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RESEARCH ARTICLE

MHD VISCOELASTIC BOUNDARY LAYER FLOW AND HEAT TRANSFER PAST A CONVECTIVELY HEATED RADIATING STRETCHING/SHRINKING SHEET WITH TEMPERATURE DEPENDENT HEAT SOURCE/SINK

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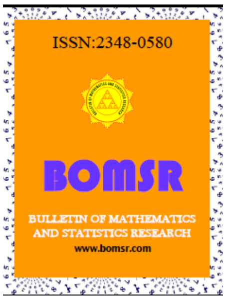

ABSTRACT

The prominent focus of this work is to analyze the convective heat transfer in a steady boundary layer viscoelastic fluid flow and heat transfer over a stretching/shrinking sheet.Three cases were considered here. That is (i)The sheet with prescribed surface temperature. (ii) The sheet with prescribed surface heat flux. (iii) Convective heating.The governing boundary value problem, which is in the form of nonlinear partial differential equations are transformed into nonlinear ordinary differential equations, using a suitable similarity transformation and are solved numerically using Runge Kutta fourth order method with shooting technique.The numerical results for flow and temperature field, are found to depend solidly on, Viscoelastic parameter (k_1),Chandrashekar number (Q), thermal radiation parameter (Nr),Prandtl number (Pr),wall temperature parameter (s), heat source/sink parameter (α),Biot number (B_i).

Key Words: Stretching/shrinking sheet, biot number, PST and PHF cases, Thermal Radiation, Chandrashekar number

INTRODUCTION

Many fluids such as blood, dyes, yoghurt, ketchup, shampoo,paint, mud, clay coatings, polymer melts, certain oils and greases etc, have nonlinear relation between stresses and strains. Such fluids do not obey the Newton's law of viscosity and are usuallycalled non-Newtonian fluids. The flows of such fluids occur in a widerange of practical applications and have key importance in polymer devolatisation, bubble columns, fermentation, composite processing,boiling, plastic foam processing, bubble absorption and many others.Therefore, non-Newtonian fluids have attracted the attention ofa large variety of researchers including the interests of experimentalists and theoreticians like engineers, modelers, physicists, computerscientists and mathematicians. However, as these fluids are inthemselves varied in nature, the constitutive equations which govern them are many taking account of the variations of rheological properties.The model and hence, the arising equations, are much morecomplicated and of higher order than the well known Navier--Stokesequations.

Study of laminar boundary layer flow caused by a moving rigid surface was initiated by Sakiadis [1] and later the work was extended to the flow due to stretching of a sheet by Crane [2]. The flow of an incompressible fluid past a moving surface has several engineering applications. The aerodynamic extrusion of plastic sheets, the cooling of a large metallic plate in a cooling bath, the boundary layer along a liquid film in condensation process and a polymer sheet or filament extruded continuously from a die, or along thread traveling between a feed roll and a wind-up roll are the examples of practical applications of a continuous flat surface.

In certain dilute polymer solution (such as 5.4% of polyisobutylenein cetane and 0.83% solution of ammonium alginate in water [3,4]),the viscoelastic fluid flow occurs over a stretching sheet. Any fluid that does not behave in accordance with the Newtonian constitutive relation is called non-Newtonian [5–12]. Non-Newtonian fluids have gained considerable importance because the power required in stretching a sheet in a viscoelastic fluid is less than when it is placed in a Newtonian fluid; and the heat transfer rate for a viscoelastic fluid is found to be less than that of Newtonian fluids

The central problem in non-Newtonian fluid dynamics is the establishment of expressions for the stress tensor T to replace the Newtonian expression. The relation between the stress tensor and various kinematic tensors is called the constitutiveequation or the rheological equation of state. Rivlin and Ericksenand Coleman and Noll have presented constitutive relations for the stress tensor as a function of the symmetric part of the velocity gradient and its higher (total) derivatives.

Another class of models is the rare-type fluidmodels, such as Oldroyd model, which has been modified by Walters.This modified model is referred to as the Walters’ liquid B. The steady two-dimensional boundary layer equations for Walters’ liquidB were derived by Beard and Walters [10] to first-order in elasticity(i.e., for short memory fluids with short relaxation times).Walters’ liquid B considered by Sidappa and Abel [13] exhibit normal stress-differences in simple shear flows. Rajagopal et al. [14]analyzed the effects of viscoelasticity on the flow of a second-order fluid with gradually fading memory and arrived to the boundarylayer equations as that in Ref. [13]. H.I.Andersson[15] considered MHD flow of a viscoelastic fluid past a stretching sheet.An exact analytical solution of the governing nonlinear boundary layer equation was obtained illustrating, that the effect of magnetic field is same as that of viscoelasticity, on flow and heat transfer charecteristics.

On the other hand, Abel and Veena [16] investigated a viscoelastic fluid flow and heat transfer in a porous medium over a stretchingsheet and observed that the dimensionless surfacetemperature profiles increases with an increase in viscoelasticparameter $k1$, however, later, Abel et al. [17]studied the effect of heat transfer on MHD viscoelasticfluid over a stretching surface and an importantfinding was that the effect of viscoelasticity is to decrease dimensionless surface temperature profiles inthat flow. Furthermore, Char [18] studied MHD flowof a viscoelastic fluid over a stretching sheet, however,only the thermal diffusion is considered in the energyequation; later, Sarma and Rao [19] analysed the effects of work due to deformation in that equation.

