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RESEARCH ARTICLE

STOCHASTIC BEHAVIOR OF A COLD STANDBY SYSTEM WITH IMPERFECT SWITCH UNDER POISSON SHOCKS AND REPAIRMAN VACATION

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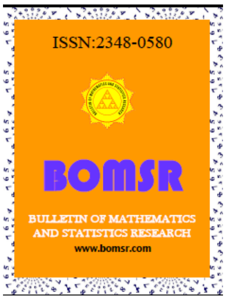

ABSTRACT

The main aim of the present paper is to analyze the stochastic behavior of a non-identical two unit cold standby system with imperfect switch under Poisson shocks. For this purpose, one of the two units is active and other is kept as cold standby. The system can fail due to external factors such as shocks. The shocks can infect the active unit. The repairman may exist in the system or leave for a vacation. The time shocks arrive follow a Poisson process while the distributions of all times are arbitrary. Various measures of reliability of the system are obtained using the supplementary variable technique. Finally special case is presented to illustrate the results.

Keywords: Poisson shock, Imperfect Switch, Reliability Indices.

1. INTRODUCTION

Poisson shocks play an important role in reliability engineering. Most researchers studied the effects of shocks on some different systems. [3] considered that the operating environment of the component is random, the working component may be influenced by other causes, they extend threshold on the component which is a random variable, and this assumption is more realistic. [5] studied a two-unit repairable system with a variant vacation policy. By using the supplementary variable technique and the vector Markov process theory. [7] derived the reliability indices of a cold standby system with unreliable repair facility and one repairman who can take multi-vacation under Poisson shocks. Some measures of repairable system with an unreliable repair facility and one repairman who can take single vacation considered by [6]. [1] derived some important reliability indexes of a warm standby repairable system consisting of two repairable components and a repairman who may take multiple vacations by using the supplementary variable technique. [9] considered the reliability analysis of a two-component cold standby system with a repairman who may have vacation. By using the vector Markov process theory, the supplementary variable method, Laplace transform, and Tauberian theory. Some measures of reliability are derived. The repair-replacement problem for a repairable cold standby system that is composed of two similar components with preventive maintenance under Poisson shocks is discussed by [4]. The simple repairable system with a warning device and a repairman who can have delayed-multiple vacations is investigated by [8]. [2] studied of the stochastic analysis of a two-unit cold standby system considering hardware failure, human error failure and preventive maintenance (PM). Various measures of reliability of the system are obtained using the regenerative point technique and all these measures are also derived based on time t_0.

This paper is more general than the paper of Wu and Wu [3]. Other meaning, we discuss the imperfect switching system with two components. All results this paper are studied with perfect switch as special case. The structure of this paper is organized as follows. The model description and assumptions are presented in section 2. In section 3, we construct the state equations of the system and obtain the Laplace transforms of the state probabilities of the system. Section 4 presents some reliability measure of the system. Section 5 discusses some special cases to illustrate the effects of parameters on the system performance. Conclusions are given in section 6.

2. Model and Assumptions

The following assumptions are associated with the system.

1. The system consists of two non-identical units and switch. Initially one unit is operating and the other is in standby case (cold standby).
2. The switch between operating unit and standby unit is imperfect.
3. The system is subject to shocks. The arrivals of the shocks follow a Poisson process $\left[N(t), t \geq 0\right]$ with the intensity $\lambda > 0$. The magnitude of each shock $\hat{X}$, is an independent random variable with distribution function F.
4. When a shock arrives, it only affects the operating unit. The operating unit will fail when the magnitude of a shock exceeds a threshold. The threshold of unit i is a non-negative random variable τ_i with a distribution function Φ_i (i = 1, 2).
5. The switch is perfect with probability $p = 1 - q$.
6. When a unit fails with the presence of the repairman, it will be repaired immediately. The switch has the priority to be repaired.
7. Service discipline is a first-come, first-served (FCFS). A single repair facility is available for repair (switch and unit).
8. If a unit fails when the other is being repaired, the newly failed unit must wait for repair and the system is down. If two units are waiting for repair when the repairman comes to the system, unit 1 has the priority to be repaired.
9. Once the failed unit is repaired, the repairman takes a vacation.
10. All random variables are independent. At the beginning, the two units are new, one unit starts to work, the other unit is in cold standby state and the repairman be in vacation. The units and switching device can be repaired as good as new.

Nomenclature

$Y_i\ (i = 1,2)$	:	Unit i 's repair time, where $H_i(y_i) = \int_0^t h_i(y_i)\, dy_i = 1 - e^{-\int_0^t \mu_i(y_i) dy_i}$.
X	:	Vacation time when the repairman is taking a vacation. Its distribution is $V(x) = \int_0^t v(x)\, dx = 1 - e^{-\int_0^t \alpha(x) dx}$.
Z	:	Switching device's repair time. Its distribution is $A(z) = \int_0^t A(z)\, dz = 1 - e^{-\int_0^t \gamma(z) dz}$.

*	:	Laplace transforms; $f^*(s) = L_s[f(x)] = \int_0^\infty f(x)e^{-xs}\,dx\,; s > 0$.
AV	:	The steady-state availability of the system.
R(t)	:	The reliability of the system.
MTTF	:	Mean time to the system failure.
r_i	:	The probability that one shock causes unit i to fail, $(i = 1,2)$

3. Model Analysis

With the model assumptions given in the preceding section, the failure probability of unit i $(i = 1,2)$, given the shock value $\hat{x}$, is $\Phi_i\,(\hat{x}) = P(\tau < \hat{x})$.Since the magnitude of a shock is a random variable $\hat{X}$, the conditional failure probability of unit i is a random $\Phi_i\,(\hat{x})$ with $(i = 1,2)$, respectively, and its probability distribution can be written by: $P_i(x) = P\big(\Phi_i\,(\hat{X}) \le x\big) = P\left(\hat{X} \le \Phi_i^{-1}\,(x)\right) = F\left(\Phi_i^{-1}\,(x)\right), 0 < x < 1, (i = 1,2)$.

