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RESEARCH ARTICLE

ROUGH NEUTROSOPHIC RELATION ON TWO UNIVERSAL SETS

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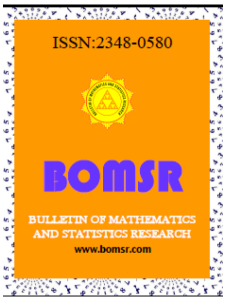

ABSTRACT

In this paper, we define the rough neutrosophic relation of two universe sets and study the algebraic properties of two rough neutrosophic relations that are interesting in the theory of rough sets. Finally, we present the similarity rough neutrosophic relation with an example.

I. INTRODUCTION

Rough set theory [12] proposed by Pawlak in about 1980s, is used to handle the redundancies, uncertainties and incorrectness in data mining, as design databases [7] or information systems [8]. This theory has been well developed in both theories and applications. Along with the study of the individual properties of rough set theory, the rough set theory in combination with neutrosophic set theory [18] also gains great interest of researchers and becomes a useful tool in exploring the feature selection, the clustering, the control problem, etc. The combination of fuzzy sets and rough sets lead to two concepts [17]: rough fuzzy sets and fuzzy rough sets. Rough fuzzy sets [6, 15, 17, 18] are the fuzzy sets approximated in the crisp approximation spaces and fuzzy rough sets [8] are the crisp sets approximated in the fuzzy approximation spaces. One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [13], [14].

Neutrosophic sets described by three functions: a membership function indeterminacy function and a non-membership function that are independently related. The theory of neutrosophic set have achieved great success in various areas such as medical diagnosis [1], database [5], [6], topology [3, 9], image processing [10], [11], [19], and decision making problem [17]. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness. Neutrosophic sets and rough sets are two different topics, none conflicts the other. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness. By combining the Neutrosophic sets and rough sets the rough sets in neutrosophic approximation space [2] and Neutrosophic neutrosophic rough sets [4] were introduced. The main objective of this study is to introduce a new hybrid intelligent structure called rough neutrosophic relations on the Cartesian product of two universe sets, and subsequently their properties are examined. The significance of introducing hybrid set structures is that the computational techniques based on any one of these structures alone will not always yield the best results but a fusion of two or more of them can often give better results.

The remaining parts of this paper are organized as following: rough neutrosophic sets are re-introduced. After studying the rough neutrosophic relation and its properties composition of two rough neutrosophic relation and inverse rough neutrosophic relation are presented and at last the reflexive, symmetric, transitive rough neutrosophic relations are studied. Finally, we propose the similarity rough neutrosophic relation.

2 PRELIMINARIES

Definition 2.1[13] A neutrosophic set A on the universe of discourse X is defined as A = $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$, Where $T, I, F: X \to]^-0,1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2.2:[12] Let U be any non empty set. Suppose R is an equivalence relation over U. For any non null subset X of U, the sets

$A_1(X) = \{x: [x]_R \subseteq X\}$

$A_2(X) = \{x: [x]_R \cap X \neq \emptyset\}$

are called lower approximation and upper approximation respectively of X and the pair S= (U, R) is called approximation space. The equivalence relation R is called indiscernibility relation. The pair A(X) = ($A_1(X)$, $A_2(X)$) is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x

Definition 2.3[4]: Let U be a non-null set and R be an equivalence relation on U. Let A be a neutrosophic set in U with the truth value $T_A(x)$, indeterminate value $I_A(x)$ and false value $F_A(x)$. The lower and the upper approximations of A in the approximation (U, R) denoted by $\underline{R}(A)$ and $\overline{R}(A)$ are respectively defined as follows:

$\overline{R}(A) = \{x, T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x) / y \in [x]_R, x \in U\}$

$\underline{R}(A) = \{x, T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) / y \in [x]_R, x \in U\}$

where :

$T_{\underline{R}(A)}(x) = \wedge_{y \in [x]R} T_A(y)$, , $I_{\underline{R}(A)}(x) = \wedge_{y \in [x]R} I_A(y)$, $F_{\underline{R}(A)}(x) \vee_{y \in [x]R} F_A(y)$

$T_{\overline{R}(A)}(x) = \vee_{y \in [x]R} T_A(y)$, $I_{\overline{R}(A)}(x) = \vee_{y \in [x]R} I_A(y)$, $F_{\overline{R}(A)}(x) = \wedge_{y \in [x]R} F_A(y)$

So $0 \le T_{\overline{R}(A)}(x) + I_{\overline{R}(A)}(x) + F_{\overline{R}(A)}(x) \le 3$ and $0 \le, T_{\underline{R}(A)}(x) + I_{\underline{R}(A)}(x) + F_{\underline{R}(A)}(x) \le 3$ and $, T_{\overline{R}(A)}(x), I_{\overline{R}(A)}(x), F_{\overline{R}(A)}(x)\,), T_{\underline{R}(A)}(x), I_{\underline{R}(A)}(x), F_{\underline{R}(A)}(x) : A \to [0,1]$

Where "V "and "Λ " mean "max" and "min " operators respectively, , and are the truth, indeterminacy and false values of y with respect to A. It is easy to see that $\underline{R}(A)$ and $\overline{R}(A)$ are two neutrosophic sets in U ,thus NS mapping.

