \mathrm{a(t)}\mathcal{N}(\mathrm{x},\mathrm{Sx},\mathrm{Sx},\mathrm{t}) + \mathrm{b(t)}\mathcal{N}(\mathrm{y},\mathrm{Ty},\mathrm{Ty},\mathrm{t}) + \\ \mathrm{c(t)}\mathcal{N}(\mathrm{x},\mathrm{Ty},\mathrm{Ty},\alpha\mathrm{t}) + \mathrm{d(t)}\mathcal{N}(\mathrm{y},\mathrm{Sx},\mathrm{Sx},(2-\alpha)\mathrm{t}) + \\ \mathrm{e(t)}\,\mathcal{N}(x,y,y,t) \end{array}\right\} \qquad (3.4.1)$$

for all $x, y \in X$ and $\alpha \in (0, 2)$, where a, b, c, d, e : $[0, \infty) \to [0, 1]$ are five functions such that $a(t) + b(t) + c(t) + d(t) + e(t) = 1$ for all $t \in [0, \infty)$ Then S and T have a unique common fixed point in X.

Proof :

Let $x_0 \in X$ be an arbitrary point. Then there exist $x_1, x_2 \in X$, such that $x_1 = Sx_0$ and $x_2 = Tx_1$. Inductively, we can construct a sequence $\{x_n\}$ in X such that

$$\begin{cases} x_{2n+1} = Sx_{2n}, \\ x_{2n+2} = Tx_{2n+1}, \end{cases} \text{for all } n \geq 0 \qquad (3.4.2)$$

Now, we show that $\{x_n\}$ is a Cauchy sequence in X. If we get

$d_m(t) = \mathcal{M}(x_m, x_{m+1}, x_{m+1}, t)$ and $\delta_m(t) = \mathcal{N}(x_m, x_{m+1}, x_{m+1}, t)$, for all $t > 0$ (3.4.3)

then we prove that $\{d_m(t)\}$ and $\{\delta_m(t)\}$ are increasing with respect to $m \in N$.

In fact, for any odd $m = 2n+1 \in N$, we have

$$\begin{aligned} d_{2n+1}(kt) &= \mathcal{M}(x_{2n+1}, x_{2n+2}, x_{2n+2}, kt) \\ &= \mathcal{M}(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}, kt) \\ &\geq \{ a(t)\mathcal{M}(x_{2n}, Sx_{2n}, Sx_{2n}, t) + b(t)\,\mathcal{M}(x_{2n+1}, Tx_{2n+1}, Tx_{2n+1}, t) + \\ &\quad c(t)\,\mathcal{M}(x_{2n}, Tx_{2n+1}, Tx_{2n+1}, \alpha t) + d(t)\,\mathcal{M}(x_{2n+1}, Sx_{2n}, Sx_{2n}, (2-\alpha)t) + \\ &\quad e(t)\,\mathcal{M}(x_{2n}, x_{2n+1}, x_{2n+1}, t) \} \\ &= \{ a(t)\mathcal{M}(x_{2n}, x_{2n+1}, x_{2n+1}, t) + b(t)\,\mathcal{M}(x_{2n+1}, x_{2n+2}, x_{2n+2}, t) + \\ &\quad c(t)\,\mathcal{M}(x_{2n}, x_{2n+2}, x_{2n+2}, \alpha t) + d(t)\,\mathcal{M}(x_{2n+1}, x_{2n+1}, x_{2n+1}, (2-\alpha)t) + \\ &\quad e(t)\,\mathcal{M}(x_{2n}, x_{2n+1}, x_{2n+1}, t) \} \text{ and so} \end{aligned}$$

$$d_{2n+1}(t) \geq a(t)d_{2n}(t) + b(t)d_{2n+1}(t) + c(t)d_{2n}(t) * d_{2n+1}(qt) + d(t) + e(t)d_{2n}(t) \qquad (3.4.4)$$