However all of the above research dealing with non-newtonian fluids attributes only to the most general heat transfer cases of PST and PHF and none of the above problems considered the most prominent aspect of convective heating.As a result this research attempts to solve this much more complicated problem involving convective heat transfer in a boundary layer.The effects of viscoelastic parameter and Biot number on flow and heat transfer charecteristics is a salint feature of this study.

MATHEMATICAL FORMULATION

Consider a steady, laminar free convective flow of an incompressible and electrically conducting visco-elastic fluid over continuously moving stretching surface embedded in a porous medium. Two equal and opposite forces are introduced along the x-axis so that sheet is stretched with a speed proportional to the distance from the origin. The resulting motion of the otherwise quiescent fluid is thus caused solely by the moving surface. A uniform magnetic field of strength B_0 is imposed along y-axis. This flow satisfies the rheological equation of state derived by Beard and Walters in 1964.

The steady two-dimensional boundary layer equations for this in usual notation are,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_o\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma B_o^2 u}{\rho} \qquad (2)$$

Here x and y are respectively the directions along and perpendicular to the surface, u, v are the velocity components along x & y directions respectively and other symbols have their usual meanings.In deriving the equations, it is assumed, in addition to the usual boundary layer approximations that the contribution due to the normal stress is of the same order of magnitude as the shear stress.

The boundary conditions applicable to the flow problem are,

$$\begin{aligned} &u=bx, v=0, \ at \ \ y=0\\ &u \to 0, as \quad y\to\infty \end{aligned} \qquad (3)$$

b<0 (Shrinking sheet), b>0(stetching sheet)

Equations (1) and (2) admit self-similar solution of the form,

$$u=bxf', \quad v=-\sqrt{b\gamma}\, f, Where\ \eta=\sqrt{\frac{b}{\gamma}}y \qquad (4)$$

where prime denotes the derivative with respect to η. Clearly u & v satisfy the equation (1) identically. Substituting these new variables in equation (2), we obtain,

$$f'^2-f\,f''=f'''-k_1\left\{2f'\,f'''-f\,f^{IV}-f''^2\right\}-Qf' \qquad (5)$$

Where $k_1=\frac{k_0 b}{\gamma}$, $Q=\frac{\sigma B_0^2}{b\rho}$, Where k_1 and Q are the viscoelastic parameter and Chandrashekar number respectively.

Similarly boundary condition (3) takes the form

$$\begin{aligned} &f'(0)=1, \quad f(0)=0 \qquad at \quad \eta=0\\ &f'(\eta)\to 0, \quad f''(\eta)\to 0 \qquad as \quad \eta\to\infty \end{aligned} \qquad (6)$$

HEAT TRANSFER ANALYSIS

The energy equation in the presence of radiation and internal heat generation / absorption for two-dimensional flow is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=K\frac{\partial^2 T}{\partial y^2}+\frac{\lambda}{\rho C_p}(T-T_\infty)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{7}$$

Where is K thermal conductivity, λ is heat source/sink, q_r is radiative heat flux.

By using Rosseland approximation , the radiative heat flux is given by

$$q_r=-\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial y} \tag{8}$$

Where σ^* and K^* are respectively, the Stephan-Boltzman constant and the mean absorption coefficient. We assume the differences within the flow are such that T^4 can be expressed as a linear function of temperature. Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms thus,

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{9}$$

The boundary conditions are

$$T=T_w=T_\infty+A\left(\frac{x}{l}\right)^s \text{ at y=0 and } T\to T_\infty, as, y\to\infty \quad \text{PST Case} \tag{10}$$

$$-K\frac{\partial T}{\partial y}=Q_w=D\left(\frac{x}{l}\right)^s \text{ at y=0 and } T\to T_\infty, as, y\to\infty \quad \text{PHF Case} \tag{11}$$

$$-K\frac{\partial T}{\partial y}=h(T_f-T) \text{ at,y=0and} T\to T_\infty, as, y\to\infty \quad \text{Convective Case} \tag{12}$$

The fluid temperature which is charecterised by T_f , heat transfer coefficient h

And s is wall temperature parameter.

The similarity transformations are

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \quad where \quad T_w-T_\infty=A\left(\frac{x}{l}\right)^s \quad \text{PST Case} \tag{13}$$

$$g(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \quad T_w-T_\infty=\frac{D}{k}\sqrt{\frac{\gamma}{b}}\left(\frac{x}{l}\right)^s \quad \text{PHF Case} \tag{14}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty} \quad \text{Convective Case} \tag{15}$$

PST and Convective Case

Now using equations (13),and(15) equation (7) becomes

$$(1+Nr)\theta''+\Pr f\theta'-\Pr(f'-\alpha)\theta=0 \tag{16}$$

Where $\Pr=\frac{\mu C_p}{K}$, $Nr=\frac{16\sigma^* T_\infty^3}{3K^* K_\infty}$, $\alpha=\frac{\lambda}{b\rho C_p}$

are the Prandtl Number,Radiation parameter and heat source / Sink Parameter respectively.