From assumption, we can see that, the probability that one shock causes unit i to fail is:

$r_i\; = P\big(\hat{X} > \tau_i\big) = \int_0^\infty P\big(\tau_i < \hat{x}\big|\hat{X} = \hat{x}\big)dP\big(\hat{X} \le \hat{x}\big) = \int_0^\infty \Phi_i\,(\hat{x})dF(\hat{x}).\ (i = 1,2)$.

Let $S(t)$be the system state at time t, then

State 0: at time t, unit 1 is working, unit 2 is on cold standby and the repairman is taking a vacation.

State 1: at time t, unit 2 is working, unit 1 is on cold standby and the repairman is taking a vacation.

State 2: at time t, unit 2 is working, unit 1 is waiting for repair and the repairman is taking a vacation.

State 3: at time t, unit 1 is working, unit 2 is waiting for repair and the repairman is taking a vacation.

State 4: at time t, the switching device is waiting for repair, unit 1 is also waiting for repair, unit 2 is on cold standby and the repairman is taking a vacation.

State 5: at time t, the switching device is waiting for repair, unit 2 is also waiting for repair, unit 1 is on cold standby and the repairman is taking a vacation.

State 6: at time t, unit 1 is working, unit 2 is on cold standby and the repairman is idle.

State 7: at time t, unit 2 is working, unit 1 is on cold standby and the repairman is idle.

State 8: at time t, the switching device is being repaired, unit 1 is waiting for repair and unit 2 is on cold standby.

State 9: at time t, the switching device is being repaired, unit 2 is waiting for repair and unit 1 is on cold standby.

State 10: at time t, unit 2 is working and unit 1 is being repaired.

State 11: at time t, unit 1 is working and unit 2 is being repaired.

State 12: at time t, the two units are waiting for repair and the repairman is taking a vacation.

State 13: at time t, unit 1 is being repaired and unit 2 is waiting for repair.

State 14: at time t, unit 2 is being repaired and unit 1 is waiting for repair.

The state space is $\Omega = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14\}$ where the up state set is $\mathrm{W} = \{0,1,2,3,6,7,10,11\}$ and the down state set $\mathrm{F} = \{4,5,8,9,12,13,14\}$. As the repair, inspect and post repair time have a general continuous distribution, $\{S(t), t \ge 0\}$ is not a Markov process. So we introduce supplementary variable,

- X(t) : if $S(t) = 0,1,2,3,,4,5,12$,then X(t) is the elapsed vacation time when the repairman is taking a vacation at time t.
- Y_1 (t) : if $S(t) = 10,13$, then Y_1 (t) is the elapsed repair time of unit 1 being repaired at time t.
- Y_2 (t) : if $S(t) = 11,14$, then Y_2 (t) is the elapsed repair time of unit 2 being repaired at time t
- Z(t) : if $S(t) = 8,9$, then Z(t) is the elapsed repair time of the switching device being repaired at time t.

Then $\{(S(t), X(t), \mathrm{Y}_1(\mathrm{t}), \mathrm{Y}_2(\mathrm{t}), \mathrm{Z}(\mathrm{t})), \mathrm{t} \geq 0\}$ is a generalized Markov process. Denote:

$Q_i(t,x) = P(S(t) = i, X(t) \leq x), \quad (i = 0,1,2,3,4,5,12).$

$Q_i(t,y_1) = P(S(t) = i, Y_1(t) \leq y_1), \quad (i = 10,13).$

$Q_i(t,y_2) = P(S(t) = i, Y_2(t) \leq y_2), \quad (i = 11,14).$

$Q_i(t,z) = P(S(t) = i, Z(t) \leq z), \quad (i = 8,9).$

where $P(B)$is probability of event B. consider:

$P_i(t,w) = \frac{d}{dw} Q_i(t,w) \quad (i = \{0,1,2,3,4,5,8,9,10,11,12,13,14\}).$

Using the probability arguments and limiting transitions, we have the following integro-differential equations:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_1\lambda\right) P_0(t,x) = 0\,, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_2\lambda\right) P_1(t,x) = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_2\lambda\right) P_2(t,x) = p(r_1\lambda)P_0(t,x), \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_1\lambda\right) P_3(t,x) = p(r_2\lambda)P_1(t,x), \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x)\right) P_4(t,x) = q(r_1\lambda)P_0(t,x), \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x)\right) P_5(t,x) = q(r_2\lambda)P_1(t,x), \tag{6}$$

$$\left(\frac{d}{dt} + r_1\lambda\right) P_6(t) = \int_0^\infty \alpha(x)P_0(t,x)dx, \tag{7}$$

$$\left(\frac{d}{dt} + r_2\lambda\right) P_7(t) = \int_0^\infty \alpha(x)P_1(t,x)dx, \tag{8}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \gamma(z)\right) P_8(t,z) = 0, \tag{9}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \gamma(z)\right) P_9(t,z) = 0, \tag{10}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + \mu_1(y_1) + r_2\lambda\right) P_{10}(t,y_1) = 0, \tag{11}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \mu_2(y_2) + r_1\lambda\right) P_{11}(t,y_2) = 0, \tag{12}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x)\right) P_{12}(t,x) = r_2\lambda P_2(t,x) + r_1\lambda P_3(t,x), \tag{13}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + \mu_1(y_1)\right) P_{13}(t,y_1) = r_2\lambda P_{10}(t,y_1), \tag{14}$$

and

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_2} + \mu_2(y_2)\right) P_{14}(t,y_2) = r_1\lambda P_{11}(t,y_2). \tag{15}$$

