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RESEARCH ARTICLE

THEOREMS ON SOME COMMON FIXED POINT FOR GENERALIZED NONEXPANSIVE TYPE FUZZY MAPPINGS

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ABSTRACT

The aim of this paper is to prove some fixed point theorems for associated multimaps.Our Results extend and improve the result of Bijendra Singh and M.S. Chauhan[2].

Key Words: Fixed point, fuzzy mapping, non expansive mapping, Associated multimap.

MSC 2010 : 47H10 ,54H25

INTRODUCTION

Fixed point theory is very important in mathematics and has applications in many fields. The study of fixed point for multivalued mapping was originally initiated by van Neumann [5] .The development of geometric fixed point theory for multivalued mapping was initiated with the work of Nadler[12].He combined the ideas of multivalued mapping and Lipschitz mapping and used the concept of Hausdorff metric to establish the multivalued contraction principle, usually referred as Nadler's contraction mapping principle. Several researchers were conducted on the generalizations of the concept of Nadler's contraction mapping principle.

On the other hand, in the year1965, Zadeh [6] introduced the concept of fuzzy set which motivated a lot of mathematical activities on generalization of the notion of fuzzy set.Heilpern[13] introduced the concept of fuzzy mapping and proved a fixed point theorem for fuzzy contraction mappings, which was successively generalized by Estruch and Vidal[15]. C.S.Sen[3]defined an associated multimaps of fuzzy mappings and proved significant results. Singh and Chauhan[2] proved some results for associated multimaps of fuzzy mappings taking a new type of contractive inequality. Afterwards, a number of papers appeared in which fixed points of fuzzy mappings satisfying contractive inequalities have been discussed [1, 3, 6-8, 10, 14]and references there in.

Preliminaries: Throughout this paper we will be using the terminology and notations of Heilpern [13].

Definitions 2.1: A fuzzy set A in complete metric space X is a function from X into [0,1]. If $x \in X$, the function value $A(x)$ is called the grade of member of X in A. The $\alpha - level$ set of a denoted by

$A_\alpha = \{x : A(x) \geq \alpha\}$, if $\alpha \in [0,1]$, $A_0 = \{x : A(x) > 0\}$.

Definition 2.2 : A Fuzzy set A is said to be an approximate quantity if and only if A_α is compact and convex for each $\alpha \in [0,1]$ and $\sup_{x \in X} A(x) = 1$.

When A is an approximate quantity and $A(x)_0 = 1$ for some $x_0 \in X$,A is identified with an approximation of x_0.the collection of all fuzzy set in X is denoted by $F(X)$ and $w(X)$ is the subcollection of all approximated quantities. $c(X)$ be the set of compact subset of X , $(c(X), H)$ as defined by

$$H(A,B) = \max\{\sup_{a \in A} D(a,B), \sup_{b \in B} D(b,A)\}$$

For any $A, B \in c(X)$,where $D(a,B) = \inf_{b \in B} d(a,b)$, and $D(A,B) = \inf_{x \in A, y \in B} d(x,y)$.

Definition 2.3: Let $A, B \in w(X)$,the A is said to be more accurate than B,denoted by $A \subset B$ if and only if $A(X) \leq B(X)$ for each $x \in X$ .The relation $"\subset"$ induces a partial ordering on the family $w(X)$.

Definition 2.4: Let X and Y be two complete linear metric space. F is called fuzzy mapping if F is mapping from X into $w(X)$.

A fuzzy mapping F is a fuzzy subset $X \times Y$ with membership function $F(x,y)$ .the function value $F(x,y)$ is the grade of membership of Y in $F(X)$.Each fuzzy mapping is a set valued mapping.

Definition 2.5:A mapping $T : X \to w(X)$ is said to be nonexpansive if for all $x, y \in X$, $H(Tx, Ty) \leq d(x,y)$

Definition2.6: If $F' : X \to w(X)$ is a fuzzy map,we define an associated multimap $F : X \to c(X)$ as $F(X) = \{y \in X : F'_x(y) = \max_{u \in X} F'_x(u)\}$ point p of X is called a fixed point of the fuzzy map F' if $F'_p(p) \geq F'_p(x)$ For all $x \in X$.

Lemma 2.7: $F'_p(p) \geq F_p(x)$ For all $x \in X$ if and only if $p \in F'(p)$.