PHF CASE

Now using equations (14), equation (7) becomes

$$(1+ Nr)g'' + \Pr f\, g' - \Pr(f' - \alpha)g = 0 \quad (17)$$

The boundary conditions takes the form:

$\theta(0)=1, \quad \theta(\eta)\to 0 \quad as \quad \eta\to\infty$ PST Case

$\theta'(0)=-1, \quad \theta(\eta)\to 0 \quad as \quad \eta\to\infty$ PHF Case

$\theta'(0)=-B_i\left(1-\theta(0)\right), \ \theta(\eta)\to 0, as \ \eta\to\infty$ Convective Case (18)

NUMERICAL SOLUTION

Because of the non-linearity and couplings between the momentum and the thermal boundary layer equations, exact solutions do not seem feasible for complete set of equations (16),(17) and (18), therefore solution must be sought numerically. In order to solve them, we employ most efficient shooting technique with fourth order Runge-Kutta integration scheme.

Selection of an appropriate finite value of η_∞ is most important aspect in this method. To select η_∞, we begin with some initial guess value and solve the problem with some particular set of parameters to obtain $f''(0)$ and $\theta'(0)$. The solution process is repeated with another larger (or smaller, as the case may be) value of η_∞. The values of $f''(0)$ and $\theta'(0)$ compared to their respective previous values, if they agreed to about six significant digits, the last value of η_∞ used was considered the appropriate value for that particular set of parameters; otherwise the procedure was repeated until further changes in η_∞ did not lead to any more change in the values of $f''(0)$ and $\theta'(0)$. The initial step size employed was h=0.01. The convergence criterion was largely depends on fairly good guesses of the initial conditions in the shooting technique.

RESULTS AND DISCUSSION

The present study considers the flow of viscoelastic incompressible electrically conducting fluid flow past a stretching/shrinking sheet in prescence of Magnetic field, uniform heat source/sink and convective heat transfer.The aim of the following discussion is to bring about the effect of magnetic field, heat source /sink, and convective heat transfer over stretching/shrinking sheet on flow and heat transfer charecteristics.

In Fig (1) it is noticed that the effect of Chandrashekar number Q is to accelerate motion in case of shrinking sheet.This is due to the fact that the prescence of viscoelasticity contribute to stored energy by obstructing energy loss, as one is aware of the fact that in viscoelastic fluid flows, a fixed amount of energy is stored up in the material as stored energy. Because of this the resistive force due to magnetic field is overcome , resulting in enhancement in magnitude of velocity. Where as in case of stretching sheet the effect of Q is to retard flow velocity within the boundary layer.

Fig 2 is a graph concerns to the effect of viscoelastic parameter k1 on flow velocity for both stretching/shrinking sheet.Here this Fig 2 depicts that for an increase in viscoelastic parameter k1 results in decrease of velocity in boundary layer in case of shrinking sheet. This result is consistent with the fact that the introduction of tensile stress due to visco-elasticity cause transverse contraction of the boundary and hence velocity decreases. Where as for stretching sheet the opposite effect is noticed.

The effect of Prandtl Number(Pr) is analysed in view of Fig 3, for both PST as well as PHF cases.This figure illustrates that increase in Prandtl Number(Pr) results in decrease of temperature distribution in thermal boundary layer region, which obviously a means for decrease of boundary layer thickness.Decrease of boundary layer thickness results slow rate of thermal diffusion.It is also noticed that wall temperature distribution is at unity in case of PST , where as in PHF case it is other than unity, due to adiabatic boundary condition.

The effect of Chandrashekar number Q, on heat transfer is depicted in Fig. 4 in case of PST and PHF respectively. Here it is noticed that the contribution of transverse magnetic field, is to thicken thermal boundary layer. This is due to the fact the applied transverse magnetic field produces a body force, in the form of Lorentz force, which enhances temperature distribution in flow region.The enhancement in temperature distribution in flow region is because of resistance offered by Lorentz force on flow velocity.

Fig 5 shows the effect of viscoelastic parameter K1 on temperature profile, and it is noticed that Temperature profile increases with the increase of viscoelastic parameter K1, in both PST and PHF cases.

An increase in temperature distribution due to the presence of elastic elements may be attributed to the fact that when a viscoelastic fluid is in flow, a certain amount of energy is stored up in the material as strain energy, which is responsible for enhancement of temperature distribution in thermal boundary layer region.

Fig 6 reveals the influence of radiation parameter Nr on temperature profile, where in it produces a significant increase in the thickness of thermal boundary layer, resulting in enhancement of temperature in thermal boundary layer region in both PST and PHF cases.The prominent effect of Nr is to enhance heat transfer,therefore Nr shold be kept at minimum value to fecilitate the cooling process of polymer extrudate in polymer industry.