Their boundary conditions are:-

$$P_2(t,0) = P_3(t,0) = P_4(t,0) = P_5(t,0) = P_{12}(t,0) = P_{14}(t,0) = 0, \tag{16}$$

$$P_0(t,0) = \int_0^\infty \mu_2(y_2)P_{11}(t,y_2)dy_2 + \delta(t)\,, \tag{17}$$

$$P_1(t,0) = \int_0^\infty \mu_1(y_1)P_{10}(t,y_1)dy_1, \tag{18}$$

$$P_8(t,0) = \int_0^\infty \alpha(x)P_4(t,x)dx + q(r_1\lambda)P_6(t), \tag{19}$$

$$P_9(t,0) = \int_0^\infty \alpha(x)P_5(t,x)dx + q(r_2\lambda)P_7(t), \tag{20}$$

$$P_{10}(t,0) = p(r_1\lambda)P_6(t) + \int_0^\infty \alpha(x)P_2(t,x)dx + \int_0^\infty \gamma(z)P_8(t,z)dz + \int_0^\infty \mu_2(y_2)P_{14}(t,y_2)dy_2\,, \tag{21}$$

$$P_{11}(t,0) = p(r_2\lambda)P_7(t) + \int_0^\infty \alpha(x)P_3(t,x)dx + \int_0^\infty \gamma(z)P_9(t,z)dz + \int_0^\infty \mu_1(y_1)P_{13}(t,y_1)dy_1\,, \tag{22}$$

and,

$$P_{13}(t,0) = \int_0^\infty \alpha(x)P_{12}(t,x)dx. \tag{23}$$

The initial conditions are

$$P_1(0,x) = \delta(x) = \begin{cases} 1, & x = 0 \\ 0, & x \neq 0 \end{cases}, \tag{24}$$

Otherwise is zero

According to the formula of the total probability, we have:

$$\sum_{i=0}^{5} \int_0^\infty P_i(t,x)dx + P_6(t) + P_7(t) + \sum_{i=8}^{14} \int_0^\infty P_i(t,x)dx = 1. \tag{25}$$

4.Model analysis:-

4.1. steady-state availability:

Let the following steady-probability:

$P_i = \lim_{t\to\infty} P_i(t) \quad , i = \{0,1,2,\dots,14\},$

$g_i(u) = \lim_{t\to\infty} P_i(t,u) \quad , i = \{0,1,2,3,4,5,8,9,10,11,12,13,14\}.$

This follows the following relations:

$P_i = \int_0^\infty g_i(u)du \ ,(i = \{0,1,2,3,4,5,8,9,10,11,12,13,14\}).$

By taking the limit of Eq.(1-25) as $t \to \infty$, The following equations can be obtained:

$$\left(\frac{d}{dx} + \alpha(x) + r_1\lambda\right) g_0(x) = 0, \tag{26}$$

$$\left(\frac{d}{dx} + \alpha(x) + r_2\lambda\right) g_1(x) = 0, \tag{27}$$

$$\left(\frac{d}{dx} + \alpha(x) + r_2\lambda\right) g_2(x) = p(r_1\lambda)g_0(x), \tag{28}$$

$$\left(\frac{d}{dx} + \alpha(x) + r_1\lambda\right) g_3(x) = p(r_2\lambda)g_1(x), \tag{29}$$

$$\left(\frac{d}{dx} + \alpha(x)\right) g_4(x) = q(r_1\lambda)g_0(x), \tag{30}$$

$$\left(\frac{d}{dx} + \alpha(x)\right) g_5(x) = q(r_2\lambda)g_1(x), \tag{31}$$

$$r_1\lambda P_6 = \int_0^\infty \alpha(x)g_0(x)\,dx, \tag{32}$$

$$r_2\lambda P_7 = \int_0^\infty \alpha(x)g_1(x)\,dx, \tag{33}$$

$$\left(\frac{d}{dz} + \gamma(z)\right) g_8(z) = 0, \tag{34}$$

$$\left(\frac{d}{dz} + \gamma(z)\right) g_9(z) = 0, \tag{35}$$

$$\left(\frac{d}{dy_1} + \mu_1(y_1) + r_2\lambda\right) g_{10}(y_1) = 0, \tag{36}$$

$$\left(\frac{d}{dy_2} + \mu_2(y_2) + r_1\lambda\right) g_{11}(y_2) = 0, \tag{37}$$

$$\left(\frac{d}{dx} + \alpha(x)\right) g_{12}(x) = r_2\lambda g_2(x) + (r_1\lambda)g_3(x), \tag{38}$$

$$\left(\frac{d}{dy_1} + \mu_1(y_1)\right) g_{13}(y_1) = r_2\lambda g_{10}(y_1), \tag{39}$$

and

$$\left(\frac{d}{dy_2} + \mu_2(y_2)\right) g_{14}(y_2) = r_1\lambda g_{11}(y_2). \tag{40}$$