Singh and Chauhan[02] proved the following results

Theorem-2.8: let (X,d) be a complete metric space and let $F', G' : X \to w(X)$ be two fuzzy mappings and F and G be their associated multimaps defined from X into $c(X)$ satisfying

$$H(Fx.Gy) \leq \max\{d(x,y), D(x,Fx), D(y,Gy), \frac{1}{2}[D(x,Gy + D(y,Fx)]\}$$

$$- w[\max\{d(x,y), D(x,Fx), D(y,Gy), \frac{1}{2}[D(x,Gy + D(y,Fx)]\}]$$

For all $x, y \in X$, $w : R^+ \to R^+$ is a continuous function such that $0 < w(r) < r$ for all $r > 0$.

Then there exists a common fixed point of F & G and F' and G' have also a common fixed point.

Singh and Chauhan generalized theorem 2.8 for two sequence of fuzzy mappings:

Theorem 2.9: let (X,d) be a complete metric space and $\{F_i\}_{i=1}^{\infty}$ and $\{G_i\}_{i=1}^{\infty}: X \to w(X)$, the sequence of fuzzy maps and $\{F_i\}_{i=1}^{\infty}$ and $\{G_i\}_{i=1}^{\infty}: X \to c(X)$ the sequence of their associated multimaps converging pointwise to the associated multimaps F, G of fuzzy map $F^{'}$ and $G^{'}$,respectively satisfying

$$H(F_n x, G_n y) \le a \max\{d(x,y), D(y,G_n y)\} + b \max\{D(x,Fx_n), D(y,Gy_n), D(y,F_n x)\}$$
$$+c[D(x,G_n y) + D(y,F_n x)] - w[a \max\{d(x,y), D(y,G_n y)\}$$
$$+b \max\{D(x,Fx_n), D(y,Gy_n), D(y,F_n x)\} + c[D(x,G_n y) + D(y,F_n x)]]$$

Then F and G have a common fixed point.

Main Results

Theorem3.1 let (X,d) be a complete metric space and let $F^{'}, G^{'}: X \to w(X)$ be two fuzzy mappings and F and G be their associated multimaps defined from X into $c(X)$ satisfying

$$\begin{aligned} &H(Fx.Gy) \le a \max\{d(x,y), D(y,Gy)\} + b \max\{D(x,Fx), D(y,Gy), D(y,Fx)\} + \\ &c[D(x,Gy + D(y,Fx)] - w[a \max\{d(x,y), D(y,Gy)\}] + \\ &b \max\{D(x,Fx), D(y,Gy), D(y,Fx)\} + c[D(x,Gy) + D(y,Fx)] \end{aligned} \tag{3.1}$$

Where $a,b,c > 0$ such that $a+b+2c=1$ and $w: R^+ \to R^+$ is a continuous function such that $0 < w(r) < r$ for all $r > 0$ .then F & G have a common fixed point.

Proof : Let x_0 be an arbitrary point in X.Since C(X) is compact there we can construct a sequence $\{x_n\}$ such that $x_n \in Fx_{n-1}$ with

$$d(x_{n-1}, x_n) = D(x_{n-1}, Fx_{n-1})$$
$$d(x_n, x_{n+1}) = D(x_n, Gx_n) \quad \text{and}$$
$$d(x_n, x_{n+1}) = D(Gx_n, Fx_{n-1})$$

If $x_n = x_{n+1}$ then the result holds good. If $x_n \ne x_{n+1}$ then we have from (3.1)

$$d(x_1,x_2) \le H(Fx_0, Gx_1)$$

$$\le a \max\{d(x_0,x_1), D(x_1,Gx_1)\} + b \max\{D(x_0,Fx_0), D(x_1,Gx_1)., D(x_1,Fx_0)\}$$
$$+c[D(x_0,Gx_1) + D(x_1,Fx_0)] - w[a \max\{d(x_0,x_1), D(x_1,Gx_1)\} + b \max\{D(x_0,Fx_0)$$
$$, D(x_1,Gx_1), D(x_1,Fx_0)\} + c[D(x_0,Gx_1) + D(x_1,Fx_0)]]$$

$$\le a \max\{d(x_0,x_1), d(x_1,x_2)\} + b \max\{d(x_0,x_1), d(x_1,x_2)\} + c[d(x_0,x_1) + d(x_1,x_2)]$$
$$-w[a \max\{d(x_0,x_1), d(x_1,x_2)\} + b \max\{d(x_0,x_1)\} + c[d(x_0,x_1) + d(x_1,x_2)]]$$