$$\begin{aligned} \delta_{2n+1}(kt) &= \mathcal{N}(x_{2n+1}, x_{2n+2}, x_{2n+2}, kt) \\ &= \mathcal{N}(Sx_{2n}, Tx_{2n+1}, Tx_{2n+1}, kt) \\ &\leq \{ a(t)\,\mathcal{N}(x_{2n}, Sx_{2n}, Sx_{2n}, t) + b(t)\,\mathcal{N}(x_{2n+1}, Tx_{2n+1}, Tx_{2n+1}, t) + \\ &\quad c(t)\,\mathcal{N}(x_{2n}, Tx_{2n+1}, Tx_{2n+1}, \alpha t) + d(t)\,\mathcal{N}(x_{2n+1}, Sx_{2n}, Sx_{2n}, (2-\alpha)t) + \\ &\quad e(t)\,\mathcal{N}(x_{2n}, x_{2n+1}, x_{2n+1}, t) \}. \\ &= \{ a(t)\,\mathcal{N}(x_{2n}, x_{2n+1}, x_{2n+1}, t) + b(t)\,\mathcal{N}(x_{2n+1}, x_{2n+2}, x_{2n+2}, t) + \\ &\quad c(t)\,\mathcal{N}(x_{2n}, x_{2n+2}, x_{2n+2}, \alpha t) + d(t)\,\mathcal{N}(x_{2n+1}, x_{2n+1}, x_{2n+1}, (2-\alpha)t) + \\ &\quad e(t)\,\mathcal{N}(x_{2n}, x_{2n+1}, x_{2n+1}, t) \} \text{ and so} \end{aligned}$$

$$\delta_{2n+1}(kt) \leq \{ a(t)d_{2n}(t) + b(t)d_{2n+1}(t) + c(t)d_{2n}(t) * d_{2n+1}(qt) + d(t) + e(t)d_{2n}(t) \qquad (3.4.5)$$

The equality in (3.4.4), (3.4.5) are true because, if get $\alpha = 1+ q$ for any $q \in (k,1)$, then

$$\begin{aligned} \mathcal{M}(x_{2n}, x_{2n+2}, x_{2n+2}, (1+q)t) &= \mathcal{M}(x_{2n}, x_{2n}, x_{2n+2}, (1+q)t) \\ &\geq \mathcal{M}(x_{2n}, x_{2n}, x_{2n+1}, t) * \mathcal{M}(x_{2n+1}, x_{2n+2}, x_{2n+2}, qt) \\ &= d_{2n}(t) * d_{2n+1}(qt) \text{ and} \end{aligned}$$

$$\begin{aligned} \mathcal{N}(x_{2n}, x_{2n+2}, x_{2n+2}, (1+q)t) &= \mathcal{N}(x_{2n}, x_{2n}, x_{2n+2}, (1+q)t) \\ &\leq \mathcal{N}(x_{2n}, x_{2n}, x_{2n+1}, t) * \mathcal{N}(x_{2n+1}, x_{2n+2}, x_{2n+2}, qt) \\ &= \delta_{2n}(t) * \delta_{2n+1}(qt). \end{aligned}$$

Now, we claim that

$d_{2n+1}(t) \geq d_{2n}(t)$ and $\delta_{2n+1}(t) \leq \delta_{2n}(t)$, for all $n \geq 1$

Infact, if $d_{2n+1}(t) < d_{2n}(t)$ and $\delta_{2n+1}(t) > \delta_{2n}(t)$ then, since

$d_{2n+1}(qt) * d_{2n}(t) \geq d_{2n+1}(qt) * d_{2n+1}(qt) = d_{2n+1}(qt)$ and

$\delta_{2n+1}(qt) * \delta_{2n}(t) \leq \delta_{2n+1}(qt) * \delta_{2n+1}(qt) = \delta_{2n+1}(qt)$, in (3.4.4)and (3.4.5), we have

$d_{2n+1}(kt) > a(t)\, d_{2n+1}(qt) + b(t)\, d_{2n+1}(qt) + c(t)d_{2n+1}(qt) + d(t)d_{2n+1}(qt) + e(t)d_{2n+1}(qt) = d_{2n+1}(qt)$