Their boundary conditions are:-

$$g_2(0) = g_3(0) = g_4(0) = g_5(0) = g_{12}(0) = g_{14}(0) = 0, \tag{41}$$

$$g_0(0) = \int_0^\infty \mu_2(y_2)g_{11}(y_2)\,dy_2, \tag{42}$$

$$g_1(0) = \int_0^\infty \mu_1(y_1)g_{10}(y_1)\,dy_1, \tag{43}$$

$$g_8(0) = \int_0^\infty \alpha(x)g_4(x)\,dx + q(r_1\lambda)P_6, \tag{44}$$

$$g_9(0) = \int_0^\infty \alpha(x)g_5(x)\,dx + q(r_2\lambda)P_7, \tag{45}$$

$$g_{10}(0) = p(r_1\lambda)P_6 + \int_0^\infty \alpha(x)g_2(x)\,dx + \int_0^\infty \gamma(z)g_8(z)\,dz + \int_0^\infty \mu_2(y_2)g_{14}(y_2)dy_2\ , \tag{46}$$

$g_{11}(0) = p(r_2\lambda)P_7 + \int_0^\infty \alpha(x)g_3(x)\,dx + \int_0^\infty \gamma(z)g_9(z)dz + \int_0^\infty \mu_1(y_1)g_{13}(y_1)dy_1$, (47)

and

$g_{13}(0) = \int_0^\infty \alpha(x)g_{12}(x)\,dx.$ (48)

Hence the solutions $g_i(u)$,i = 0,1,2,3,4,5,8,9,10,11,12,13,14 ,P_6 and P_7 can be obtained. Formula $P_i = \int_0^\infty g_i(u)du$, $i = 0,1,2,3,4,5,8,9,10,11,12,13,14$,the following steady-state probabilities can be calculated as follows:

$P_0 = c_0\bar{V}^*[r_1\lambda]$, (49)

$P_1 = c_0\, c_1\, \bar{V}^*[\lambda r_2]\, {h_1}^*[\lambda r_2]$, (50)

$P_2 = c_0\, c_2\;\, p\, r_1,$ (51)

$P_3 = c_0\;\, c_1\, c_2\;\, p\, r_2\, {h_1}^*[\lambda r_2]$, (52)

$P_4 = c_0\, \frac{(1-p)(1-\alpha\,\bar{V}^*[\lambda r_1])}{\alpha}$, (53)

$P_5 = c_0\, \frac{c_1\,(1-p)(1-\alpha\bar{V}^*[\lambda r_2]){h_1}^*[\lambda r_2]}{\alpha}$, (54)

$P_6 = c_0\, \frac{v^*[\lambda r_1]}{\lambda r_1},$ (55)

$P_7 = c_0\;\, c_1 \frac{v^*[\lambda r_2]{h_1}^*[\lambda r_2]}{\lambda r_2},$ (56)

$P_8 = c_0\,(1-p)\frac{1}{\gamma},$ (57)

$P_9 = c_0\, \frac{c_1(1-p){h_1}^*[\lambda r_2]}{\gamma},$ (58)

$P_{10} = \frac{c_0\;\,(r_1-r_2+p(r_2(1-v^*[\lambda r_1])-r_1(1-v^*[\lambda r_2])){h_2}^*[\lambda r_1])\bar{H_1}^*[\lambda r_2]}{(r_1-r_2){h_2}^*[\lambda r_1]}$, (59)

$P_{11} = \frac{c_0\;\,\bar{H_2}^*[\lambda r_1]}{{h_2}^*[\lambda r_1]},$ (60)

$P_{12} = c_0\, c_3\, p(1 + c_1\, {h_1}^*[\lambda r_2])$, (61)

$$P_{13} = \frac{c_0}{\mu_1}\left(c_4(p + pc_1{h_1}^*[\lambda r_2]) - (1-\mu_1\bar{H_1}^*[\lambda r_2])\left(\frac{(r_2-r_1)(1+c_1\,{h_1}^*[\lambda r_2])}{(r_1-r_2)({h_1}^*[\lambda r_2](1-{h_2}^*[\lambda r_1])+{h_2}^*[\lambda r_1])} + \frac{(p(r_1(1-v^*[\lambda r_2])-r_2(1-v^*[\lambda r_1]))+c_1(r_1-r_2)(h_1)^*[\lambda r_2])(h_2)^*[\lambda r_1]}{(r_1-r_2)({h_1}^*[\lambda r_2](1-{h_2}^*[\lambda r_1])+{h_2}^*[\lambda r_1])}\right)\right), \tag{62}$$

$P_{14} = \frac{c_0\,(\frac{1}{\mu_2}-\bar{H_2}^*[\lambda r_1])}{{h_2}^*[\lambda r_1]},$ (63)

The steady-state availability of the system can be written as :

$$\mathrm{AV} = P_0 + P_1 + P_2 + P_3 + P_6 + P_7 + P_{10} + P_{11}$$

After some simplifications, we get

$$\mathrm{Av} = c_0(pc_2r_1 + \frac{v^*[\lambda r_1]}{\lambda r_1} + \bar{V}^*[\lambda r_1] + pc_1c_2r_2{h_1}^*[\lambda r_2] + \frac{c_1v^*[\lambda r_2]{h_1}^*[\lambda r_2]}{\lambda r_2} + c_1\bar{V}^*[\lambda r_2]{h_1}^*[\lambda r_2] + \frac{(r_1-r_2+p(r_2(1-v^*[\lambda r_1])-r_1(1-v^*[\lambda r_2])){h_2}^*[\lambda r_1])\bar{H_1}^*[\lambda r_2]}{(r_1-r_2){h_2}^*[\lambda r_1]} + \frac{\bar{H_2}^*[\lambda r_1]}{{h_2}^*[\lambda r_1]}), \tag{64}$$

where,

$c_1 = (\frac{p(r_2(1-v^*[\lambda r_1])-r_1(1-v^*[\lambda r_2]))}{r_1-r_2} + \frac{1}{(h_2)^*[\lambda r_1]})$,