The influence of wall temperature parameter s for both PST as well as PHF cases on temperature distribution is depicted in Fig 7. Numerical solutions are sought in the range of values of s as mentioned follows,i.e $-2.0 \leq s \leq 2.0$ and $-2.0 \leq s \leq 2.0$ for PST and PHF cases.
Here we notice that as the value of s is incremented from negative values to positive values, temperature distribution decreases in thermal boundary layer.

The effect of heat source/sink parameter α on temperature profile within the boundary layer is depicted in Fig 8.In this figure it is noticed that the direction of heat transfer depends on temperature difference $(T_w - T_\infty)$ and dimensionless rate of heat transfer $\theta'(0)$.

To interpret the heat transfer result physically, we discuss the result of positive α and negative α separately. For positive α, we have a heat source in the boundary layer when $T_w < T_\infty$ and heat sink when $T_w > T_\infty$. Physically, these correspond, respectively, recombination and dissociation within the boundary layer. For the case of cooled wall ($T_w < T_\infty$), there is heat transfer from the fluid to the wall even without a heat source. The presence of heat source $(\alpha > 0)$ will further increase the heat flow to the wall.

When α is negative, this indicates a heat source for $T_w > T_\infty$ and a heat sink for $T_w < T_\infty$. This corresponds to combustion and an endothermic chemical reaction. For the case of heated wall ($T_w > T_\infty$), the presence of a heat source $(\alpha < 0)$ creates a layer of hot fluid adjacent to the surface and therefore the heat from the wall decreases. For cooled wall case ($T_w < T_\infty$), the presence of heat sink $(\alpha < 0)$ blankets the surface with a layer of cool fluid and therefore heat flow in to the surface decreases.

The effect of Biot number Bi on temperature profile is depicted in fig 9. Here it is noticed that an increase in biot number Bi results in increase in rate of heat transfer in thermal boundary layer region.Further it is noticed that there is increase in thickness of thermal boundary layer .

Concluding Remarks

The governing boundary layer equations of flow and heat transfer for a steady, flow of an incompressible and electrically conducting visco-elastic fluid over continuously moving stretching surface with combined effect of thermal radiation and convective heating is analysed. The governing boundary value problem , which is in the form of nonlinear partial differential equations are converted into nonlinear ordinary differential equations and are solved numerically using Runge-Kutta fourth order method with shooting technique.. Numerical evaluations were performed and graphical results were obtained to demonstrate the details of flow and heat transfer characteristics and their dependence on some of the physical parameters.

The important findings of our investigations are

- The increase of Chandrashekar number leads to the enhanced deceleration of the flow and hence the velocity decreases but increases temperature in the boundary layer.
- The effect of increase in Viscoelastic parameter k_1 leads to decrease the horizontal velocity profile but increase the temperature in the boundary layer.
- The effect of heat source in the boundary layer generates energy, which causes the temperature to increase while the presence of heat absorption effects caused reductions in the fluid temperature, which results in decreasing the fluid velocity, in both PST and PHF cases.
- The effect of thermal radiation parameter Nr produces a significant increase in the thickness of the thermal boundary layer of the fluid and so as the temperature increases in presence/ absence of thermal conductivity parameter, in both PST and PHF cases.
- An increase in biot number Bi results in increase in rate of heat transfer in thermal boundary layer region, resulting in increase of thickness of thermal boundary layer .

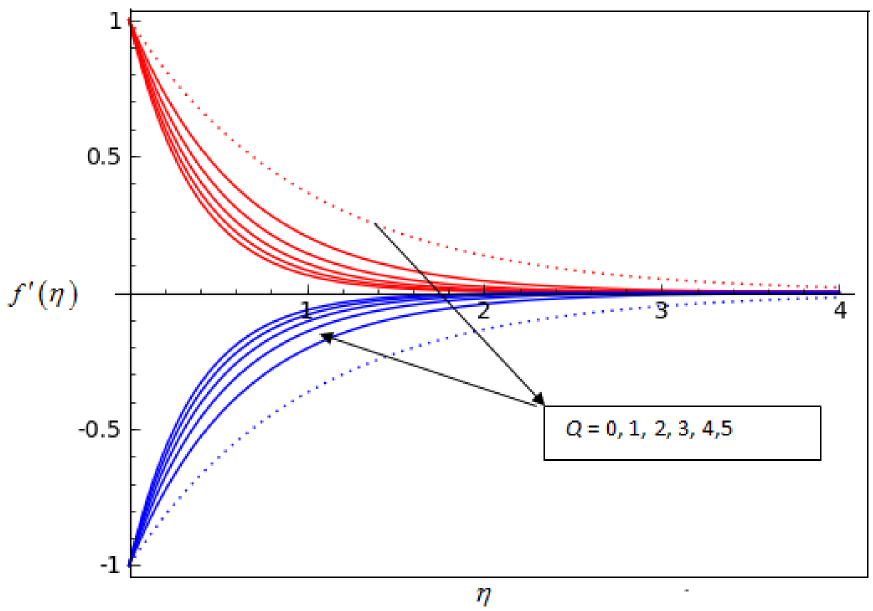

Figure 1: Plot of axial $\left(f_\eta(\eta)\right)$ velocity versus η for different values of Chandrasekhar number Q with K_1 = 0.2.