$\delta_{2n+1}(kt) < a(t)\,\delta_{2n+1}(qt) + b(t)\,\delta_{2n+1}(qt)+ c(t)\,\delta_{2n+1}(qt)+ d(t)\,\delta_{2n+1}(qt)+ e(t)\,\delta_{2n+1}(qt) = \delta_{2n+1}(qt)$

and so $d_{2n+1}(kt) > d_{2n+1}(qt)$ and $\delta_{2n+1}(kt) < \delta_{2n+1}(qt)$, which is contradiction

Hence $d_{2n+1}(t) \geq d_{2n}(t)$ and $\delta_{2n+1}(t) \leq \delta_{2n}(t)$ for all n ∈ N and t > 0.

By (3.4.4) and (3.4.5), we have

$d_{2n+1}(kt) \geq a(t)\, d_{2n}(qt) + b(t)\, d_{2n}(qt) + c(t)d_{2n}(qt) + d(t)d_{2n}(qt) + e(t)\, d_{2n}(qt) = d_{2n}(qt)$

$\delta_{2n+1}(kt) \leq a(t)\,\delta_{2n}(qt) + b(t)\,\delta_{2n}(qt)+ c(t)\,\delta_{2n}(qt)+ \delta(t)\,\delta_{2n}(qt)+ e(t)\,\delta_{2n}(qt) = \delta_{2n}(qt)$

Now, if m = 2n, then by (3.4.3) we have

$$\begin{aligned}
d_{2n}(kt) &= \mathcal{M}(x_{2n}, x_{2n+1}, x_{2n+1}, kt)\\
&= \mathcal{M}(Sx_{2n-1}, Tx_{2n}, Tx_{2n}, kt)\\
&\geq \{ a(t)\mathcal{M}(x_{2n-1}, Sx_{2n-1}, Sx_{2n-1}, t) + b(t)\mathcal{M}(x_{2n}, Tx_{2n}, Tx_{2n}, t) +\\
&\quad c(t)\mathcal{M}(x_{2n-1}, Tx_{2n}, Tx_{2n}, \alpha t) + d(t)\mathcal{M}(x_{2n}, Sx_{2n-1}, Sx_{2n-1}, (2-\alpha)t) +\\
&\quad e(t)\mathcal{M}(x_{2n-1}, x_{2n}, x_{2n}, t)\}\\
&= \{a(t)\mathcal{M}(x_{2n-1}, x_{2n}, x_{2n}, t) + b(t)\mathcal{M}(x_{2n}, x_{2n+1}, x_{2n+1}, t) +\\
&\quad c(t)\mathcal{M}(x_{2n-1}, x_{2n+1}, x_{2n+1}, \alpha t) + d(t)\mathcal{M}(x_{2n}, x_{2n}, x_{2n}, (2-\alpha)t) +\\
&\quad e(t)\mathcal{M}(x_{2n-1}, x_{2n}, x_{2n}, t)\} \text{ and so,}
\end{aligned}$$

$$d_{2n}(kt) \geq a(t)d_{2n-1}(t)+ b(t)d_{2n}(t)+ c(t)\, d_{2n-1}(t) * d_{2n}(qt)+ d(t) + e(t)\, d_{2n-1}(t) \qquad (3.4.6)$$

