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RESEARCH ARTICLE

A Peer Reviewed International Research Journal



BIPOLAR VALUED FUZZY RW-CLOSED AND BIPOLAR VALUED FUZZY RW-OPEN SETS IN BIPOLAR VALUED FUZZY TOPOLOGICAL SPACES

M.AZHAGAPPAN¹, M.KAMARAJ²

¹Department of Mathematics, Yadava College, Madurai, Tamilnadu, India.

Email: mazhagappan72@gmail.com

²Principal(i/c), Department of Mathematics, Government Arts and Science College, Sivakasi, Tamilnadu, India

Email: kamarajm17366@gmail.com



ABSTRACT

In this paper, we introduce some properties of bipolar valued fuzzy rw-closed and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces and study some important properties on these.

2000 AMS SUBJECT CLASSIFICATION: 03F55, 08A72, 20N25.

KEY WORDS: Bipolar valued fuzzy subset, bipolar valued fuzzy topological spaces, bipolar valued fuzzy rw-closed set, bipolar valued fuzzy rw-open set.

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INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [20] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers are useful to us to work on this paper. C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Tapas kumar mondal and S.K.Samanta [18] have introduced the topology of interval valued fuzzy sets. We are motivated to introduce the concept of bipolar valued fuzzy rw-closed sets and bipolar valued fuzzy rw-open sets in bipolar valued fuzzy topological spaces and established some results.

1.PRELIMINARIES

1.1 Definition: A bipolar valued fuzzy set A in X is defined as an object of the form A = {< x, A+(x), A-(x) >/ x∈X}, where A+ : X→ [0, 1] and A- : X→ [-1, 0]. The positive membership degree A+(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree A-(x) denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A. If A+(x) ≠ 0 and A-(x) = 0, it is the situation that x is regarded as having only positive satisfaction for A and if A+(x) = 0

and $A^-(x) \neq 0$, it is the situation that x does not satisfy the property of A , but somewhat satisfies the counter property of A . It is possible for an element x to be such that $A^+(x) \neq 0$ and $A^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X .

1.2 Example: $A = \{ \langle a, 0.6, -0.4 \rangle, \langle b, 0.8, -0.3 \rangle, \langle c, 0.5, -0.5 \rangle \}$ is a bipolar valued fuzzy subset of $X = \{ a, b, c \}$.

1.3 Definition: Let A and B be two bipolar valued fuzzy subsets of a set X . We define the following relations and operations:

- (i) $A \subset B$ if and only if $A^+(x) \leq B^+(x)$ and $A^-(x) \geq B^-(x)$, for all $x \in X$.
- (ii) $A = B$ if and only if $A^+(x) = B^+(x)$ and $A^-(x) = B^-(x)$, for all $x \in X$.
- (iii) $A \cup B = \{ \langle x, \max(A^+(x), B^+(x)), \min(A^-(x), B^-(x)) \rangle / x \in X \}$.
- (iv) $A \cap B = \{ \langle x, \min(A^+(x), B^+(x)), \max(A^-(x), B^-(x)) \rangle / x \in X \}$.
- (v) $A^c = \{ \langle x, 1 - A^+(x), -1 - A^-(x) \rangle / x \in X \}$.

1.4 Example: Let $X = \{ a, b, c \}$ be a set. Let $A = \{ \langle a, 0.5, -0.4 \rangle, \langle b, 0.1, -0.6 \rangle, \langle c, 0.6, -0.2 \rangle \}$ and $B = \{ \langle a, 0.4, -0.4 \rangle, \langle b, 0.6, -0.3 \rangle, \langle c, 0.8, -0.5 \rangle \}$ be any two bipolar valued fuzzy subsets of X . Then

- (i) $A \cup B = \{ \langle a, 0.5, -0.4 \rangle, \langle b, 0.6, -0.6 \rangle, \langle c, 0.8, -0.5 \rangle \}$.
- (ii) $A \cap B = \{ \langle a, 0.4, -0.4 \rangle, \langle b, 0.1, -0.3 \rangle, \langle c, 0.6, -0.2 \rangle \}$.
- (iii) $A^c = \{ \langle a, 0.5, -0.6 \rangle, \langle b, 0.9, -0.4 \rangle, \langle c, 0.4, -0.8 \rangle \}$.