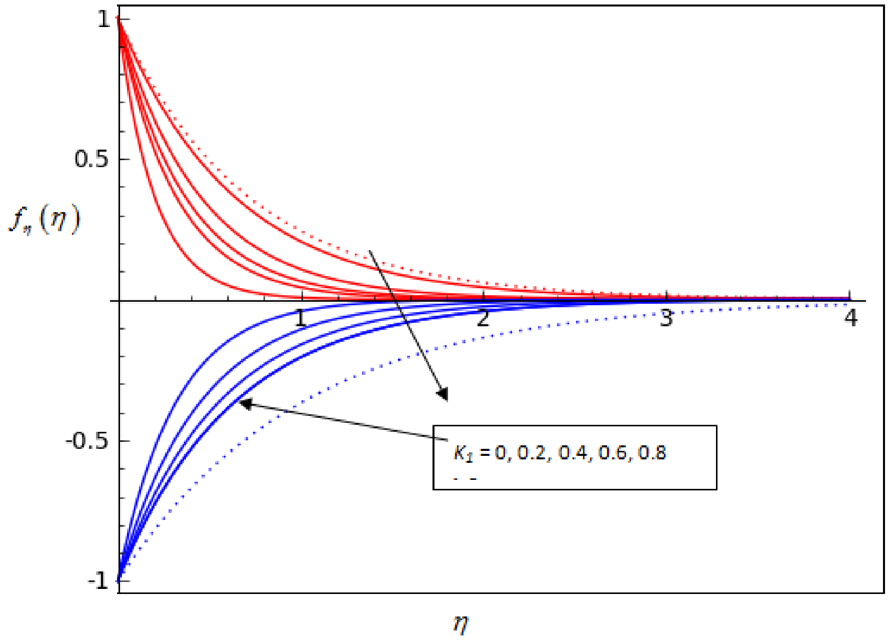

Figure 2: Plot of axial velocity versus η for different values of viscoelastic parameter K_1 with Q =1.

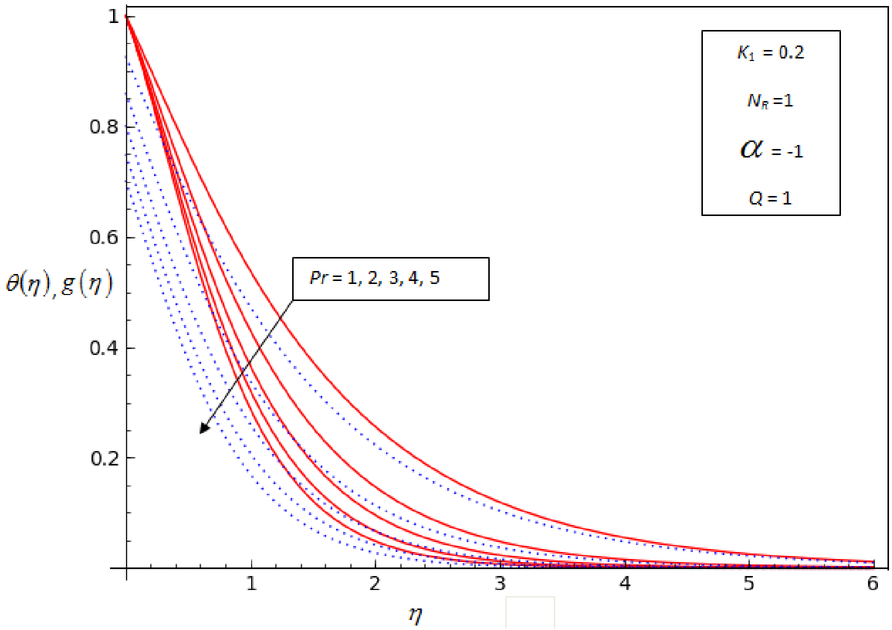

Figure 3:Variation of the non-dimensional temperature θ with η the transformation co-ordinate normal to the surface for different values of Prandtl number *Pr* for the cases PST and PHF.