If $d(x_0,x_1) < d(x_1,x_2)$,then

$$d(x_1,x_2) < (a+b)d(x_1,x_2) + 2cd(x_1,x_2) - w[(a+b)d(x_1,x_2) + 2cd(x_1,x_2)]$$
$$= (a+b+2c)d(x_1,x_2) - w[(a+b+2c)d(x_1,x_2)]$$

$= d(x_1, x_2) - wd(x_1, x_2)$

$< d(x_1, x_2),$

A contradiction.Hence

$$d(x_1, x_2) \le (a+b)d(x_0, x_1) + 2cd(x_0, x_1) - w[(a+b)d(x_0, x_1) + 2cd(x_0, x_1)]$$
$$\le (a+b+2c)d(x_0, x_1) - w[(a+b+2c)d(x_0, x_1)]$$
$$\le d(x_0, x_1) - w(d(x_0, x_1)) \quad (3.2)$$

Similarly

$$d(x_2, x_3) \le d(x_1, x_2) - w(d(x_1, x_2)) \quad (3.3)$$

And hence inductively

$$d(x_n, x_{n+1}) \le d(x_{n-1}, x_n) - w(d(x_{n-1}, x_n)) \quad (3.4)$$

Adding (3.2) to (3.4),we have

$$\sum_{i=0}^{n} w(d(x_i, x_{i+1})) \le d(x_0, x_1) - d(x_n, x_{n+1})$$
$$\le d(x_0, x_1)$$

Therefore

$\sum_{i=0}^{n} w(d(x_i, x_{i+1})) < \infty$ and

$$\lim_{n\to\infty} w(d(x_n, x_{n+1})) = 0 \quad (3.5)$$

Since $\{d(x_n, x_{n+1})\}$ is a decreasing sequence of non negative terms, therefore (3.5) implies that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0 \quad (3.6)$$

Now suppose that $\{x_n\}$ is not a Cauchy sequence .Then there is an $\in > 0$ such that for each positive integer K,there are positive integers $2m(k)$ and $2n(k)$ with $2m(k) > 2n(k) > 2k$ such that

$$d(x_{2m(k)}, x_{2n(k)}) > \in \quad (3.7)$$

For each positive integer k,let $2m(k)$ be the least positive exceeding $2n(k)$ satisfying the above inequality, so that

$$d(x_{2n(k)}, x_{2m(k)-2}) \le \in \quad (3.8)$$

Using (3.6),(3.7) and(3.8) we have

$$\in < d(x_{2m(k)}, x_{2n(k)})$$
$$\le d(x_{2n(k)}, x_{2m(k)-2}) + d(x_{2m(k)-2}, x_{2m(k)})$$
$$\le \in + d(x_{2m(k)-2}, x_{2m(k)-1}) + d(x_{2m(k)-1}, x_{2m(k)})$$

Which implies $d(x_{2m(k)}, x_{2n(k)}) \to \in$ as $k \to \infty$

Now by using triangular inequality, we have the following

$$d(x_{2m(k)}, x_{2n(k)+1}) - d(x_{2m(k),}, x_{2n(k)}) \le d(x_{2n(k),} x_{2n(k)+1}),$$
$$d(x_{2m(k)+1}, x_{2n(k)+1}) - d(x_{2m(k)} x_{2n(k)+1}) \le d(x_{2m(k)}, x_{2m(k)+1})$$
$$d(x_{2m(k),} x_{2n(k)+2}) - d(x_{2m(k),} x_{2n(k)+1}) \le d(x_{2n(k)+1}, x_{2n(k)+2})$$
$$d(x_{2m(k+1)}, x_{2n(k)+2}) - d(x_{2m(k)+1} x_{2n(k)+1}) \le d(x_{2n(k)+1}, x_{2n(k)+2})$$