$$\begin{aligned}
\delta_{2n}(kt) &= \mathcal{N}(x_{2n}, x_{2n+1}, x_{2n+1}, kt)\\
&= \mathcal{N}(Sx_{2n-1}, Tx_{2n}, Tx_{2n}, kt)\\
&\leq a(t)\,\mathcal{N}(x_{2n-1}, Sx_{2n-1}, Sx_{2n-1}, t)+b(t)\,\mathcal{N}(x_{2n}, Tx_{2n}, Tx_{2n}, t)+\\
&\quad c(t)\,\mathcal{N}(x_{2n-1}, Tx_{2n}, Tx_{2n}, \alpha t)+d(t)\,\mathcal{N}(x_{2n}, Sx_{2n-1}, Sx_{2n-1}, (2-\alpha)t)+\\
&\quad e(t)\,\mathcal{N}(x_{2n-1}, x_{2n}, x_{2n}, t)\\
&= a(t)\,\mathcal{N}(x_{2n-1}, x_{2n}, x_{2n}, t)+ b(t)\,\mathcal{N}(x_{2n}, x_{2n+1}, x_{2n+1}, t) +\\
&\quad c(t)\,\mathcal{N}(x_{2n-1}, x_{2n+1}, x_{2n+1}, \alpha t)+d(t)\,\mathcal{N}(x_{2n}, x_{2n}, x_{2n}, (2-\alpha)t) +\\
&\quad e(t)\,\mathcal{N}(x_{2n-1}, x_{2n}, x_{2n}, t) \text{ and so,}
\end{aligned}$$

$$\delta_{2n}(kt) \leq a(t)\,\delta_{2n-1}(t) + b(t)\,\delta_{2n}(t) + c(t)\,\delta_{2n-1}(t) * \delta_{2n}(qt) + d(t) + e(t)\,\delta_{2n-1}(t) \qquad (3.4.7)$$

The equality in (3.4.6) and (3.4.7) are true because if α =1+q for any q ∈ (k,1), then

$$\begin{aligned}
\mathcal{M}(x_{2n-1}, x_{2n+1}, x_{2n+1}, (1+q)t) &= \mathcal{M}(x_{2n-1}, x_{2n-1}, x_{2n+1}, (1+q)t)\\
&\geq \mathcal{M}(x_{2n-1}, x_{2n-1}, x_{2n}, t) * \mathcal{M}(x_{2n}, x_{2n+1}, x_{2n+1}, qt)\\
&= d_{2n-1}(t) * d_{2n}(qt).
\end{aligned}$$

$$\begin{aligned}
\mathcal{N}(x_{2n-1}, x_{2n+1}, x_{2n+1}, (1+q)t) &= \mathcal{N}(x_{2n-1}, x_{2n-1}, x_{2n+1}, (1+q)t)\\
&\leq \mathcal{N}(x_{2n-1}, x_{2n-1}, x_{2n}, t) * \mathcal{N}(x_{2n}, x_{2n+1}, x_{2n+1}, qt)\\
&= \delta_{2n-1}(t) * \delta_{2n}(qt).
\end{aligned}$$

Now, we also claim that

$d_{2n}(t) \geq d_{2n-1}(t)$ and $\delta_{2n}(t) \leq \delta_{2n-1}(t)$, for all n ≥ 1.

In fact, if $d_{2n}(t) < d_{2n-1}(t)$ and $\delta_{2n}(t) < \delta_{2n-1}(t)$ then, since

$d_{2n}(qt) * d_{2n-1}(t) \geq d_{2n}(qt) * d_{2n}(qt) = d_{2n}(qt)$ and

$\delta_{2n}(qt) * \delta_{2n-1}(t) \leq \delta_{2n}(qt) * \delta_{2n}(qt) = \delta_{2n}(qt)$, in (3.4.6), (3.4.7), we have

$d_{2n}(kt) > a(t)d_{2n}(qt) + b(t)d_{2n}(qt) + c(t)\, d_{2n}(qt) + d(t)\, d_{2n}(qt) + e(t)\, d_{2n}(qt) = d_{2n}(qt)$

$\delta_{2n}(kt) < a(t)\,\delta_{2n}(qt)+ b(t)\,\delta_{2n}(qt)+ c(t)\,\delta_{2n}(qt)+ d(t)\,\delta_{2n}(qt)+ e(t)\,\delta_{2n}(qt) = \delta_{2n}(qt)$

and so $d_{2n}(kt) > d_{2n}(qt)$ and $\delta_{2n}(kt) < \delta_{2n}(qt)$, which is a contradiction.