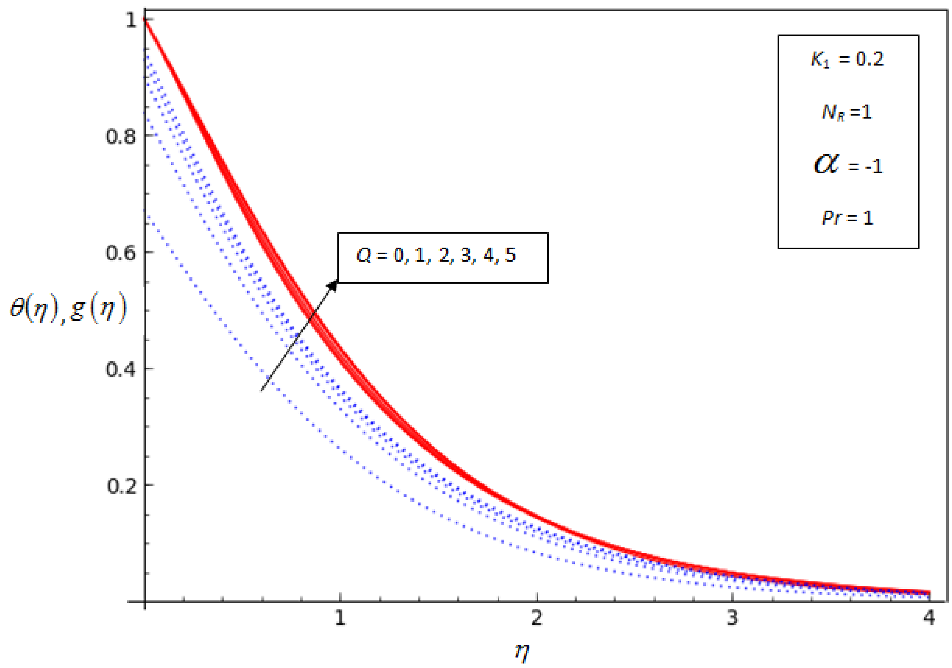

Figure 4:Variation of the non-dimensional temperature θ with η the transformation co-ordinate normal to the surface for different values of Chandrasekhar number Q for the cases PST and PHF.

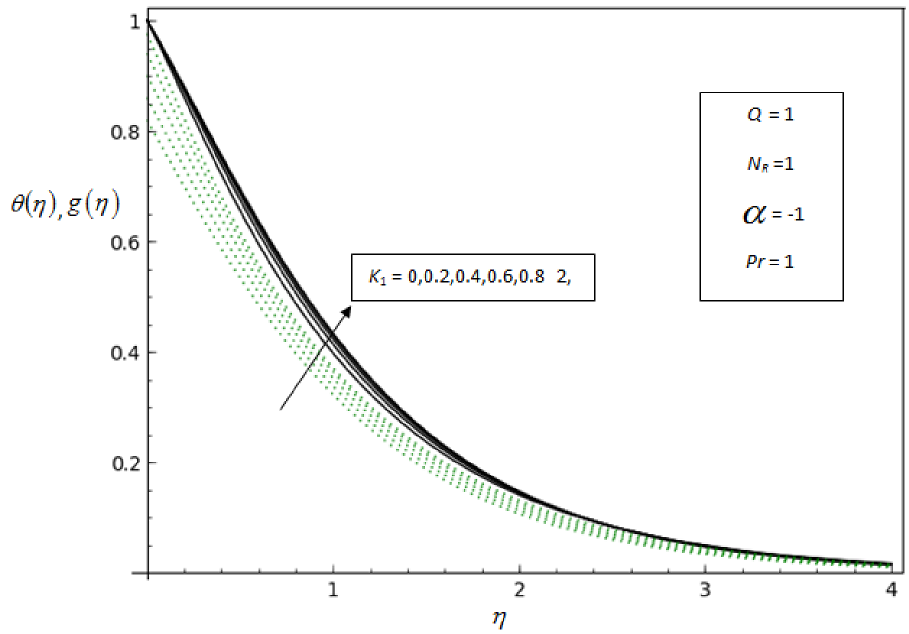

Figure 5:Variation of the non-dimensional temperature θ with η the transformation co-ordinate normal to the surface for different values of viscoelastic parameter K_1 for the cases PST and PHF.

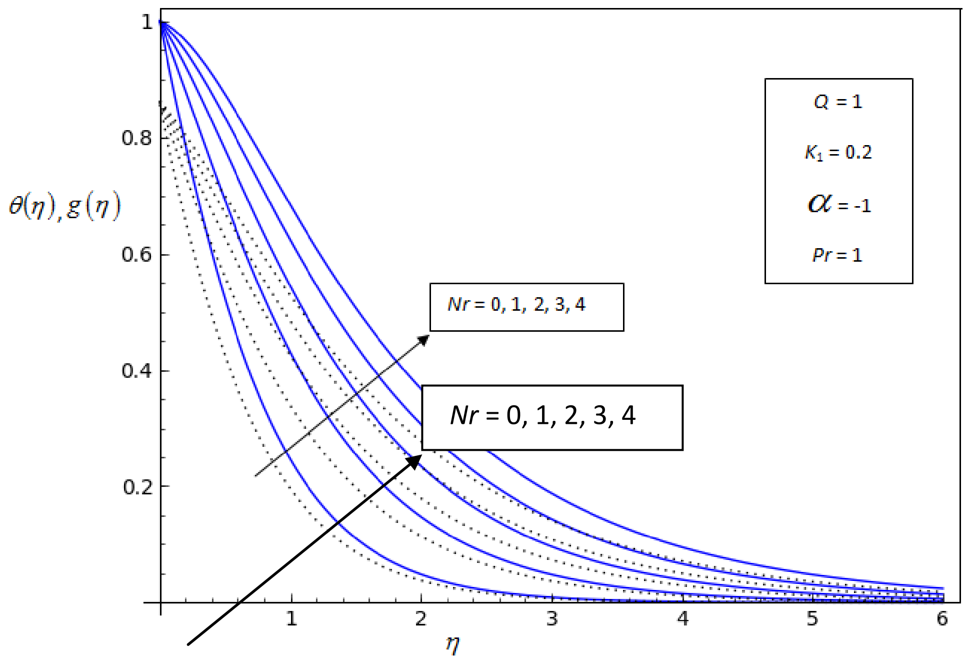

Figure 6:Variation of the non-dimensional temperature θ with η the transformation co-ordinate normal to the surface for different values of radiation parameter *Nr* for the cases PST and PHF.