$c_2 = \frac{(\bar{V}^*[r_2\lambda]-\bar{V}^*[r_1\lambda])}{r_1-r_2}$,

$c_3 = \frac{((1-\alpha(\bar{V})^*[\lambda r_2])r_1-(1-\alpha(\bar{V})^*[\lambda r_1])r_2)}{\alpha(r_1-r_2)}$,

$c_4 = \frac{((1-(v)^*[\lambda r_2])r_1-(1-(v)^*[\lambda r_1])r_2)}{r_1-r_2},$

and,

$$c_0{}^{-1} = \frac{q}{\gamma} + \frac{qc_1h_1{}^*[\lambda r_2]}{\gamma} + \frac{c_1v^*[\lambda r_2]h_1{}^*[\lambda r_2]}{\lambda r_2} + pc_3(1 + c_1h_1{}^*[\lambda r_2]) + \frac{p\alpha\lambda\, c_2r_1^2 + \alpha v^*[\lambda r_1] + \lambda r_1(q + p\alpha\bar{V}^*[\lambda r_1] + c_1(q + p\alpha(c_2r_2 + \bar{V}^*[\lambda r_2]))h_1{}^*[\lambda r_2])}{\alpha\lambda r_1} + \frac{(r_1 - r_2 + p(r_2(1 - v^*[\lambda r_1]) - r_1(1 - v^*[\lambda r_2]))h_2{}^*[\lambda r_1])\bar{H}_1{}^*[\lambda r_2]}{(r_1 - r_2)h_2{}^*[\lambda r_1]} + \frac{c_4(p + pc_1(h_1)^*[\lambda r_2])}{\mu_1} + \frac{((r_2 - r_1)(1 + c_1h_1{}^*[\lambda r_2]) + (p(r_1(1 - v^*[\lambda r_2]) - r_2(1 - v^*[\lambda r_1])) + c_1(r_1 - r_2)h_1{}^*[\lambda r_2])h_2{}^*[\lambda r_1])(1 - \mu_1\bar{H}_1{}^*[\lambda r_2])}{\mu_1(r_1 - r_2)(h_1{}^*[\lambda r_2](-1 + h_2{}^*[\lambda r_1]) - h_2{}^*[\lambda r_1])} + \frac{\bar{H}_2{}^*[\lambda r_1]}{h_2{}^*[\lambda r_1]} + \frac{1 - \mu_2\bar{H}_2{}^*[\lambda r_1]}{\mu_2h_2{}^*[\lambda r_1]}.$$

4.2. The steady-state probability of the repairman vacation:

Since the steady-state probability of the repairman vacation is

$$\mathrm{V} = P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_{12}.$$

then

$$\mathrm{V} = \frac{c_0}{\alpha}(q + p\alpha c_3 + p\alpha c_2r_1 + p\alpha\bar{V}^*[\lambda r_1] + c_1(q + p\alpha(c_3 + c_2r_2 + \bar{V}^*[\lambda r_2]))(h_1)^*[\lambda r_2]) . \quad (65)$$

4.3. The steady-state probability when the system is waiting:

The steady-state probability when the system is waiting can be written as

$$\mathrm{W} = P_4 + P_5 + P_{12}$$

then

$$\mathrm{W} = \frac{c_0}{\alpha}(p\alpha c_3(1 + c_1(h_1)^*[\lambda r_2]) + q(1 - \alpha\bar{V}^*[\lambda r_1] + c_1(1 - \alpha\bar{V}^*[\lambda r_2])(h_1)^*[\lambda r_2])) . \quad (66)$$

4.4. The steady-state failure frequency of the system:

The steady-state failure frequency of the system is

$$F_f = \lambda r_2P_2 + \lambda r_1P_3 + \lambda r_2P_{10} + \lambda r_1P_{11}$$

then

$$F_f = \lambda c_0(pc_2r_1r_2(1 + c_1h_1{}^*[\lambda r_2]) + \frac{r_2}{(r_1 - r_2)h_2{}^*[\lambda r_1]}((r_1 - r_2 + p(r_2(1 - v^*[\lambda r_1]) - r_1(1 - v^*[\lambda r_2]))h_2{}^*[\lambda r_1])\bar{H}_1{}^*[\lambda r_2] + r_1(r_1 - r_2)\bar{H}_2{}^*[\lambda r_1])) \quad (67)$$

4.5. The total profit of system:

The total profit of system is given by

$$\mathrm{I} = k_1\mathrm{AV} - k_2F_f - k_3\mathrm{V}$$

where, k_1 , k_2 and k_3 represent the dividend of the system for working unit per unit time, the loss of the system for failed unit per unit time, and the dividend of the system for repairman vacation per unit time, respectively.

4.6. Reliability of the system :

Using the method similar to that in Sec. (4.1), we have the following partial-differential equations:

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_1\lambda\right)L_0(t,x) = 0, \quad (68)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_2\lambda\right)L_1(t,x) = 0, \quad (69)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_2\lambda\right)L_2(t,x) = p(r_1\lambda)L_0(t,x), \quad (70)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \alpha(x) + r_1\lambda\right)L_3(t,x) = p(r_2\lambda)L_1(t,x), \quad (71)$$

$$\left(\frac{d}{dt} + r_1\lambda\right)L_6(t) = \int_0^\infty \alpha(x)L_0(t,x)dx, \quad (72)$$

$$\left(\frac{d}{dt} + r_2\lambda\right)L_7(t) = \int_0^\infty \alpha(x)L_1(t,x)dx, \quad (73)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y_1} + \mu_1(y_1) + r_2\lambda\right)L_{10}(t,y_1) = 0, \quad (74)$$

and

$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y_2}+\mu_2(y_2)+r_1\lambda\right)L_{11}(t,y_2)=0.$ (75)