1.5 Definition: Let X be any set. Let 0_x and 1_x be the bipolar valued fuzzy sets on X defined as follows $0_x = \{ \langle x, 0, 0 \rangle / \text{for all } x \in X \}$ and $1_x = \{ \langle x, 1, -1 \rangle / \text{for all } x \in X \}$.

1.6 Definition: Let X be a set and \mathfrak{T} be a family of bipolar valued fuzzy subsets of X . The family \mathfrak{T} is called bipolar valued fuzzy topology on X if and only if \mathfrak{T} satisfies the following axioms (i) $0_x, 1_x \in \mathfrak{T}$,

(ii) If $\{A_i; i \in I\} \subseteq \mathfrak{T}$, then $\bigcup_{i \in I} A_i \in \mathfrak{T}$, (iii) If $A_1, A_2, A_3, \dots, A_n \in \mathfrak{T}$, then $\bigcap_{i=1}^n A_i \in \mathfrak{T}$. The pair (X, \mathfrak{T}) is called

bipolar valued fuzzy topological space. The members of \mathfrak{T} are called bipolar valued fuzzy open sets in X . Bipolar valued fuzzy set A in X is said to be bipolar valued fuzzy closed set in X if and only if A^c is bipolar valued fuzzy open set in X .

1.7 Definition: Let (X, \mathfrak{T}) be bipolar valued fuzzy topological space and A be bipolar valued fuzzy set in X . Then $\bigcap \{B : B^c \in \mathfrak{T} \text{ and } B \supseteq A\}$ is called bipolar valued fuzzy closure of A and is denoted by $\text{bcl}(A)$.

1.8 Theorem: Let A and B be two bipolar valued fuzzy sets in bipolar valued fuzzy topological space (X, \mathfrak{T}) . Then the following results are trivial,

- (i) $\text{bcl}(A)$ is bipolar valued fuzzy closed set in X . (ii) $\text{bcl}(A)$ is the smallest bipolar valued fuzzy closed set containing A . (iii) A is bipolar valued fuzzy closed if and only if $A = \text{bcl}(A)$. (iv) $\text{bcl}(0_x) = 0_x$, 0_x is the empty bipolar valued fuzzy set. (v) $\text{bcl}(\text{bcl}(A)) = \text{bcl}(A)$. (vi) $\text{bcl}(A \cup B) = \text{bcl}(A) \cup \text{bcl}(B)$. (vii) $\text{bcl}(A) \cap \text{bcl}(B) \supseteq \text{bcl}(A \cap B)$.

1.9 Definition: Let (X, \mathfrak{T}) be bipolar valued fuzzy topological space and A be a bipolar valued fuzzy set in X . Then $\bigcup \{B : B \in \mathfrak{T} \text{ and } B \subseteq A\}$ is called bipolar valued fuzzy interior of A and is denoted by $\text{bint}(A)$.

1.10 Theorem: Let (X, \mathfrak{T}) be bipolar valued fuzzy topological space, A and B be two bipolar valued fuzzy sets in X . The following results hold good,

- (i) $\text{bint}(A)$ is bipolar valued fuzzy open set in X . (ii) $\text{bint}(A)$ is the largest bipolar valued fuzzy open set in X which is contained in A . (iii) A is bipolar valued fuzzy open set if and only if $A = \text{bint}(A)$. (iv) $A \subseteq B$ implies $\text{bint}(A) \subseteq \text{bint}(B)$. (v) $\text{bint}(\text{bint}(A)) = \text{bint}(A)$. (vi) $\text{bint}(A \cap B) = \text{bint}(A) \cap \text{bint}(B)$. (vii) $\text{bint}(A) \cup \text{bint}(B) \subseteq \text{bint}(A \cup B)$. (viii) $\text{bint}(A^c) = (\text{bcl}(A))^c$. (ix) $\text{bcl}(A^c) = (\text{bint}(A))^c$.