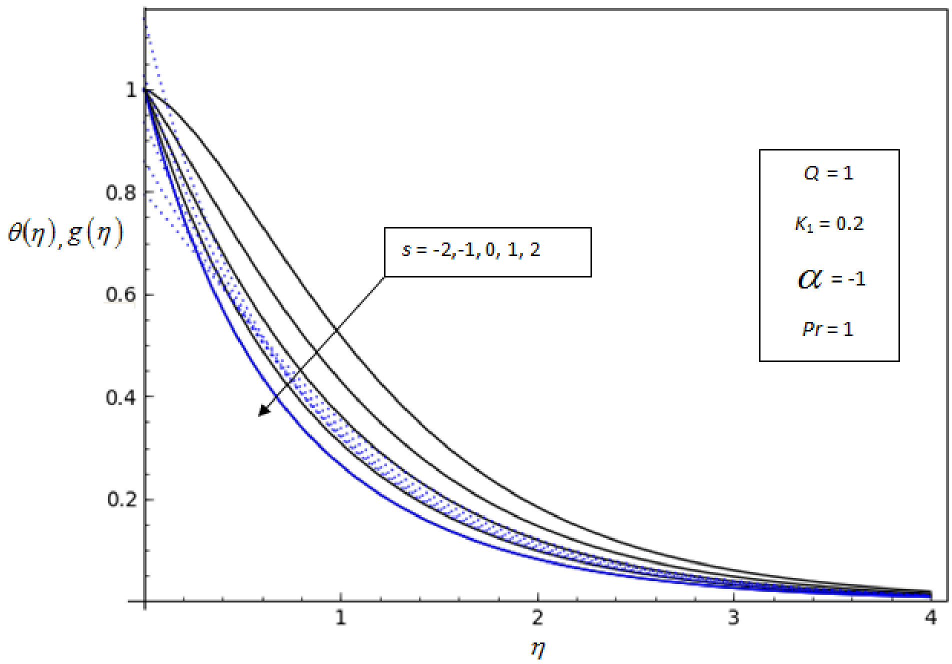

Figure 7:Variation of the non-dimensional temperature θ with η the transformation co-ordinate normal to the surface for different values of wall temperature parameter *s* for the cases PST and PHF.

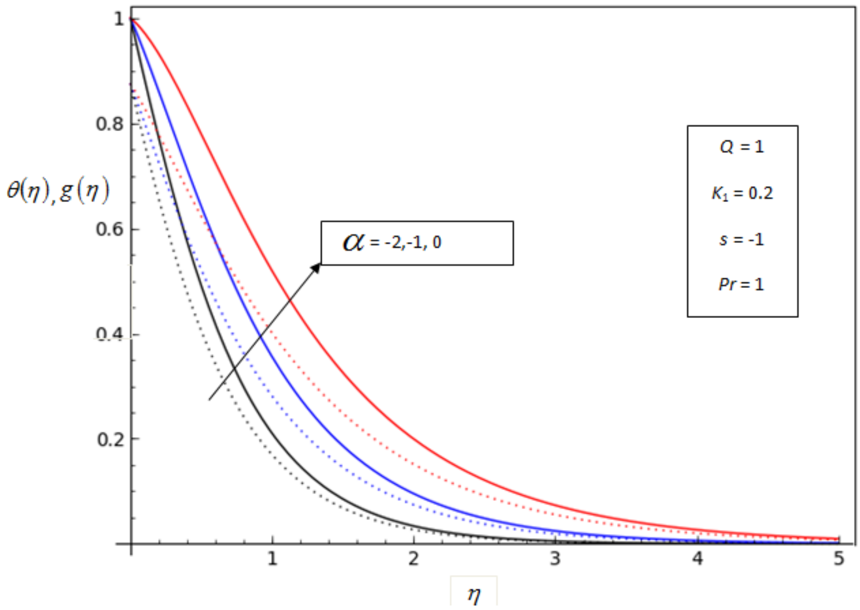

Figure 8:Variation of the non-dimensional temperature θ with η the transformation co-ordinate normal to the surface for different values of heat source parameter N_l for the cases PST and PHF.