$\underline{R}(A)$ and $\overline{R}(A)$: $A^N \to A^N$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $\underline{R}(A)$ and $\overline{R}(A)$ is called the rough neutrosophic set in (U, R).

From the above definition, we can see that $\underline{R}(A)$ and $\overline{R}(A)$ have constant membership on the equivalence classes of U.

3. NEUTROSOPHIC ROUGH SETS REDEFINED

Definition 3.1: Let U be a non-empty set of objects, R is an equivalence relation on U. Then the space (U,R) is called an approximation space. Let X be a neutrosophic set on U. We define the lower approximation and upper approximation of X, respectively,

$$T_Y(\underline{R}(X))=1,\ T_Y(U-\overline{R}(X))=0,\ 0<T_Y(\overline{R}(X)-\underline{R}(X))<1$$

$$I_Y(\underline{R}(X))=1,\ I_Y(U-\overline{R}(X))=0,\ 0<I_Y(\overline{R}(X)-\underline{R}(X))<1$$

$$F_Y(\underline{R}(X))=0,\ F_Y(U-\overline{R}(X))=1,\ 0<F_Y(\overline{R}(X)-\underline{R}(X))<1.$$

The neutrosophic set X is called a rough neutrosophic set if $\text{Boundry}(X)\neq\phi$.

Definition 3.2: Let X,Y be two rough neutrosophic sets then the Cartesian product of X and Y is defined as

$X\times Y=\{(x,y)\}:x\in U,y\in V\}$, where

$T_{X\times Y}(x,y)=\min\{T_X(x),T_y(y)\}$, $I_{X\times Y}(x,y)=\min\{I_X(x),I_y(y)\}$ and

$F_{X\times Y}(x,y)=\max\{F_X(x),F_y(y)\}$

Definition 3.3: Let $X\times Y$ be the rough neutrosophic sets on U,V respectively. We call $R\subseteq U\times V$ is a neutrosophic rough relation on based on $U\times V$ if it satisfy $X\times Y$

1. $T_Y(x,y)=1$, for all $(x,y)\in\underline{X\times Y}$, where $\underline{X\times Y}=\underline{R_U}(X)\times\underline{Rv}(Y)$,

   $T_Y(x,y)=0$, for all $(x,y)\in\overline{X\times Y}$, where $\overline{X\times Y}=\overline{R_U}(X)\times\overline{R_V}(Y)$,

   $0<T_Y(x,y)<1$, for all $(x,y)\in\overline{X\times Y}-\underline{X\times Y}$.
2. $I_Y(x,y)=1$, for all $(x,y)\in\underline{X\times Y}$, where $\underline{X\times Y}=\underline{R_U}(X)\times\underline{Rv}(Y)$,

   $I_Y(x,y)=0$, for all $(x,y)\in\overline{X\times Y}$, where $\overline{X\times Y}=\overline{R_U}(X)\times\overline{R_V}(Y)$,

   $0<I_Y(x,y)<1$, for all $(x,y)\in\overline{X\times Y}-\underline{X\times Y}$.
3. $F_Y(x,y)=0$, for all $(x,y)\in\underline{X\times Y}$, where $\underline{X\times Y}=\underline{R_U}(X)\times\underline{Rv}(Y)$,

   $F_Y(x,y)=1$, for all $(x,y)\in\overline{X\times Y}$, where $\overline{X\times Y}=\overline{R_U}(X)\times\overline{R_V}(Y)$,

   $0<F_Y(x,y)<1$, for all $(x,y)\in\overline{X\times Y}-\underline{X\times Y}$.

The conditions (1), (2) show that $X\times Y$ is a rough neutrosophic set on $U\times V$.

Definition 3.4: let R be a rough neutrosophic set on $U\times V$ based on $X\times Y$. For any real numbers $\alpha,\beta,\gamma-cut$ of relation R is defined as:

$$R_\alpha(x,y)=1\Leftrightarrow R(x,y)\geq\alpha\quad R_\beta(x,y)=1\Leftrightarrow R(x,y)\geq\beta\quad R_\gamma(x,y)=0\Leftrightarrow R(x,y)\geq\gamma$$

$$R_\alpha(x,y)=0\Leftrightarrow R(x,y)<\alpha'\quad R_\beta(x,y)=0\Leftrightarrow R(x,y)<\beta'\quad R_\gamma(x,y)=1\Leftrightarrow R(x,y)<\gamma.$$

Example 3.6: Let $U=\{1,2,3,4\}$ and $V=\{16,7,8,9,10\}$ be two universal sets and