Hence $d_{2n}(t) \geq d_{2n-1}(t)$ and $\delta_{2n}(t) \leq \delta_{2n-1}(t)$ for all n ∈ N and t > 0. By (3.4.6), (3.4.7), we have

$$\begin{aligned}
d_{2n}(kt) &\geq a(t)d_{2n-1}(qt) + b(t)d_{2n-1}(qt) + c(t)\, d_{2n-1}(qt) * d_{2n-1}(qt) + d(t)\, d_{2n-1}(qt) + e(t)\, d_{2n-1}(qt)\\
&= d_{2n-1}(qt)
\end{aligned}$$

$$\begin{aligned}
\delta_{2n}(kt) &\leq a(t)\,\delta_{2n-1}(qt)+ b(t)\,\delta_{2n-1}(qt) + c(t)\,\delta_{2n-1}(qt) * \delta_{2n-1}(qt) + d(t)\,\delta_{2n-1}(qt)+ e(t)\,\delta_{2n-1}(qt)\\
&= \delta_{2n-1}(qt)
\end{aligned}$$

and so $d_{2n}(kt) \geq d_{2n-1}(qt)$ and $\delta_{2n}(kt) \leq \delta_{2n-1}(qt)$

Thus we have $d_n(kt) \geq d_{n-1}(qt)$ and $\delta_n(kt) \leq \delta_{n-1}(qt)$, for all n ≥1. Therefore, it follows that
$\mathcal{M}(x_n, x_{n+1}, x_{n+1}, t) \geq \mathcal{M}\left(x_{n-1}, x_n, x_n, \frac{q}{k}t\right) \geq \ldots \geq \mathcal{M}(x_0, x_1, x_1, \left(\frac{q}{k}\right)^n t)$ and
$\mathcal{N}(x_n, x_{n+1}, x_{n+1}, t) \leq \mathcal{N}\left(x_{n-1}, x_n, x_n, \frac{q}{k}t\right) \leq \ldots \leq \mathcal{N}(x_0, x_1, x_1, \left(\frac{q}{k}\right)^n t)$.
Hence, by Lemma (2.8), $\{x_n\}$ is a Cauchy sequence in X and by the completeness of X.
$\{x_n\}$ converges to a point x ∈ X and $\lim_{n\to\infty} x_{2n+1} = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} x_{2n+2} = x$.
Now, we prove that Sx = x.
Letting α = 1, x = x and y = x_{2n+1} in (3.4.1),(3.4.2) respectively, we obtain

$$\mathcal{M}(Sx, Tx_{2n+1}, Tx_{2n+1}, kt) \geq a(t)\, \mathcal{M}(x, Sx, Sx, t) + b(t)\, \mathcal{M}(x_{2n+1}, Tx_{2n+1}, Tx_{2n+1}, t) + c(t)\, \mathcal{M}(x, Tx_{2n+1}, Tx_{2n+1}, t) + d(t)\, \mathcal{M}(x_{2n+1}, Sx, Sx, t) + e(t)\, \mathcal{M}(x, x_{2n+1}, x_{2n+1}, t) \qquad (3.4.8)$$

$$\mathcal{N}(Sx, Tx_{2n+1}, Tx_{2n+1}, kt) \leq a(t)\, \mathcal{N}(x, Sx, Sx, t) + b(t)\, \mathcal{N}(x_{2n+1}, Tx_{2n+1}, Tx_{2n+1}, t) + c(t)\, \mathcal{N}(x, Tx_{2n+1}, Tx_{2n+1}, t) + d(t)\, \mathcal{N}(x_{2n+1}, Sx, Sx, t) + e(t)\, \mathcal{N}(x, x_{2n+1}, x_{2n+1}, t) \qquad (3.4.9)$$

If Sx ≠ x, then letting n→∞ in (3.4.8),(3.4.9), we have

$$\mathcal{M}(Sx, x, x, kt) \geq a(t)\, \mathcal{M}(x, Sx, Sx, t) + b(t)\, \mathcal{M}(x, x, x, t) + c(t)\, \mathcal{M}((x, x, x, t) + d(t)\, \mathcal{M}(x, Sx, Sx, t) + e(t)\, \mathcal{M}(x, x, x, t) > \mathcal{M}(x, x, Sx, t)$$ and

$$\mathcal{N}(Sx, x, x, kt) \leq a(t)\, \mathcal{N}(x, Sx, Sx, t) + b(t)\, \mathcal{N}(x, x, x, t) + c(t)\, \mathcal{N}((x, x, x, t) + d(t)\, \mathcal{N}(x, Sx, Sx, t) + e(t)\, \mathcal{N}(x, x, x, t) < \mathcal{N}(x, x, Sx, t),$$ which is a contradiction.