Their boundary conditions are:-

$L_2(t,0)=L_3(t,0)=0,$ (76)

$L_0(t,0)=\int_0^\infty \mu_2(y_2)L_{11}(t,y_2)dy_2+\delta(t)\,,$ (77)

$L_1(t,0)=\int_0^\infty \mu_1(y_1)L_{10}(t,y_1)dy_1,$ (78)

$L_{10}(t,0)=p(r_1\lambda)L_6(t)+\int_0^\infty \alpha(x)L_2(t,x)dx,$ (79)

and

$L_{11}(t,0)=p(r_2\lambda)L_7(t)+\int_0^\infty \alpha(x)L_3(t,x)dx.$ (80)

The initial conditions are

$L_1(0,x)=\delta(x)=\begin{cases}1, & x=0\\ 0, & x\neq 0'\end{cases}$ (81)

Otherwise is 0

Making use of Laplace transform with respect to *t* to Equ(68-81) , gives

$\left(\frac{d}{dx}+s+\alpha(x)+r_1\lambda\right)L_0{}^*(s,x)=0,$ (82)

$\left(\frac{d}{dx}+s+\alpha(x)+r_2\lambda\right)L_1{}^*(s,x)=0,$ (83)

$\left(\frac{d}{dx}+s+\alpha(x)+r_2\lambda\right)L_2{}^*(s,x)=p(r_1\lambda)L_0{}^*(s,x),$ (84)

$\left(\frac{d}{dx}+s+\alpha(x)+r_1\lambda\right)L_3{}^*(s,x)=p(r_2\lambda)L_1{}^*(s,x),$ (85)

$(s+r_1\lambda)L_6{}^*(s)=\int_0^\infty \alpha(x)L_0{}^*(s,x)dx,$ (86)

$(s+r_2\lambda)L_7{}^*(s)=\int_0^\infty \alpha(x)L_1{}^*(s,x)\,dx,$ (87)

$\left(\frac{d}{dy_1}+s+\mu_1(y_1)+r_2\lambda\right)L_{10}{}^*(s,y_1)=0,$ (88)

and

$\left(\frac{d}{dy_2}+s+\mu_2(y_2)+r_1\lambda\right)L_{11}{}^*(s,y_2)=0.$ (89)

Their boundary conditions are:-

$L_2{}^*(s,0)=L_3{}^*(s,0)=0,$ (90)

$L_0{}^*(s,0)=\int_0^\infty \mu_2(y_2)L_{11}{}^*(s,y_2)\,dy_2+1,$ (91)

$L_1{}^*(s,0)=\int_0^\infty \mu_1(y_1)L_{10}{}^*(s,y_1)\,dy_1,$ (92)

$L_{10}{}^*(s,0)=p(r_1\lambda)L_6{}^*(s)+\int_0^\infty \alpha(x)L_2{}^*(s,x)dx\,,$ (93)

and

$L_{11}{}^*(s,0)=p(r_2\lambda)L_7{}^*(s)+\int_0^\infty \alpha(x)L_3{}^*(s,x)dx\,.$ (94)

By solution the above equation, we find:

$L_0{}^*(s,x)=C_5e^{-x(s+\lambda r_1)}\bar{V}[x]\,,$ (95)

$L_1{}^*(s,x)=C_6e^{-x(s+\lambda r_2)}\bar{V}[x],$ (96)

$L_2{}^*(s,x)=c_5r_1\,p\,\frac{(e^{-x(s+\lambda r_2)}-e^{-x(s+\lambda r_1)})\bar{V}[x]}{r_1-r_2}\,,$ (97)

$L_3{}^*(s,x)=c_6r_2\,p\,\frac{(e^{-x(s+\lambda r_2)}-e^{-x(s+\lambda r_1)})\bar{V}[x]}{r_1-r_2}\,,$ (98)

$L_6{}^*(s)=c_5\frac{v^*[s+\lambda r_1]}{s+\lambda r_1}\,,$ (99)

$L_7{}^*(s)=c_6\frac{v^*[s+\lambda r_2]}{s+\lambda r_2},$ (100)

$L_{10}{}^*(s,x)=pc_5r_1\bar{H}_1[x]\frac{e^{-x(s+\lambda r_2)}\,((s+\lambda r_1)v^*[s+\lambda r_2]-(s+\lambda r_2)v^*[s+\lambda r_1])}{(s+\lambda r_1)(r_1-r_2)}\,,$ (101)

and

$$L_{11}{}^{*}(s,x) = pc_6 r_2 \bar{H}_2[x] \frac{e^{-x(s+\lambda r_1)}((s+\lambda r_1)v^*[s+\lambda r_2]-(s+\lambda r_2)v^*[s+\lambda r_1])}{(r_1-r_2)(s+\lambda r_2)} . \quad (102)$$

Where,

$$C_5^{-1} = 1 - \frac{r_1 r_2 (p(s+\lambda r_2)v^*[s+\lambda r_1] - p(s+\lambda r_1)v^*[s+\lambda r_2])^2 h_1{}^*[s+\lambda r_2] h_2{}^*[s+\lambda r_1]}{(s+\lambda r_1)(r_1-r_2)^2(s+\lambda r_2)}$$

$$c_6 = \frac{pc_5 r_1((s+\lambda r_1)v^*[s+\lambda r_2] - (s+\lambda r_2)v^*[s+\lambda r_1])(h_1)^*[s+\lambda r_2]}{(s+\lambda r_1)(r_1-r_2)}$$