$R_U=\{(x,y):xR_Uy\}\text{ if }x\equiv y(\text{mod }2)\}$ and $R_V=\{(x,y):xR_Vy\}\text{ if }z\equiv t(\text{mod }3)\}$, are equivalent relations on respectively.

$$X=\frac{(1,1,0)}{1}+\frac{(0.5,0.4,0.3)}{2}+\frac{(1,1,0)}{3}+\frac{(0.8,0.7,0.3)}{4}$$

$$Y=\frac{(0.4,0.3,0.1)}{6}+\frac{(0.5,0.5,0.5)}{7}+\frac{(1,1,0)}{8}+\frac{(0.7,0.6,0..8)}{9}+\frac{(1,1,0)}{10}$$

are rough neutrosophic sets on U,V respectively.

$$\overline{R_U}X = \frac{(1,1,0)}{1} + \frac{(0.5,0.4,0.3)}{2} + \frac{(1,1,0)}{3} + \frac{(0.5,0.4,0.3)}{4}$$

$$\overline{R_V}Y = \frac{(0.7,0.6,0.1)}{6} + \frac{(1,1,0)}{7} + \frac{(1,1,0)}{8} + \frac{(0.7,0.6,0.1)}{9} + \frac{(1,1,0)}{10}$$

And

$$\underline{R_U}(X) = \frac{(1,1,0)}{1} + \frac{(0.8,0.7,0.3)}{2} + \frac{(1,1,0)}{3} + \frac{(0.8,0.7,0.3)}{4}$$

$$\underline{R_V}(Y) = \frac{(0.4,0.3,0.8)}{6} + \frac{(0.5,0.5,0.5)}{7} + \frac{(1,1,0)}{8} + \frac{(0.4,0.3,0.8)}{9} + \frac{(0.5,0.5,0.5)}{10}.$$

Here we can define a rough neutrosophic relation R by a matrix:

$$M(R) = \begin{bmatrix} (0,0,1) & (0.5,0.5,0.5) & (1,1,0) & (0,0,1) & (0.5,0.5,0.5) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0.5,0.5,0.5) & (1,1,0) & (0,0,1) & (0.5,0.5,0.5) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{bmatrix}$$

$$R_{0.5} = \begin{bmatrix} (0,0,1) & (1,1,0) & (1,1,0) & (1,1,0) & (1,1,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0,0,1) & (1,1,0) & (1,1,0) & (0,0,1) & (1,1,0) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \end{bmatrix}$$

Proposition 3.7: Let R_1, R_2 be two rough neutrosophic relations on $U \times V$ based on $X \times Y$. Then $R_1 \wedge R_2$ where

$T_{R_1 \wedge R_2}(x,y) = \min\{T_{R_1}(x,y), T_{R_2}(x,y)\}$, $I_{R_1 \wedge R_2}(x,y) = \min\{I_{R_1}(x,y), I_{R_2}(x,y)\}$,

$F_{R_1 \wedge R_2}(x,y) = \max\{F_{R_1}(x,y), F_{R_2}(x,y)\}$ for all $(x,y) \in U \times V$,is a rough neutrosophic set on $U \times V$ based on $X \times Y$.

Proof:

We show $R_1 \wedge R_2$ that satisfy definition 2.4

1. Since, $T_{R_1}(x,y) = T_{R_2}(x,y) = 1$, for all $(x,y) \in \underline{X \times Y}$, then

   $T_{R_1 \wedge R_2}(x,y) = \min\{T_{R_1}(x,y), T_{R_2}(x,y)\} = 1$ for all $(x,y) \in \underline{X \times Y}$.

   Since, $T_{R_1}(x,y) = T_{R_2}(x,y) = 0$, for all $(x,y) \in U \times V - \underline{X \times Y}$, then

   $T_{R_1 \wedge R_2}(x,y) = \min\{T_{R_1}(x,y), T_{R_2}(x,y)\} = 0$ for all $(x,y) \in U \times V - \underline{X \times Y}$.

   Since, $0 < \min\{T_{R_1}(x,y), T_{R_2}(x,y)\} < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$, then

   $0 < T_{R_1 \wedge R_2}(x,y) = \min\{T_{R_1}(x,y), T_{R_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.

2. Since, $I_{R_1}(x,y) = I_{R_2}(x,y) = 1$, for all $(x,y) \in \underline{X \times Y}$, then

   $I_{R_1 \wedge R_2}(x,y) = \min\{I_{R_1}(x,y), I_{R_2}(x,y)\} = 1$ for all $(x,y) \in \underline{X \times Y}$.

   Since, $I_{R_1}(x,y) = I_{R_2}(x,y) = 0$, for all $(x,y) \in U \times V - \underline{X \times Y}$, then

   $I_{R_1 \wedge R_2}(x,y) = \min\{I_{R_1}(x,y), I_{R_2}(x,y)\} = 0$ for all $(x,y) \in U \times V - \underline{X \times Y}$.