Thus it follows that Sx = x.
Similarly, we can prove that Tx = x. In fact, again, put x = x_{2n} and y = x in (3.4.1) and (3.4.2) respectively, for $\alpha = 1$, we have

$$\mathcal{M}(Sx_{2n}, Tx, Tx, kt) \geq a(t)\mathcal{M}(x_{2n}, Sx_{2n}, Sx_{2n}, t) + b(t)\, \mathcal{M}(x, Tx, Tx, t) + c(t)\, \mathcal{M}(x_{2n}, Tx, Tx, t) + d(t)\, \mathcal{M}(x, Sx_{2n}, Sx_{2n}, t) + e(t)\, \mathcal{M}((x_{2n}, x, x, t) \qquad (3.4.10)$$

$$\mathcal{N}(Sx_{2n}, Tx, Tx, kt) \leq a(t)\mathcal{N}(x_{2n}, Sx_{2n}, Sx_{2n}, t) + b(t)\, \mathcal{M}(x, Tx, Tx, t) + c(t)\, \mathcal{N}(x_{2n}, Tx, Tx, t) + d(t)\, \mathcal{N}(x, Sx_{2n}, Sx_{2n}, t) + e(t)\, \mathcal{N}((x_{2n}, x, x, t) \qquad (3.4.11)$$

and so, if Tx ≠ x, letting n → ∞ in (3.4.10), (3.4.11), we have

$$\mathcal{M}(x, Tx, Tx, kt) \geq a(t)\, \mathcal{M}(x, x, x, t) + b(t)\, \mathcal{M}(x, Tx, Tx, t) + c(t)\, \mathcal{M}(x, Tx, Tx, t) + d(t)\, \mathcal{M}(x, x, x, t) + e(t)\, \mathcal{M}(x, x, x, t) > \mathcal{M}(x, Tx, Tx, t)$$ and

$$\mathcal{N}(x, Tx, Tx, kt) \leq a(t)\, \mathcal{N}(x, x, x, t) + b(t)\, \mathcal{N}(x, Tx, Tx, t) + c(t)\, \mathcal{N}(x, Tx, Tx, t) + d(t)\, \mathcal{N}(x, x, x, t) + e(t)\, \mathcal{N}(x, x, x, t) < N(x, Tx, Tx, t),$$ which implies that Tx = x.

Therefore Sx = Tx = x and x is a common fixed point of the self-mappings S and T of X.
Uniqueness: If x' be another fixed point of S and T.
Then, for α = 1, by (3.4.1)and (3.4.2), we have

$$\mathcal{M}(x, x', x', kt) = \mathcal{M}(Sx, Tx', Tx', kt) \geq a(t)\, \mathcal{M}(x, Sx, Sx, t) + b(t)\, \mathcal{M}(x', Tx', Tx', t) + c(t)\, \mathcal{M}(x, Tx', Tx', t) + d(t)\, \mathcal{M}(x', Sx, Sx, t) + e(t)\, \mathcal{M}(x, x', x', t) > \mathcal{M}(x, x', x', t)$$

$\mathcal{N}(x, x', x', kt) = \mathcal{N}(Sx, Tx', Tx', kt)$
$\leq a(t)\, \mathcal{N}(x, Sx, Sx, t) + b(t)\, \mathcal{N}(x', Tx', Tx', t) + c(t)\, \mathcal{N}(x, Tx', Tx', t) +$
$d(t)\, \mathcal{N}(x', Sx, Sx, t) + e(t)\, \mathcal{N}(x, x', x', t)$
$< \mathcal{N}(x, x', x', t)$

and so $x = x'$.