The reliability of the system is

$$R(t) = \sum_{\mathrm{i}=0}^{3} \int_0^\infty L_i(\mathrm{t,x})\mathrm{dx} + L_6(\mathrm{t}) + L_7(\mathrm{t}) + \sum_{\mathrm{j}=10}^{11} \int_0^\infty L_i(\mathrm{t,x})\mathrm{dx} \quad (103)$$

The laplace transformation of $R(t)$

$$R^*(s) = \sum_{\mathrm{i}=0}^{3} \int_0^\infty L_{\mathrm{i}}{}^*(\mathrm{s,x})\mathrm{dx} + L_6{}^*(\mathrm{s}) + L_7{}^*(s) + \sum_{\mathrm{j}=10}^{11} \int_0^\infty L_j{}^*(\mathrm{s,x})\mathrm{dx}$$

$$R^*(s) = \frac{c_5 v^*[s+\lambda r_1]}{s+\lambda r_1} + \frac{c_6 v^*[s+\lambda r_2]}{s+\lambda r_2} + c_5\bar{V}^*[s+\lambda r_1] + c_6\bar{V}^*[s+\lambda r_2] + \frac{pc_5 r_1(-\bar{V}^*[s+\lambda r_1]+\bar{V}^*[s+\lambda r_2])}{r_1-r_2} + \frac{pc_6 r_2(-\bar{V}^*[s+\lambda r_1]+\bar{V}^*[s+\lambda r_2])}{r_1-r_2} + \frac{pc_5 r_1((s+\lambda r_2)v^*[s+\lambda r_1]-(s+\lambda r_1)v^*[s+\lambda r_2])(\bar{H}_1)^*[s+\lambda r_2]}{(-r_1+r_2)(s+\lambda r_2)} + \frac{pc_5 r_1(-(s+\lambda r_2)v^*[s+\lambda r_1]+(s+\lambda r_1)v^*[s+\lambda r_2])(\bar{H}_2)^*[s+\lambda r_1]}{(s+\lambda r_1)(r_1-r_2)} \quad (104)$$

The mean time of first failure of the system *(MTTFF*) is:

$$MTTF = \int_0^\infty R(t)dt = \lim_{s\to 0} R^*(s).$$

Hence,

$$MTTF = \left(1 - \frac{p^2(r_2 v^*[\lambda r_1] - r_1 v^*[\lambda r_2])^2 (h_1)^*[\lambda r_2](h_2)^*[\lambda r_1]}{(r_1-r_2)^2}\right)^{-1} \left(\frac{v^*[\lambda r_1]}{\lambda r_1} + \bar{V}^*[\lambda r_1] + \frac{pr_1(-\bar{V}^*[\lambda r_1]+\bar{V}^*[\lambda r_2])}{r_1-r_2} + \frac{pv^*[\lambda r_2](-r_2 v^*[\lambda r_1]+r_1 v^*[\lambda r_2])(h_1)^*[\lambda r_2]}{\lambda(r_1-r_2)r_2} + \frac{p^2 r_2(r_2 v^*[\lambda r_1]-r_1 v^*[\lambda r_2])(\bar{V}^*[\lambda r_1]-\bar{V}^*[\lambda r_2])(h_1)^*[\lambda r_2]}{(r_1-r_2)^2} + \frac{p(-r_2 v^*[\lambda r_1]+r_1 v^*[\lambda r_2])\bar{V}^*[\lambda r_2](h_1)^*[\lambda r_2]}{r_1-r_2} + \frac{pr_1(-r_2 v^*[\lambda r_1]+r_1 v^*[\lambda r_2])(\bar{H}_1)^*[\lambda r_2]}{(r_1-r_2)r_2} + \frac{p(-r_2 v^*[\lambda r_1]+r_1 v^*[\lambda r_2])(\bar{H}_2)^*[\lambda r_1]}{r_1-r_2}\right) \quad (105)$$

5. Special Case

The following special cases are obtained as special case from our results:

- Case 1: if $P = 1, q = 0$, then the results of [3] can be obtained.
- Case 2: if $r_1 = r_2 = 0$, then the working unit will never fail.
- Case3: if $r_1 = r_2 = 1$, $P(X = 0) = 1$ and $q \neq 0$, then each shock will cause the working unit to fail , the switch is imperfect and the repairman will repair once the failed occur.

6. Numerical Example and Study of System Behavior Through Graphs:

Let $\alpha(x) = \alpha, \gamma(z) = \gamma, \mu_i(y_i) = \mu_i$ and $\mathrm{k_i(t_i) = k_i}$; where$(i = 1,2)$.

We plot the steady- state availability and mean time to system failure for the system model. One can note that:

- In Fig(1), the steady- state availability is increasing if the vacation rate α is increasing and λ is decreasing .
- In Fig(2), the steady- state availability is increasing if the repair rate of unit1and 2 are increasing.
- In Fig(3), the mean time to system failure is increasing if the vacation rate α is increasing and λ is decreasing .
- In Fig(4), the mean time to system failure is increasing if the repair rate of unit1and 2are increasing.