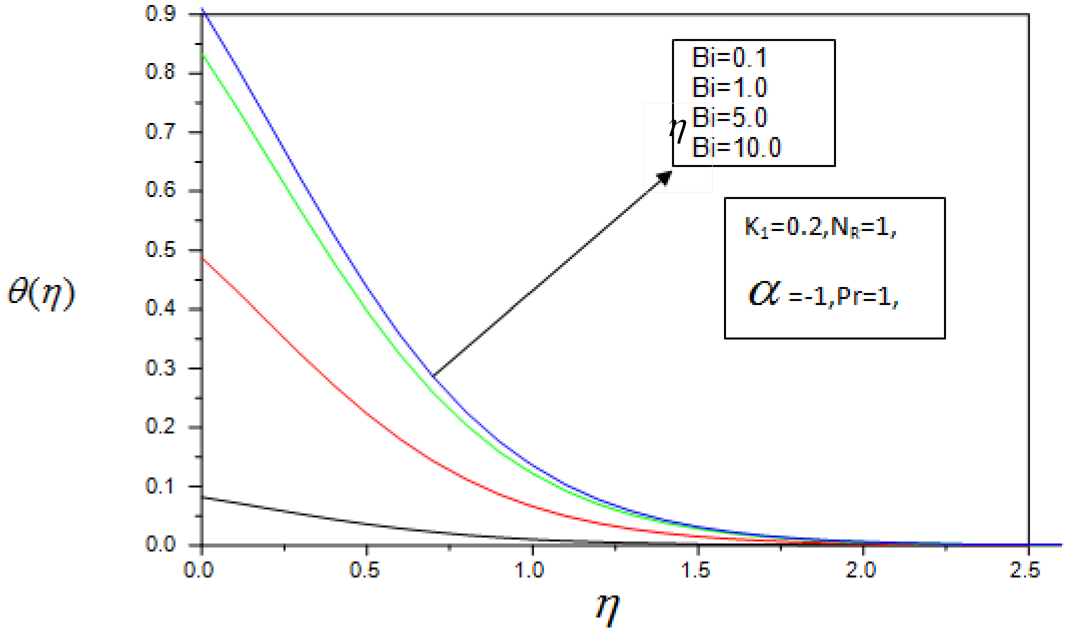

Figure 9:Variation of the non-dimensional temperature θ with η the transformation co-ordinate normal to the surface for different values of Biot number Bi for the cases PST and PHF

References

[1]. B.C. Sakiadis, Boundary layer behavior on continuous solid surfaces, Am. Inst.Chem. Eng. J. 7 (1961) 26–28.

[2]. L.J. Crane, Flow past a stretching plate, Z. Angew. Math. Phys. 21 (1970) 645–647.

[3]. H. Markovitz, B.D. Coleman, Advances in Applied Mechanics, Academic Press,New York,1964.

[4]. A. Acrovos, A theoretical analysis of laminar natural convection heat transfer to non Newtonian fluids, Am. Inst. Chem. Eng. J 6 (1961) 584–590.

[5]. G. Astarita, G. Marrucci, Principles of Non-Newtonian Fluid Mechanics, McGraw-Hill, London, 1974.

[6]. R. Darby, Viscoelastic Fluids: An Introduction to their Properties andBehaviour, Marcel Dekker, New York, 1976.

[7]. W.R. Schowalter, Mechanics of Non-Newtonian Fluids, Pergammon press,Oxford, 1978.

[8]. M.J. Crochet, A.R. Davies, K. Walters, Numerical Simulation of Non-NewtonianFlows, Elsevier, Amsterdam, 1984.

[9]. G.K. Rajeswari, S.L. Rathna, Flow of a particular class of non-Newtonian viscoelastic and visco-inelastic fluids near a stagnation point, Z. Angew. Math. Phys. 13 (1962) 43–57.

[10]. D.W. Beard, K. Walters, Elastico-viscous boundary layer flows: part I. Two dimensional flow near a stagnation point, in: Proceedings of Cambridge Philos. Soc, 1964, pp. 667–674.

[11]. A. Acrivos, M.J. Shah, E.E. Peterson, Momentum and heat transfer in laminar boundary flow of non-Newtonian fluids past external surfaces, Am. Inst. Chem. Eng. J. 6 (1961) 312–317.

[12]. V.G. Fox, L.E. Erickson, L.T. Fan, The laminar boundary layer on a moving continuous flat sheet immersed in a non-Newtonian fluid, Am. Inst. Chem. Eng. J. 15 (1969) 327–336.

[13]. B. Siddappa, S. Abel, Non-Newtonian flow past a stretching plate, Z. Angew. Math. Phys. 36 (1985) 890–892.

[14]. K.R. Rajagopal, T.Y. Na, A.S. Gupta, Flow of a visco-elastic fluid over a stretching sheet, Rheol. Acta 23 (1984) 213–215.

[15]. H.I. Andersson, MHD flow of a viscoelastic fluid past a stretching surface, Acta Mech. 95 (1992) 227–230.

[16]. S. Abel, P.H. Veena, Viscoelastic fluid flow and heat transfer in a porous medium over a stretching sheet, Int. J. Non-LinearMech. 33 (1998) 531–540.

[17]. S. Abel, M. Joshi, R.M. Sonth, Heat transfer in MHD viscoelastic fluid over a stretching surface, Z. Angew. Math. Mech.81 (2001) 691–698.

[18]. M.I. Char, Heat and mass transfer in a hydromagnetic flow of the viscoelastic fluid over a stretching sheet, J. Math. Anal.Appl. 186 (1994) 674–689.

[19]. M.S. Sarma, B.N. Rao, Heat transfer in a viscoelastic fluidover a stretching sheet, J. Math. Anal. Appl. 222 (1998)268–275.