Example 3.5 :

Let (X, $\mathcal{M}, \mathcal{N}$, $*$, $\Diamond$) be an generalized intuitionistic fuzzy metric space, where X = [0, 1] with t-norm defined a $*$ b = min{a, b} and t-conorm defined a $\Diamond$ b = max {a, b} for all a, b ∈ [0, 1] , $\mathcal{M}(x, y, z, t) = \frac{t}{t+|x-y|+|y-z|+|x-z|}$ and $\mathcal{N}(x, y, z, t) = \frac{|x-y|+|y-z|+|z-x|}{t+|x-y|+|y-z|+|z-x|}$ for all t > 0, x, y, z ∈ X. Define the self-mappings T and S on X as follows:

$Tx = 1,\ Sx = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$

We can find the functions a, b, c, d, e : [0, ∞) →[0, 1] such that a(t) + b(t) + c(t) + d(t) + e(t) = 1 and the following inequality holds:

$\mathcal{M}(Sx, Ty, Ty, kt) \geq a(t)\, \mathcal{M}(x, Sx, Sx, t) + b(t)\, \mathcal{M}(y, Ty, Ty, t) + c(t)\, \mathcal{M}(x, Ty, Ty, \alpha t) +$
$d(t)\, \mathcal{M}(y, Sx, Sx, (2-\alpha)t) + e(t)\mathcal{M}(x, y, y, t)$ and
$\mathcal{N}(Sx, Ty, Ty, kt) \leq a(t)\, \mathcal{N}(x, Sx, Sx, t) + b(t)\, \mathcal{N}(y, Ty, Ty, t) + c(t)\, \mathcal{N}(x, Ty, Ty, \alpha t) +$
$d(t)\, \mathcal{N}(y, Sx, Sx, (2-\alpha)t) + e(t)\, \mathcal{N}(x, y, y, t)$.

It is easy to see that the all the conditions of theorem (3.4) hold and is a unique common fixed point of S and T.

Corollary 3.6 :

Let (X, $\mathcal{M}, \mathcal{N}$, $*$, $\Diamond$) be a complete generalized intuitionistic fuzzy metric space with $t * t \geq t$ and (1-t) $\Diamond$ (1-t) ≤ 1-t for all t ∈ [0, 1]. Let S be a mapping from X into itself such that there exists k ∈ (0, 1) such that

$\mathcal{M}(Sx, Sy, Sy, kt) \geq a(t)\, \mathcal{M}(x, Sx, Sx, t) + b(t)\, \mathcal{M}(y, Sy, Sy, t) + c(t)\, \mathcal{M}(x, Sy, Sy, \alpha t) +$
$d(t)\, \mathcal{M}(y, Sx, Sx, (2-\alpha)t) + e(t)\, \mathcal{M}(x, y, y, t)$
$\mathcal{N}(Sx, Sy, Sy, kt) \leq a(t)\, \mathcal{N}(x, Sx, Sx, t) + b(t)\, \mathcal{N}(y, Sy, Sy, t) + c(t)\, \mathcal{N}(x, Sy, Sy, \alpha t) +$
$d(t)\, \mathcal{N}(y, Sx, Sx, (2-\alpha)t) + e(t)\, \mathcal{N}(x, y, y, t)$

for all x, y ∈ X and α = (0, 2), where a, b, c, d, e : [0, ∞) →[0, 1] are five functions such that a(t) + b(t) + c(t) + d(t)+e(t) = 1 for all t ∈ [0, ∞]. Then S have a unique common fixed point in X.