Since, $0 < \min\{I_{R_1}(x,y), I_{R_2}(x,y)\} < 1$, for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$, then

$0 < I_{R_1\wedge R_2}(x,y) = \min I_{R_1}(x,y), I_{R_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$.

3. Since, $F_Y(x,y) = F_Y(x,y) = 0$, for all $(x,y) \in \underline{X\times Y}$, then

   $F_{R_1\wedge R_2}(x,y) = \min\{F_{R_1}(x,y), F_{R_2}(x,y)\} = 0$ for all $(x,y) \in \underline{X\times Y}$.

   Since, $F_Y(x,y) = F_Y(x,y) = 1$, for all $(x,y) \in U\times V - \underline{X\times Y}$, then

   $F_{R_1\wedge R_2}(x,y) = \min\{F_{R_1}(x,y), F_{R_2}(x,y)\} = 1$ for all $(x,y) \in U\times V - \underline{X\times Y}$.

   Since, $0 < \min\{I_{R_1}(x,y), I_{R_2}(x,y)\} < 1$, for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$, then

   $0 < F_{R_1\wedge R_2}(x,y) = \min\{F_{R_1}(x,y), F_{R_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$.

Proposition 3.8: Let R_1, R_2 be two rough neutrosophic relations on $U\times V$ based on $X\times Y$. Then $R_1 \vee R_2$ where

$T_{R_1\vee R_2}(x,y) = \max\{T_{R_1}(x,y), T_{R_2}(x,y)\}$, $I_{R_1\vee R_2}(x,y) = \max\{I_{R_1}(x,y), I_{R_2}(x,y)\}$,

$F_{R_1\vee R_2}(x,y) = \min\{F_{R_1}(x,y), F_{R_2}(x,y)\}$ for all $(x,y) \in U\times V$ is a rough neutrosophic set on $U\times V$ based on $X\times Y$.

We show $R_1 \vee R_2$ is satisfied in Definition 3.4 indeed:

1. Since, $T_{R_1}(x,y) = T_{R_2}(x,y) = 1$, for all $(x,y) \in \underline{X\times Y}$, then

   $T_{R_1\vee R_2}(x,y) = \max\{T_{R_1}(x,y), T_{R_2}(x,y)\} = 1$ for all $(x,y) \in \underline{X\times Y}$.

   Since, $T_{R_1}(x,y) = T_{R_2}(x,y) = 0$, for all $(x,y) \in U\times V - \underline{X\times Y}$, then

   $T_{R_1\vee R_2}(x,y) = \max\{T_{R_1}(x,y), T_{R_2}(x,y)\} = 0$ for all $(x,y) \in U\times V - \underline{X\times Y}$.

   Since, $0 < \max\{T_{R_1}(x,y), T_{R_2}(x,y)\} < 1$, for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$, then

   $0 < T_{R_1\vee R_2}(x,y) = \max\{T_{R_1}(x,y), T_{R_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$.

2. Since, $I_{R_1}(x,y) = I_{R_2}(x,y) = 1$, for all $(x,y) \in \underline{X\times Y}$, then

   $I_{R_1\wedge R_2}(x,y) = \min\{I_{R_1}(x,y), I_{R_2}(x,y)\} = 1$ for all $(x,y) \in \underline{X\times Y}$.

   Since, $I_{R_1}(x,y) = I_{R_2}(x,y) = 0$, for all $(x,y) \in U\times V - \underline{X\times Y}$, then

   $I_{R_1\vee R_2}(x,y) = \max\{I_{R_1}(x,y), I_{R_2}(x,y)\} = 0$ for all $(x,y) \in U\times V - \underline{X\times Y}$.

   Since, $0 < \max\{I_{R_1}(x,y), I_{R_2}(x,y)\} < 1$, for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$, then

   $0 < I_{R_1\vee R_2}(x,y) = \max\{I_{R_1}(x,y), I_{R_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$.

3. Since, $F_Y(x,y) = F_Y(x,y) = 0$, for all $(x,y) \in \underline{X\times Y}$, then

   $F_{R_1\vee R_2}(x,y) = \max\{F_{R_1}(x,y), F_{R_2}(x,y)\} = 0$ for all $(x,y) \in \underline{X\times Y}$.

   Since, $F_Y(x,y) = F_Y(x,y) = 1$, for all $(x,y) \in U\times V - \underline{X\times Y}$, then

   $F_{R_1\vee R_2}(x,y) = \max\{F_{R_1}(x,y), F_{R_2}(x,y)\} = 1$ for all $(x,y) \in U\times V - \underline{X\times Y}$.

   Since, $0 < \max\{F_{R_1}(x,y), F_{R_2}(x,y)\} < 1$, for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$, then

   $0 < F_{R_1\vee R_2}(x,y) = \max\{F_{R_1}(x,y), F_{R_2}(x,y)\} < 1$ for all $(x,y) \in \overline{X\times Y} - \underline{X\times Y}$.