Letting limit as $k \to \infty$, we have

$$\begin{aligned} & d(x_{2m(k)}, x_{2n(k+1)}) \to \in \\ & d(x_{2m(k)+1}, x_{2n(k)+1}) \to \in \\ & d(x_{2m(k)}, x_{2n(k)+2}) \to \in \\ & d(x_{2m(k)+1}, x_{2n(k)+2}) \to \in \end{aligned} \tag{3.9}$$

Using (3.1) & (3.9),we have

$$\begin{aligned} & d(x_{2m(k)+1}, x_{2n(k)+2}) \le H(Fx_{2m(k)}, Gx_{2n(k)+1}) \\ & \le a \max\{ d(x_{2m(k)}, x_{2n(k)+1}), D(x_{2n(k)+1}, Gx_{2n(k)+1})\} \\ & + b \max\{ d(x_{2m(k)} Fx_{2m(k)}), D(x_{2m(k)+1} Gx_{2n(k)+1}), D(x_{2n(k)+1}, Fx_{2m(k)})\} \\ & + c[D(x_{2m(k)}, Gx_{2n(k)+1}) + D(x_{2n(k)+1}, Fx_{2m(k)})] \\ & - w[a \max\{ d(x_{2m(k)}, x_{2n(k)+1}), D(x_{2n(k)+1} Gx_{2n(k)+1})\} \\ & + b\{D(x_{2m(k)}, Fx_{2m(k)}), D(x_{2m(k)+1}, Gx_{2n(k)+1}), D(x_{2n(k)+1}, Fx_{2m(k)})\} \\ & + c[D(x_{2m(k)}, Gx_{2n(k)+1}) + D(x_{2n(k)+1}, Fx_{2m(k)})]] \end{aligned}$$

$$\begin{aligned} & \le a \max\{ d(x_{2m(k)}, x_{2n(k)+1}), d(x_{2n(k)+1}, x_{2n(k)+2})\} \\ & + b \max\{ d(x_{2m(k)} x_{2m(k)+1}), d(x_{2m(k)+1} x_{2n(k)+2}), d(x_{2n(k)+1}, x_{2m(k)+1})\} \\ & + c[d(x_{2m(k)}, x_{2n(k)+2}) + d(x_{2n(k)+1}, x_{2m(k)+1})] \\ & - w[a \max\{ d(x_{2m(k)}, x_{2n(k)+1}), d(x_{2n(k)+1} x_{2n(k)+2})\} \\ & + b\{D(x_{2m(k)}, x_{2m(k)+1}), d(x_{2m(k)+1}, x_{2n(k)+2}), d(x_{2n(k)+1}, x_{2m(k)+1})\} \\ & + c[D(x_{2m(k)}, x_{2n(k)+2}) + D(x_{2n(k)+1}, x_{2m(k)+1})]] \end{aligned}$$

$k \to \infty$ and using (3.9) ,we have $\in \le \in - w(\in) \prec \in$,

a contradiction. Thus $\{x_n\}$ is a Cauchy sequence. Since X is complete, so it converges to a point $z \in X$

Now by inequality (3.1) again

$$D(x_{2n+1}, Gz) \le H(Fx_{2n}, Gz)$$

$$\leq a\max\{d(x_{2n},z),D(z,Gz)\}$$
$$+b\max\{D(x_{2n},Fx_{2n}),D(z,Gz),D(z,,Fx_{2n})\}$$
$$+c[D(x_{2n},Gz)+D(z,Fx_{2n})]$$
$$-w[a\max\{d(x_{2n},z),D(z,Gz)\}$$
$$+b\{D(x_{2n},Fx_{2n}),D(z,Gz),D(z,Fx_{2n})\}$$
$$+c[D(x_{2n},Gz)+D(z,Fx_{2n})$$

$$\leq a\max\{d(x_{2n},z),D(z,Gz)\}$$
$$+b\max\{D(x_{2n},x_{2n+1}),D(z,Gz),D(z,,x_{2n+1})\}$$
$$+c[D(x_{2n},Gz)+D(z,Fx_{2n+1})]$$
$$-w[a\max\{d(x_{2n},z),D(z,Gz)\}$$
$$+b\{D(x_{2n},x_{2n+1}),D(z,Gz),D(z,x_{2n+1})\}$$
$$+c[D(x_{2n},Gz)+D(z,x_{2n+1})$$

Letting $n\to\infty$,we get

$$D(z,Gz)=(a+b+c)D(z,Gz)-w[(a+b+c)D(z,Gz)]$$
$$<D(z,Gz)-w[D(z,Gz)]$$
$$<D(z,Gz),$$

Which implies that $z\in Gz$. Similarly, we can prove that $z\in Fz$.