Corollary 3.7 :

Let (X, $\mathcal{M}, \mathcal{N}$, $*$, $\Diamond$) be a complete generalized intuitionistic fuzzy metric space with $t * t \geq t$ and (1-t) $\Diamond$ (1-t) ≤ 1-t for all t ∈ [0, 1]. Let S be a mapping from X into itself such that there exists k ∈ (0, 1) such that

$\mathcal{M}(Sx, y, y, kt) \geq a(t)\, \mathcal{M}(x, Sx, Sx, t) + b(t)\, \mathcal{M}(x, y, y, \alpha t) + c(t)\, \mathcal{M}(y, Sx, Sx, (2-\alpha)t) +$
$d(t)\, \mathcal{M}(x, y, y, t)$
$\mathcal{N}(Sx, y, y, kt) \leq a(t)\, \mathcal{N}(x, Sx, Sx, t) + b(t)\, \mathcal{N}(x, y, y, \alpha t) + c(t)\, \mathcal{N}(y, Sx, Sx, (2-\alpha)t) +$
$d(t)\, \mathcal{N}(x, y, y, t)$

for all x, y ∈ X, α ∈ (0,2) and where a, b, c, d, e : [0,∞)→[0, 1] are five functions such that a(t) + b(t) + c(t) + d(t) =1 for all t ∈ [0, ∞). Then S have a unique common fixed point in X.

Corollary 3.8 :

Let (X, $\mathcal{M}, \mathcal{N}$, $*$, $\Diamond$) be a complete generalized intuitionistic fuzzy metric space with $t * t \geq t$ and (1-t) $\Diamond$ (1-t) ≤ 1-t for all t ∈ [0, 1]. Let S and T be mappings from X into itself such that there exists k ∈ (0, 1) such that

$\mathcal{M}(S^n x, T^m y, T^m y, kt) \geq a(t)\,\mathcal{M}(x, S^n x, S^n x, t) + b(t)\,\mathcal{M}(y, T^m y, T^m y, t) + c(t)\,\mathcal{M}(x, T^m y, T^m y, \alpha t) + d(t)\,\mathcal{M}(y, S^n x, S^n x, (2-\alpha)t) + e(t)\,\mathcal{M}(x, y, y, t)$ and

$\mathcal{N}(S^n x, T^m y, T^m y, kt) \leq a(t)\,\mathcal{N}(x, S^n x, S^n x, t) + b(t)\,\mathcal{N}(y, T^m y, T^m y, t) + c(t)\,\mathcal{N}(x, T^m y, T^m y, \alpha t) + d(t)\,\mathcal{N}(y, S^n x, S^n x, (2-\alpha)t) + e(t)\,\mathcal{N}(x, y, y, t)$

for all x, y ∈ X, α ∈ [0, 2] and n, m ≥ 2, where a, b, c, d, e: [0, ∞)→[0, 1] are five functions such that a(t) + b(t) + c(t) + d(t) + e(t) =1 for all t ∈ [0,∞).

If $S^nT = TS^n$ and $T^mS = ST^m$, then S and T have a unique common fixed point in X.

Proof :

By theorem (3.4), S^n and T^m have a unique common fixed point in X.

That is, there exists a unique point z ∈ X such that $S^n(z) = T^m(z) = z$.

Since $S(z) = S(S^n(z)) = S^n(S(z))$ and $S(z) = S(T^m(z)) = T^m(S(z))$, that is

S(z) is fixed point S^n and T^m and so S(z) = z. Similarly, Tz = z.

Corollary 3.9 :

Let (X, $\mathcal{M}, \mathcal{N}$, ∗, ◊) be a complete generalized intuitionistic fuzzy metric space with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq 1-t$ for all t ∈ [0, 1]. Let S and T be mappings from X into itself such that there exists k ∈ (0, 1) such that

$\mathcal{M}(Sx, Ty, Ty, kt) \geq a(t)\,\mathcal{M}(x, Sx, Sx, t) + b(t)\,\mathcal{M}(y, Ty, Ty, t)$,

$\mathcal{N}(Sx, Ty, Ty, kt) \leq a(t)\,\mathcal{N}(x, Sx, Sx, t) + b(t)\,\mathcal{N}(y, Ty, Ty, t)$

for all x, y ∈ X and α ∈ (0, 2), where a, b:[0, ∞)→[0, 1] are two functions such that a(t) + b(t) = 1 for all t ∈ [0, ∞). Then S and T have a unique common fixed point in X.

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