Lemma 3.9: If $0 < a, b < 1$ then

i) $0 < ab < 1$ (obvious)

ii) $0 < a + b - ab < 1$

Indeed, since $0 < a, b < 1$ then

$a + b \geq 2\sqrt{ab} > 2ab > ab > 0$,

therefore $0 < a + b - ab > 0$.

On the other hand $1 - (a + b - ab) = (1 - a)(1 - b) > 0$

then $a + b - ab < 1$.

The following properties of rough neutrosophic relations are obtained by using these algebraic results:

Proposition 3.10: Let R_1, R_2 be two rough neutrosophic relations on $U \times V$ based on $X \times Y$.

Then $R_1 \otimes R_2$ where

$T_{R_1 \otimes R_2}(x, y) = T_{R_1}(x, y) \bullet T_{R_2}(x, y)$, $I_{R_1 \otimes R_2}(x, y) = I_{R_1}(x, y) \bullet I_{R_2}(x, y)$,

$F_{R_1 \otimes R_2}(x, y) = F_{R_1}(x, y) \bullet F_{R_2}(x, y)$ for all $(x, y) \in U \times V$ is a rough neutrosophic set on $U \times V$ based on $X \times Y$.

Proof. The relation $R_1 \otimes R_2$ is satisfied in Definition 2.4.

1. Since, $T_{R_1}(x, y) = T_{R_2}(x, y) = 1$, for all $(x, y) \in \underline{X \times Y}$, then

   $T_{R_1 \otimes R_2}(x, y) = T_{R_1}(x, y) \bullet T_{R_2}(x, y) = 1$ for all $(x, y) \in \underline{X \times Y}$.

   Since, $T_{R_1}(x, y) = T_{R_2}(x, y) = 0$, for all $(x, y) \in U \times V - \underline{X \times Y}$, then

   $T_{R_1 \otimes R_2}(x, y) = T_{R_1}(x, y) \bullet T_{R_2}(x, y) = 0$ for all $(x, y) \in U \times V - \underline{X \times Y}$.

   Since, $0 < T_{R_1}(x, y), T_{R_2}(x, y) < 1$, for all $(x, y) \in \overline{X \times Y} - \underline{X \times Y}$, then

   $0 < T_{R_1 \otimes R_2}(x, y) = T_{R_1}(x, y) \bullet T_{R_2}(x, y) < 1$ for all $(x, y) \in \overline{X \times Y} - \underline{X \times Y}$.

   (by Lemma 3.9-(i))

2. Since, $I_{R_1}(x, y) = I_{R_2}(x, y) = 1$, for all $(x, y) \in \underline{X \times Y}$, then

   $I_{R_1 \otimes R_2}(x, y) = I_{R_1}(x, y) \bullet I_{R_2}(x, y) = 1$ for all $(x, y) \in \underline{X \times Y}$.

   Since, $I_{R_1}(x, y) = I_{R_2}(x, y) = 0$, for all $(x, y) \in U \times V - \underline{X \times Y}$, then

   $I_{R_1 \otimes R_2}(x, y) = I_{R_1}(x, y) \bullet I_{R_2}(x, y) = 0$ for all $(x, y) \in U \times V - \underline{X \times Y}$.

   Since, $0 < I_{R_1}(x, y), I_{R_2}(x, y) < 1$, for all $(x, y) \in \overline{X \times Y} - \underline{X \times Y}$, then

   $0 < I_{R_1 \otimes R_2}(x, y) = I_{R_1}(x, y) \bullet I_{R_2}(x, y) < 1$ for all $(x, y) \in \overline{X \times Y} - \underline{X \times Y}$.

   (by Lemma 3.9-(i))

3. Since, $F_Y(x, y) = F_Y(x, y) = 0$, for all $(x, y) \in \underline{X \times Y}$, then

   $F_{R_1 \otimes R_2}(x, y) = F_{R_1}(x, y) \bullet F_{R_2}(x, y) = 0$ for all $(x, y) \in \underline{X \times Y}$.

   Since, $F_Y(x, y) = F_Y(x, y) = 1$, for all $(x, y) \in U \times V - \underline{X \times Y}$, then

   $F_{R_1 \otimes R_2}(x, y) = F_{R_1}(x, y) \bullet F_{R_2}(x, y) = 1$ for all $(x, y) \in U \times V - \underline{X \times Y}$.

Since, $0 < F_{R_1}(x,y), F_{R_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$, then

$0 < F_{R_1 \otimes R_2}(x,y) = F_{R_1}(x,y) \bullet F_{R_2}(x,y) < 1$ for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.

(by Lemma 3.9-(i))

Proposition 3.11: Let R_1, R_2 be two rough neutrosophic relations on $U \times V$ based on $X \times Y$.