1.11 Definition: Let (X, \mathfrak{T}) be bipolar valued fuzzy topological space and A be bipolar valued fuzzy set in X . Then A is said to be (i) bipolar valued fuzzy semi open if and only if there exists bipolar valued fuzzy open set V in X such that $V \subseteq A \subseteq \text{bcl}(V)$. (ii) bipolar valued fuzzy semiclosed if and only if there exists bipolar valued fuzzy closed set V in X such that $\text{bint}(V) \subseteq A \subseteq V$. (iii) bipolar valued fuzzy regular open set of X if $\text{bint}(\text{bcl}(A)) = A$. (iv) bipolar valued fuzzy regular closed set of X if $\text{bcl}(\text{bint}(A)) = A$. (v) bipolar valued fuzzy regular semi open set of X if there exists bipolar valued fuzzy regular open set V in X such that $V \subseteq A \subseteq \text{bcl}(V)$. We denote the class of bipolar valued fuzzy regular semi open sets in bipolar valued fuzzy topological space X by $\text{BVFRSO}(X)$. (vi) bipolar valued fuzzy generalized closed if $\text{bcl}(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy open set and A is bipolar valued fuzzy generalized open if A^c is bipolar valued fuzzy generalized closed.

1.12 Theorem: The following are equivalent:

(i) A is bipolar valued fuzzy semi closed set. (ii) A^c is bipolar valued fuzzy semi open set. (iii) $\text{bint}(\text{bcl}(A)) \subseteq A$. (iv) $\text{bcl}(\text{bint}(A^c)) \supseteq A$.

1.13 Theorem: Any union of bipolar valued fuzzy semi open sets is bipolar valued fuzzy semi open set and any intersection of bipolar valued fuzzy semi closed sets is bipolar valued fuzzy semi closed.

Remark: (i) Every bipolar valued fuzzy open set is bipolar valued fuzzy semi open but not conversely. (ii) Every bipolar valued fuzzy closed set is bipolar valued fuzzy semi-closed set but not conversely. (iii) The closure of bipolar valued fuzzy open set is bipolar valued fuzzy semi open set. (iv) The interior of bipolar valued fuzzy closed set is bipolar valued fuzzy semi-closed set.

1.14 Theorem: Bipolar valued fuzzy set A of bipolar valued fuzzy topological space X is bipolar valued fuzzy regular open if and only if A^c is bipolar valued fuzzy regular closed set.

Remark: (i) Every bipolar valued fuzzy regular open set is bipolar valued fuzzy open set but not conversely. (ii) Every bipolar valued fuzzy regular closed set is bipolar valued fuzzy closed set but not conversely.

1.15 Theorem: (i) The closure of bipolar valued fuzzy open set is bipolar valued fuzzy regular closed. (ii) The interior of bipolar valued fuzzy closed set is bipolar valued fuzzy regular open set.

1.16 Theorem: (i) Every bipolar valued fuzzy regular semi open set is bipolar valued fuzzy semi open set but not conversely. (ii) Every bipolar valued fuzzy regular closed set is bipolar valued fuzzy regular semi open set but not conversely. (iii) Every bipolar valued fuzzy regular open set is bipolar valued fuzzy regular semi open set but not conversely.

1.17 Theorem: Let (X, \mathfrak{T}) be bipolar valued fuzzy topological space and A be bipolar valued fuzzy set in X . Then the following conditions are equivalent:

(i) A is bipolar valued fuzzy regular semi open. (ii) A is both bipolar valued fuzzy semi open and bipolar valued fuzzy semi-closed. (iii) A^c is bipolar valued fuzzy regular semi open in X .

1.18 Definition: Bipolar valued fuzzy set A of bipolar valued fuzzy topological space (X, \mathfrak{T}) is called (i) bipolar valued fuzzy g -closed if $\text{bcl}(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy open set in X . (ii) bipolar valued fuzzy g -open if its complement A^c is bipolar valued fuzzy g -closed set in X . (iii) bipolar valued fuzzy rg -closed if $\text{bcl}(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy regular open set in X . (iv) bipolar valued fuzzy rg -open if its complement A^c is bipolar valued fuzzy rg -closed set in X . (v) bipolar valued fuzzy w -closed if $\text{bcl}(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy semi open set in X . (vi) bipolar valued fuzzy w -open if its complement A^c is bipolar valued fuzzy w -closed set in X . (vii) bipolar valued fuzzy gpr -closed if $\text{pbcl}(A) \subseteq V$ whenever $A \subseteq V$ and V is bipolar valued fuzzy regular open set in X . (viii) bipolar valued fuzzy gpr -open if its complement A^c is bipolar valued fuzzy gpr -closed set in X .