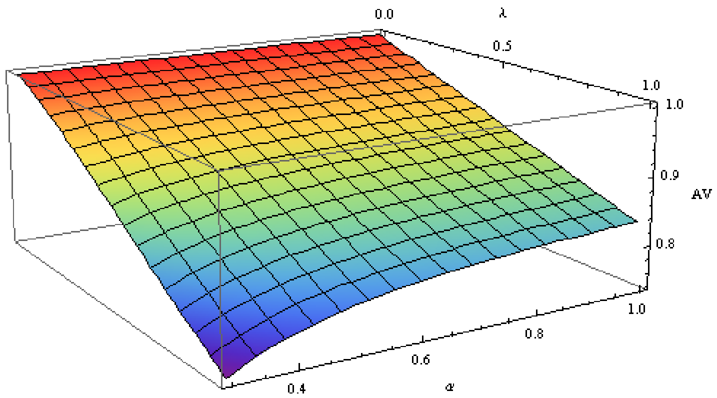

Figure (1): S.S.Availabilty vs(α, λ)
where($\mu_1 = 0.8, \mu_2 = 1.0, r_1 = 0.2, \mathrm{r}_2 = 0.25, \gamma = 0.7,$ and $q = 0.5$).

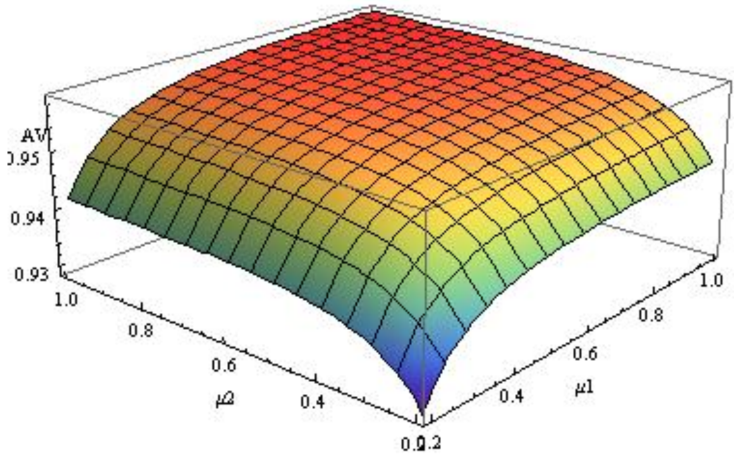

Figure (2): S.S.Availabilty vs(μ_1 , μ_2)
where($\alpha = 0.3, \lambda = 0.2, r_1 = 0.2, \mathrm{r}_2 = 0.25, \gamma = 0.7,$ and $q = 0.5$).

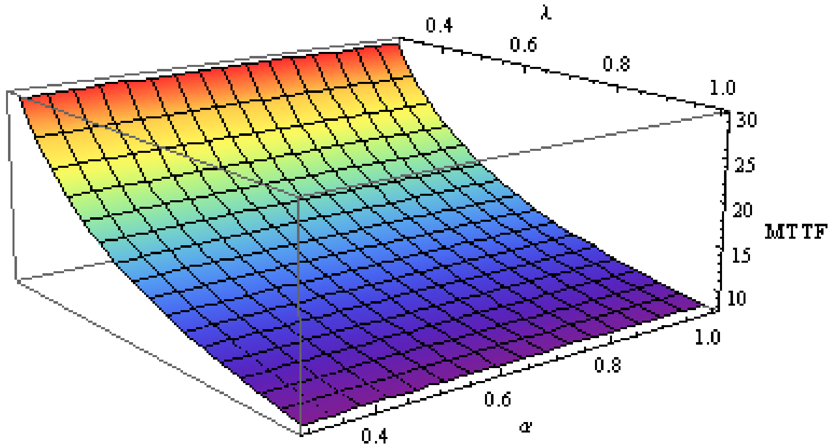

Figure (3): MTTFF vs(μ_1 , μ_2)
where($\mu_1 = 0.8, \mu_2 = 1.0, r_1 = 0.2, \mathrm{r}_2 = 0.25, \gamma = 0.7,$ and $q = 0.5$).

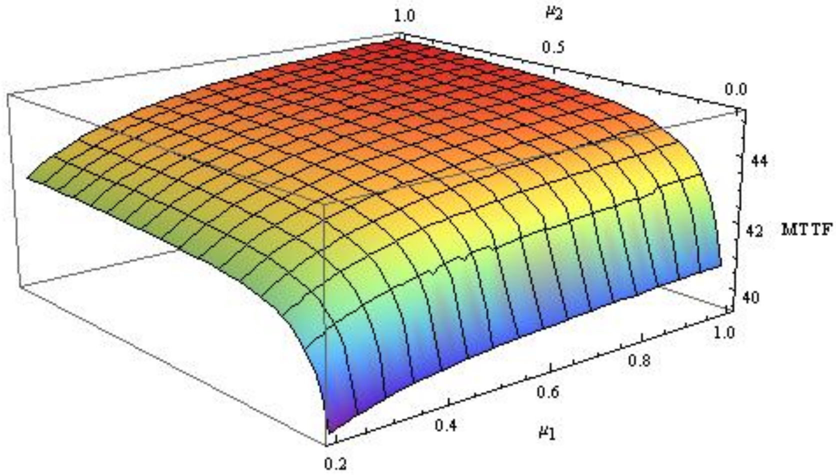

Figure (4): MTTFF vs(μ_1 , μ_2)
where($\alpha = 0.3, \lambda = 0.2, r_1 = 0.2, \mathrm{r}_2 = 0.25, \gamma = 0.7,$ and $q = 0.5$).

Conclusions

In this paper, the some reliability measures of a system consisting of two unit cold standby system with imperfect switch and single repairman are derived. The repairman can take vacation and the operating unit might be attacked by shocks. The supplementary variable method and theory of differential equation play an important role in system analysis. The relationship between the reliability measures and system parameters are showed. The numerical study shows the relationship between the reliability measures and relevant parameters. The results of [3] can be obtained as a special case of our result.

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