Then $R_1 \oplus R_2$ where

$T_{R_1 \oplus R_2}(x,y) = T_{R_1}(x,y) + T_{R_2}(x,y) - T_{R_1}(x,y) \bullet T_{R_2}(x,y)$,

$I_{R_1 \oplus R_2}(x,y) = I_{R_1}(x,y) + I_{R_2}(x,y) - I_{R_1}(x,y) \bullet I_{R_2}(x,y)$,

$F_{R_1 \oplus R_2}(x,y) = F_{R_1}(x,y) + F_{R_2}(x,y) - F_{R_1}(x,y) \bullet F_{R_2}(x,y)$

for all $(x,y) \in U \times V$ is a rough neutrosophic set on $U \times V$ based on $X \times Y$.

Proof.

The relation $R_1 \oplus R_2$ is satisfied in Definition 2.4.

1. Since, $T_{R_1}(x,y) = T_{R_2}(x,y) = 1$, for all $(x,y) \in \underline{X \times Y}$, then

   $T_{R_1 \oplus R_2}(x,y) = T_{R_1}(x,y) + T_{R_2}(x,y) - T_{R_1}(x,y) \bullet T_{R_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$.

   Since, $T_{R_1}(x,y) = T_{R_2}(x,y) = 0$, for all $(x,y) \in U \times V - \underline{X \times Y}$, then

   $T_{R_1 \oplus R_2}(x,y) = T_{R_1}(x,y) + T_{R_2}(x,y) - T_{R_1}(x,y) \bullet T_{R_2}(x,y) = 0$ for all

   $(x,y) \in U \times V - \underline{X \times Y}$.

   Since, $0 < T_{R_1}(x,y), T_{R_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$, then

   $0 < T_{R_1 \oplus R_2}(x,y) = T_{R_1}(x,y) + T_{R_2}(x,y) - T_{R_1}(x,y) \bullet T_{R_2}(x,y) < 1$ for all

   $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.(by Lemma 2.9-(ii))

2. Since, $I_{R_1}(x,y) = I_{R_2}(x,y) = 1$, for all $(x,y) \in \underline{X \times Y}$, then

   $I_{R_1 \oplus R_2}(x,y) = I_{R_1}(x,y) + I_{R_2}(x,y) - I_{R_1}(x,y) \bullet I_{R_2}(x,y) = 1$ for all $(x,y) \in \underline{X \times Y}$.

   Since, $I_{R_1}(x,y) = I_{R_2}(x,y) = 0$, for all $(x,y) \in U \times V - \underline{X \times Y}$, then

   $I_{R_1 \oplus R_2}(x,y) = I_{R_1}(x,y) + I_{R_2}(x,y) - I_{R_1}(x,y) \bullet I_{R_2}(x,y) = 0$ for all

   $(x,y) \in U \times V - \underline{X \times Y}$.

   Since, $0 < I_{R_1}(x,y), I_{R_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$, then

   $0 < I_{R_1 \oplus R_2}(x,y) = I_{R_1}(x,y) + I_{R_2}(x,y) - I_{R_1}(x,y) \bullet I_{R_2}(x,y) < 1$ for all

   $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$.(by Lemma 2.9-(ii))

3. Since, $F_Y(x,y) = F_Y(x,y) = 0$, for all $(x,y) \in \underline{X \times Y}$, then

   $F_{R_1 \oplus R_2}(x,y) = F_{R_1}(x,y) + F_{R_2}(x,y) - F_{R_1}(x,y) \bullet F_{R_2}(x,y) = 0$ for all $(x,y) \in \underline{X \times Y}$.

   Since, $F_Y(x,y) = F_Y(x,y) = 1$, for all $(x,y) \in U \times V - \underline{X \times Y}$, then

   $F_{R_1 \oplus R_2}(x,y) = F_{R_1}(x,y) + F_{R_2}(x,y) - F_{R_1}(x,y) \bullet F_{R_2}(x,y) = 1$ for all

   $(x,y) \in U \times V - \underline{X \times Y}$.

Since, $0 < F_{R_1}(x,y), F_{R_2}(x,y) < 1$, for all $(x,y) \in \overline{X \times Y} - \underline{X \times Y}$, then

$0 < F_{R_1 \oplus R_2}(x,y) = F_{R_1}(x,y) + F_{R_2}(x,y) - F_{R_1}(x,y) \bullet F_{R_2}(x,y) < 1$ for all

$(x,y) \in \overline{X \times Y} - \underline{X \times Y}$. (by Lemma 2.9-(ii)).

4. COMPOSITION OF TWO ROUGH NEUTROSOPHIC RELATIONS

Let (U,V,W) be the universal sets. R_1, R_2 are two rough neutrosophic relations on $U \times V, V \times W$ based on $X \times Y, Y \times Z$ respectively.

Definition 4.1: Composition of two rough neutrosophic relations R_1, R_2 denote $R_1 \circ R_2$ which defined on $U \times W$ based on $X \times Z$ where

$T_{R_1 \circ R_2}(x,y) = \max_{y \in V}\{\min[T_{R_1}(x,y), T_{R_2}(y,z)]\}$,

$I_{R_1 \circ R_2}(x,y) = \max_{y \in V}\{\min[I_{R_1}(x,y), I_{R_2}(y,z)]\}$,

$F_{R_1 \circ R_2}(x,y) = \min_{y \in V}\{\max[F_{R_1}(x,y), F_{R_2}(y,z)]\}$.

for all $(x,z) \in U \times W$.