2. SOME PROPERTIES

2.1 Definition: Let (X, \mathfrak{T}) be bipolar valued fuzzy topological space. Bipolar valued fuzzy set A of X is called bipolar valued fuzzy regular w -closed (briefly, bipolar valued fuzzy rw -closed) if $\text{bcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is bipolar valued fuzzy regular semi open in bipolar valued fuzzy topological space X .

NOTE: We denote the family of all bipolar valued fuzzy regular w -closed sets in bipolar valued fuzzy topological space X by $\text{BVFRWC}(X)$.

2.2 Definition: Bipolar valued fuzzy set A of bipolar valued fuzzy topological space X is called bipolar valued fuzzy regular w -open (briefly, bipolar valued fuzzy rw -open) set if its complement A^c is bipolar valued fuzzy rw -closed set in bipolar valued fuzzy topological space X .

NOTE: We denote the family of all bipolar valued fuzzy rw -open sets in bipolar valued fuzzy topological space X by $\text{BVFRWO}(X)$.

2.3 Theorem: Every bipolar valued fuzzy closed set is bipolar valued fuzzy rw -closed set in bipolar valued fuzzy topological space X .

Proof: Let A be bipolar valued fuzzy closed set in bipolar valued fuzzy topological space X . Let B be bipolar valued fuzzy regular semi open set in X such that $A \subseteq B$. Since A is bipolar valued fuzzy closed, $\text{bcl}(A) = A$. Therefore $\text{bcl}(A) = A \subseteq B$. Hence A is bipolar valued fuzzy rw -closed in bipolar valued fuzzy topological space X .

Remark: The converse of the above Theorem need not be true in general.

2.4 Example: Let $X = \{1, 2, 3\}$. Define bipolar valued fuzzy set $A = \{ \langle 1, 0.6, -0.6 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 0, 0 \rangle \}$. Let $\mathfrak{T} = \{0_x, 1_x, A\}$. Then (X, \mathfrak{T}) is bipolar valued fuzzy topological space. Define bipolar valued fuzzy set $B = \{ \langle 1, 0, 0 \rangle, \langle 2, 0.6, -0.6 \rangle, \langle 3, 0, 0 \rangle \}$. Then B is bipolar valued fuzzy rw -closed set but it is not bipolar valued fuzzy closed set in bipolar valued fuzzy topological space X .

Remark: bipolar valued fuzzy generalized closed sets and bipolar valued fuzzy rw -closed sets are independent.

2.5 Example: Let $X = \{1, 2, 3, 4\}$. Define bipolar valued fuzzy sets A, B, C in X by $A = \{ \langle 1, 1, -1 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle \}$, $B = \{ \langle 1, 0, 0 \rangle, \langle 2, 1, -1 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle \}$, $C = \{ \langle 1, 1, -1 \rangle, \langle 2, 1, -1 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle \}$. Consider $\mathfrak{T} = \{0_x, 1_x, A, B, C\}$. Then (X, \mathfrak{T}) is bipolar valued fuzzy topological space. In this bipolar valued fuzzy topological space X , the bipolar valued fuzzy set D is defined by $D = \{ \langle 1, 0, 0 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 1, -1 \rangle, \langle 4, 0, 0 \rangle \}$. Then D is bipolar valued fuzzy generalized closed set in bipolar valued fuzzy topological space X . In this bipolar valued fuzzy topological space, the bipolar valued fuzzy set E is defined by $E = \{ \langle 1, 1, -1 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 1, -1 \rangle, \langle 4, 0, 0 \rangle \}$. Then E is bipolar valued fuzzy regular semi open set containing D , but E does not contain $\text{bcl}(D)$ which is C^c . Therefore E is not bipolar valued fuzzy rw -closed set in bipolar valued fuzzy topological space X .

2.6 Example: Let $X = I = [0, 1]$. Define bipolar valued fuzzy set D in X by $D(x) = \langle x, 0.5, -0.5 \rangle$ if $x = 2/3$ = $\langle x, 0, 0 \rangle$ if otherwise.

Let $\mathfrak{T} = \{0_x, 1_x, D\}$. Then (X, \mathfrak{T}) is bipolar valued fuzzy topological space.