Hence $z\in Fz\cap Gz$ i. e. z is a common fixed point of F and G.

Using lemma 1, it is clear that $z\in F^{'}z\cap G^{'}z$ i. e. z is also the common fixed point point of $F^{'}$ and $G^{'}$.

Corollary 3.1: let $\{F_i\}_{i=1}^{\infty}:X\to w(X)$ be a sequence of fuzzy mappings and $\{F_i\}_{i=1}^{\infty}:X\to c(X)$ be a sequence of its associated multimaps.suppose for any positive integers, $i\neq j$ and $x,y\in X$.

$$H(F_ix,F_jy)\leq a\max\{d(x,y),D(y,F_jy)\}+b\max\{D(x,F_ix),D(y,F_jy),D(y,F_ix)\}$$
$$+c[D(x,F_jy)+D(y,F_ix)]-w[a\max\{d(x,y),D(y,F_jy)\} \tag{3.10}$$
$$+b\max\{D(x,F_ix),D(y,F_jy),D(y,F_ix)\}+c[D(x,F_jy)+D(y,F_ix)]]$$

Where $a,b,c\succ 0$ such that $a+b+2c=1$ and $w:R^{+}\to R^{+}$ is a continuous function such th at $0<w(r)<r$ for all $r>0$ .then there exists a fixed point of $\{F_i\}_{i=1}^{\infty}$.

Theorem 3.2: let (X,d) be a complete metric space and $\{F_i\}_{i=1}^{\infty}$ and $\{G_i\}_{i=1}^{\infty}:X\to w(X)$ are the sequence of fuzzy maps and $\{F_i\}_{i=1}^{\infty}$ and $\{G_i\}_{i=1}^{\infty}:X\to c(X)$ the sequence of their associated multimaps converging pointwise to the associated multimaps F and,G respectively satisfying

$$H(F_n x, G_n y) \le a\max\{d(x,y), D(y,G_n y)\} + b\max\{D(x,Fx_n), D(y,Gy_n), D(y,F_n x)\}$$
$$+c[D(x,G_n y)+D(y,F_n x)] - w[a\max\{d(x,y),D(y,G_n y)\}$$
$$+b\max\{D(x,Fx_n), D(y,Gy_n), D(y,F_n x)\} + c[D(x,G_n y)+D(y,F_n x)]]$$

Where $a,b,c \succ 0$ such that $a+b+2c=1$ and $w: R^+ \to R^+$ is a continuous function such that $0 < w(r) < r$ for all $r > 0$ .then F and G have a common fixed point.

Proof: Let $x_0 \in X$ .We define the sequence $\{x_n\}$ such that $x_{2n-1} \in F_n x_{2n-2}$ and $x_{2n} \in G_n x_{2n-1}$.

Now using triangular inequality, we have

$$D(y,F_n x) \le D(y,Fx) + D(F_n x, Fx)$$
$$D(y,F_n x) - D(y,Fx) \le D(F_n x, Fx) \le H(F_n x.Fx)$$

Hence

$$D(y,F_n x) - D(y,Fx) \le H(F_n x.Fx)$$

And similarly we have

$$D(y,G_n y) - D(y,Gy) \le H(G_n y, Gy)$$
$$D(x,F_n x) - D(x,Fx) \le H(F_n x.Fx)$$
$$D(x,G_n y) - D(x,Gy) \le H(G_n y, Gy)$$

Now since H is continuous and $\{F_n\}$ and $\{G_n\}$ converge point wise 'F′ to 'G' and,respectively, then(3.11) becomes

$$H(Fx.Gy) \le a\max\{d(x,y), D(y,Gy)\} + b\max\{D(x,Fx), D(y,Gy), D(y,Fx)\} +$$
$$c[D(x,Gy+D(y,Fx)] - w[a\max\{d(x,y),D(y,Gy)\}]+$$
$$b\max\{D(x,Fx), D(y,Gy), D(y,Fx)\} + c[D(x,Gy)+D(y,Fx)]$$

of the proof is identical to the proof of theorem (3.1)

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