Proposition 4.2: $R_1 \circ R_2$ is a rough neutrosophic relation which defined on $U \times W$ based on $X \times Z$.

Proof.

1. Since R_1, R_2 are two rough neutrosophic relations on $U \times V, V \times W$ based on $X \times Y, Y \times Z$ respectively. Then $T_{R_1}(x,y), T_{R_2}(x,y) = 1$, for all $(x,y) \in \underline{X \times Y}$, and $(y,z) \in \underline{Y \times Z}$. We denote

   $T_{R_1 \circ R_2}(x,y) = \max_{y \in V}\{\min_{(x,z) \in \underline{X \times Z}}[T_{R_1}(x,y), T_{R_2}(y,z)]\}$ =

   $\max_{y \in \underline{Y}}\{\min_{(x,z) \in \underline{X \times Z}}[T_{R_1}(x,y), T_{R_2}(y,z)]\} \vee \max_{v \in V - \underline{Y}}\{\min_{(x,z) \in \underline{X \times Z}}[T_{R_1}(x,y), T_{R_2}(y,z)]\}$

   = 1, for all $(x,z) \in X \times Z$.

2. Note that $T_{R_1}(x,y), T_{R_2}(x,y) = 0$ for all $(x,y) \in U \times V - \overline{X \times Y}$ and $(y,z) \in V \times W - \overline{Y \times Z}$.

   We consider

   $T_{R_1 \circ R_2}(x,z) = \max_{y \in V}\{\min_{(x,z) \in U \times W - \underline{X \times Z}}[T_{R_1}(x,y), T_{R_2}(y,z)]\}$

   For all $(x,z) \in U \times W - \overline{X \times Z}$, it exists $x \notin \overline{R}_R(x)$ so $(x,v) \in U \times V - \overline{X \times Y}$ and $T_{R_1}(x,v) = 0$

   For all $v \in V$. Similarly, it exists $z \notin \overline{R}_R(z)$ so $(x,v) \in V \times W - \overline{Y \times Z}$ and $T_{R_2}(v,z) = 0$, for all $v \in V$.

   Hence $\min_{(x,z) \in \underline{X \times Z}}[T_{R_1}(x,v), T_{R_2}(v,z)] = 0$, for all $(x,z) \in U \times W - \overline{X \times Z}$.

3. We must prove

   $0 < \max_{y \in v}\{\min[T_{R_1}(x,y), T_{R_2}(x,y)]\} < 1$, for all $\overline{X \times Z} - \underline{X \times Z}$.

   Since $(x,z) \notin \underline{X \times Z}$ then atleast exist $x \notin \underline{X} = \underline{R}_{R_U}(X)$ or $z \notin \underline{Z} = \underline{R}_{R_W}(Z)$.

   So that $\min[T_{R_1}(x,y), T_{R_2}(y,z)] < 1$, for all $y \in Y$.

   On the other hand, $T_{R_1}(x,v) = T_{R_2}(v,z) = 0$, for all $(x,z) \in U \times W - \overline{X \times Z}$ and $v \in V$

then we have $0<\min[T_{R_1}(x,y),T_{R_2}(y,z)]<1$ for all $(x,z)\in \overline{X\times Z}-\underline{X\times Z}$ and $y\in Y$.

Hence, we have $0<\max_{y\in v}\{\min[T_{R_1}(x,y),T_{R_2}(x,y)]\}<1$ or $0<T_{R_1 \circ R_2}(x,z)<1$ for all $(x,z)\in \overline{X\times Z}-\underline{X\times Z}$.

Similarly we can prove

$0<\max_{y\in v}\{\min[I_{R_1}(x,y),I_{R_2}(x,y)]\}<1$ or $0<I_{R_1 \circ R_2}(x,z)<1$ for all $(x,z)\in \overline{X\times Z}-\underline{X\times Z}$.

$0<\min_{y\in v}\{\max[F_{R_1}(x,y),F_{R_2}(x,y)]\}<1$ or $0<F_{R_1 \circ R_2}(x,z)<1$ for all $(x,z)\in \overline{X\times Z}-\underline{X\times Z}$.

Proposition 4. 3: Let U,V,W,W' be the universe sets are rough neutrosophic relations on $U\times V, V\times W, W\times W'$ based on $X\times Y, Y\times Z, Z\times Z'$ respectively. Then $(R_1 \circ R_2)\circ R_3 = R_1 \circ (R_2 \circ R_3)$.

Proof.