Let $A(x) = \langle x, 0.3, -0.3 \rangle$ if $x = 2/3$
= $\langle x, 0, 0 \rangle$ if otherwise.

Then A is bipolar valued fuzzy rw -closed set in bipolar valued fuzzy topological space X . Now $\text{bcl}(A) = D^c$ and D is bipolar valued fuzzy open set containing A but D does not contain $\text{bcl}(A)$ which is D^c . Therefore A is not bipolar valued fuzzy generalized closed.

Remark: Bipolar valued fuzzy rw -closed sets and bipolar valued fuzzy semi-closed sets are independent.

2.7 Example: Consider the bipolar valued fuzzy topological space (X, \mathfrak{T}) defined in Example 2.4. Then the bipolar valued fuzzy set $A = \{ \langle 1, 1, -1 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 0, 0 \rangle \}$ is bipolar valued fuzzy rw-closed but it is not bipolar valued fuzzy semi-closed set in bipolar valued fuzzy topological space X .

2.8 Example: Consider the bipolar valued fuzzy topological space (X, \mathfrak{T}) defined in Example 2.5. In this bipolar valued fuzzy topological space X , the bipolar valued fuzzy set μ is define by $\mu = \{ \langle 1, 1, -1 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 1, -1 \rangle, \langle 4, 0, 0 \rangle \}$. Then μ is bipolar valued fuzzy semi-closed in bipolar valued fuzzy topological space X . μ is also bipolar valued fuzzy regular semi open set containing μ which does not contain $\text{cl}(\mu) = B^c = \{ \langle 1, 1, -1 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 1, -1 \rangle, \langle 4, 1, -1 \rangle \}$. Therefore μ is not bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space X .

2.9 Theorem: Every bipolar valued fuzzy w-closed set is bipolar valued fuzzy rw-closed.

Proof: The proof follows from the Definition 2.1 and the fact that every bipolar valued fuzzy regular semi open set is bipolar valued fuzzy semi open.

Remark: The converse of Theorem 2.9 need not be true as from the following example.

2.10 Example: Let $X = \{ 1, 2 \}$ and $\mathfrak{T} = \{ 0_x, 1_x, A \}$ be bipolar valued fuzzy topology on X , where $A = \{ \langle 1, 0.7, -0.7 \rangle, \langle 2, 0.6, -0.6 \rangle \}$. Then the bipolar valued fuzzy set $B = \{ \langle 1, 0.7, -0.7 \rangle, \langle 2, 0.8, -0.8 \rangle \}$ is bipolar valued fuzzy rw-closed but it is not bipolar valued fuzzy w-closed.

2.11 Theorem: Every bipolar valued fuzzy rw-closed set is bipolar valued fuzzy rg-closed.

Proof: The proof follows from the Definition 2.1 and the fact that every bipolar valued fuzzy regular open set is bipolar valued fuzzy regular semi open.

Remark: The converse of Theorem 2.11 need not be true as from the following example.

2.12 Example: Let $X = \{ 1, 2, 3, 4 \}$ and bipolar valued fuzzy sets A, B, C, D defined as follows $A = \{ \langle 1, 0.9, -0.9 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle \}$, $B = \{ \langle 1, 0, 0 \rangle, \langle 2, 0.8, -0.8 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle \}$, $C = \{ \langle 1, 0.9, -0.9 \rangle, \langle 2, 0.8, -0.8 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle \}$, $D = \{ \langle 1, 0.9, -0.9 \rangle, \langle 2, 0.8, -0.8 \rangle, \langle 3, 0.7, -0.7 \rangle, \langle 4, 0, 0 \rangle \}$, $\mathfrak{T} = \{ 1_x, 0_x, A, B, C, D \}$ be bipolar valued fuzzy topology on X . Then the bipolar valued fuzzy set $E = \{ \langle 1, 0, 0 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 0.7, -0.7 \rangle, \langle 4, 0, 0 \rangle \}$ is bipolar valued fuzzy rg-closed but it is not bipolar valued fuzzy rw-closed.

2.13 Theorem: Every bipolar valued fuzzy rw-closed set is bipolar valued fuzzy gpr-closed.

Proof: Let A is bipolar valued fuzzy rw-closed set in bipolar valued fuzzy topological space (X, \mathfrak{T}) . Let $A \subseteq O$, where O is bipolar valued fuzzy regular open in X . Since every bipolar valued fuzzy regular open set is bipolar valued fuzzy regular semi open and A is bipolar valued fuzzy rw-closed set, we have $\text{bcl}(A) \subseteq O$. Since every bipolar valued fuzzy closed set is bipolar valued fuzzy pre closed, $\text{bpcl}(A) \subseteq \text{bcl}(A)$. Hence $\text{bpcl}(A) \subseteq O$ which implies that A is bipolar valued fuzzy gpr-closed.

Remark: The converse of Theorem 2.13 need not be true as from the following example.

2.14 Example: Let $X = \{ 1, 2, 3, 4, 5 \}$ and bipolar valued fuzzy sets A, B, C defined as follows $A = \{ \langle 1, 0.9, -0.9 \rangle, \langle 2, 0.8, -0.8 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle, \langle 5, 0, 0 \rangle \}$, $B = \{ \langle 1, 0, 0 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 0.8, -0.8 \rangle, \langle 4, 0.7, -0.7 \rangle, \langle 5, 0, 0 \rangle \}$, $C = \{ \langle 1, 0.9, -0.9 \rangle, \langle 2, 0.8, -0.8 \rangle, \langle 3, 0.8, -0.8 \rangle, \langle 4, 0.7, -0.7 \rangle, \langle 5, 0, 0 \rangle \}$. Let $\mathfrak{T} = \{ 1_x, 0_x, A, B, C \}$ be bipolar valued fuzzy topology on X . Then the bipolar valued fuzzy set $D = \{ \langle 1, 0.9, -0.9 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 0, 0 \rangle, \langle 4, 0, 0 \rangle, \langle 5, 0, 0 \rangle \}$ is bipolar valued fuzzy gpr-closed but it is not bipolar valued fuzzy rw-closed.

2.15 Theorem: If A is bipolar valued fuzzy regular open and bipolar valued fuzzy rg-closed in bipolar valued fuzzy topological space (X, \mathfrak{T}) , then A is bipolar valued fuzzy rw-closed in X .

Proof: Let A is bipolar valued fuzzy regular open and bipolar valued fuzzy rg-closed in X . We prove that A is bipolar valued fuzzy rw-closed in X . Let U be any bipolar valued fuzzy regular semi open set in X such that $A \subseteq U$. Since A is bipolar valued fuzzy regular open and bipolar valued fuzzy rg-closed, we have $\text{bcl}(A) \subseteq A$. Then $\text{bcl}(A) \subseteq A \subseteq U$. Hence A is bipolar valued fuzzy rw-closed in X .

2.16 Theorem: If A and B are bipolar valued fuzzy τ -closed sets in bipolar valued fuzzy topological space X , then union of A and B is bipolar valued fuzzy τ -closed set in bipolar valued fuzzy topological space X .

Proof: Let C be bipolar valued fuzzy regular semi open set in bipolar valued fuzzy topological space X such that $(A \cup B) \subseteq C$. Now $A \subseteq C$ and $B \subseteq C$. Since A and B are bipolar valued fuzzy τ -closed sets in bipolar valued fuzzy topological space X , $\text{bcl}(A) \subseteq C$ and $\text{bcl}(B) \subseteq C$. Therefore $(\text{bcl}(A) \cup \text{bcl}(B)) \subseteq C$. But $(\text{bcl}(A) \cup \text{bcl}(B)) = \text{bcl}(A \cup B)$. Thus $\text{bcl}(A \cup B) \subseteq C$. Hence $A \cup B$ is bipolar valued fuzzy τ -closed set in bipolar valued fuzzy topological space X .

2.17 Theorem: If A and B are bipolar valued fuzzy τ -closed sets in bipolar valued fuzzy topological space X , then the intersection of A and B need not be bipolar valued fuzzy τ -closed set in bipolar valued fuzzy topological space X .

Proof: Consider the bipolar valued fuzzy topological space (X, τ) defined in Example 2.5. In this bipolar valued fuzzy topological space X , the bipolar valued fuzzy sets G_1, G_2 are defined by $G_1 = \{ \langle 1, 0, 0 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 1, -1 \rangle, \langle 4, 1, -1 \rangle \}$ and $G_2 = \{ \langle 1, 1, -1 \rangle, \langle 2, 1, -1 \rangle, \langle 3, 1, -1 \rangle, \langle 4, 0, 0 \rangle \}$. Then G_1 and G_2 are the bipolar valued fuzzy τ -closed sets in bipolar valued fuzzy topological space X . Let $D = G_1 \cap G_2$. Then $D = \{ \langle 1, 0, 0 \rangle, \langle 2, 0, 0 \rangle, \langle 3, 1, -1 \rangle, \langle 4, 0, 0 \rangle \}$. Then $D = G_1 \cap G_2$ is not bipolar valued fuzzy τ -closed set in bipolar valued fuzzy topological space X .

REFERENCE

- [1]. Azad.K.K., On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity. *Jl.Math. Anal. Appl.* 82 No. 1 (1981), 14-32.
- [2]. Balachandran.K, Sundaram.P and Maki.H, On generalized continuous maps in topological spaces, *Mem.Fac Sci.Kochi Univ. Math.*, 12 (1991), 5-13.
- [3]. Balasubramanian.G and Sundaram.P, On some generalizations of fuzzy continuous functions, *Fuzzy sets and systems*, 86 (1997), 93-100.
- [4]. Chang.C.L., Fuzzy topological spaces, *Jl. Math. Anal. Appl.*, 24(1968), 182-190
- [5]. De, K., Biswas, R, Roy, A.R, On IFSSs, *Notes on IFSSs*, 4(2), (1998).
- [6]. Hedayati.H, Equivalence Relations Induced by (i,v) - (S, T) -fuzzy h -ideal (k -ideals) of semirings, *World Applied Sciences Journal*, 9(1) (2010), 01-13.
- [7]. Jun.Y.B and Kin.K.H, interval valued fuzzy R -subgroups of nearrings, *Indian Journal of Pure and Applied Mathematics*, 33(1) (2002), 71-80.
- [8]. Kaufmann. A, Introduction to the theory of fuzzy subsets, vol.1 Acad, Press N.Y.(1975).
- [9]. Klir.G.J and Yuan.B, Fuzzy sets and fuzzy logic, Theory and applications PHI (1997).
- [10]. Levine. N, Generalized closed sets in topology, *Rend. Circ. Mat. Palermo*, 19(1970), 89-96.
- [11]. Maki.H, Sundaram.P and Balachandran.K, On generalized continuous maps and pasting lemma in bitopological spaces, *Bull. Fukuoka Univ. Ed, part-III*, 40 (1991), 23-31.
- [12]. Malghan.S.R and Benchalli.S.S, On FTSSs, *Glasnik Matematicki*, Vol. 16(36) (1981), 313-325.
- [13]. Malghan.S.R and Benchalli.S.S, Open maps, closed maps and local compactness in FTSSs, *Jl.Math Anal. Appl* 99 No. 2(1984) 338-349.
- [14]. M.S.Anitha, K.L.Muruganantha Prasad & K.Arjunan, Notes on Bipolar valued fuzzy subgroups of a group, *Bulletin of Society for Mathematical Services and Standards*, Vol. 2, No. 3, 52-59, 2013.
- [15]. Mukherjee.M.N and Ghosh.B, Some stronger forms of fuzzy continuous mappings on FTSSs, *Fuzzy sets and systems*, 38 (1990), 375-387.
- [16]. Palaniappan.N and Rao.K.C, Regular generalized closed sets, *Kyungpook, Math. J.*, 33 (1993), 211-219.

- [17]. Solairaju.A and Nagarajan.R, Characterization of interval valued Anti fuzzy Left h-ideals over Hemirings, *Advances in fuzzy Mathematics*, Vol.4, Num. 2 (2009), 129-136.
 - [18]. Tapas Kumar Mondal and Samanta.S.K, Topology of interval valued fuzzy sets (download from net).
 - [19]. Thillaigovindan.N and Chinnadurai.V, interval valued-Fuzzy Generalized Bi-ideals of Semigroups, *Manonmaniam Sundaranar University (Algebra, Graph Theory and Their Applications)*, (2009).
 - [20]. Zadeh.L.A, Fuzzy sets, *Information and control*, Vol.8 (1965), 338-353.
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