For all $x\in U, y\in V, z\in W, t\notin W'$, we have

$$\begin{aligned}
T_{R_1 \circ (R_2 \circ R_3)}(x,t) &= \max_{y\in V}\{\min[T_{R_1}(x,y),T_{R_2 \circ R_3}(y,t)]\}\\
&= \max_{y\in V}\{\min[T_{R_1}(x,y),\max_{Z\in W}\{\min[T_{R_2}(y,z),T_{R_3}(z,t)]\}\}\}\\
&= \max_{Z\in W}\{\min\{\max_{y\in V}\{\min[T_{R_2}(y,z),T_{R_3}(z,t)]\}\}\}\\
&= \max_{Z\in W}\{\min[T_{R_1 \circ R_2}(x,y),T_{R_3}(y,t)]\}\\
&= T_{(R_1 \circ R_2)\circ R_3}(x,t)
\end{aligned}$$

Similarly,

$I_{R_1 \circ (R_2 \circ R_3)}(x,t) = I_{(R_1 \circ R_2)\circ R_3}(x,t)$.

$F_{R_1 \circ (R_2 \circ R_3)}(x,t) = F_{(R_1 \circ R_2)\circ R_3}(x,t)$.

We note that $R_1 \circ R_2 \neq R_2 \circ R_1$, because the composition of two rough neutrosophic relations R_1, R_2 exists but the composition of two rough neutrosophic relations R_2, R_1 does not exist necessarily.

5. INVERSE ROUGH NEUTROSOPHIC RELATION

Let X and Y be the two rough neutrosophic sets on U and V, respectively. $R\subset U\times V$ is a rough neutrosophic relation on $U\times V$ based on $X\times Y$. Then we define $R^{-1}\subset V\times U$ is a rough neutrosophic relation on $V\times U$ based on $Y\times X$ as following:

$T_{R^{-1}}(y,x)=T_R(x,y)$, $I_{R^{-1}}(y,x)=I_R(x,y)$ and $F_{R^{-1}}(y,x)=F_R(x,y)$ for all $(x,y)\in V\times U$.

Definition 5.1: The relation R^{-1} is called the inverse rough neutrosophic relation of R.

Proposition 5.2:

(i) $(R^{-1})^{-1}=R$.

(ii) Let R_1, R_2 be two rough neutrosophic relations on $U\times V, V\times W$ based on $X\times Y, Y\times Z$, respectively. Then $(R_1 \circ R_2)^{-1} = R_2^{-1}\circ R_1^{-1}$.

Proof.

(i) $T_{(R^{-1})^{-1}}(x,y)=T_{R^{-1}}(y,x)=T_R(x,y)=1$, for all $(x,y)\in \underline{X\times Y}$

$T_{(R^{-1})^{-1}}(x,y) = T_{R^{-1}}(y,x) = T_R(x,y) = 0$, for all $(x,y) \in U \times V - \overline{X \times Y}$.

$0 < T_{(R^{-1})^{-1}}(x,y) = T_{R^{-1}}(y,x) = T_R(x,y) < 1$

It means $(R^{-1})^{-1} = R$.

(ii) For all $x \in U, y \in V, z \in W$, we have

$T_{(R_1 \circ R_2)^{-1}}(z,x) = T_{(R_1 \circ R_2)}(x,z)$

$= \max_{y \in U} \{\min[T_{R_1}(x,y,), T_{R_2}(y,z)]\}$

$= \max_{y \in U} \{\min[T_{R_2}{}^{-1}(z,y,), T_{R_1}{}^{-1}(y,x)]\}$

$= T_{R_2^{-1} \circ R_1^{-1}}(z,x)$

That means $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$.

Similarly we can prove for indeterminacy and falsity functions.

In the same way, the representation of neutrosphic relation , we can represent the rough neutrosophic relation R by using matrix $M(R)$. So that, the inverse rough neutrosphic relation R^{-1} of rough neutrosphic relation R by using matrix $M(R)^t$, it is the transposition of the matrix $M(R)$.

Example 5.3

Consider the Example 3.5,

we define a rough neutrosophic relation R by a matrix:

$$M(R) = \begin{bmatrix} 0 & (0.5,0.5,0.5) & (1,1,0) & 0 & (0.5,0.5,0.5) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & (0.5,0.5,0.5) & (1,1,0) & 0 & (0.5,0.5,0.5) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Then R^{-1} is an inverse rough neutrosophic relation of R , it is represented by using matrix

$$M(R^{-1}) = M(R)^t = \begin{bmatrix} (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0.5,0.5,0.5) & (0,0,1) & (0.5,0.5,0.5) & (0,0,1) & (0,0,1) \\ (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) & (0,0,1) \\ (0.5,0.5,0.5) & (0,0,1) & (0.5,0.5,0.5) & (0,0,1) & (0,0,1) \end{bmatrix}$$

6. THE REFLEXIVE, SYMMETRIC, TRANSITIVE ROUGH NEUTROSOPHIC RELATION

In this section, we consider some properties of rough neutrosophic relation on a set, such as reflexive, symmetric, transitive properties.

Let (U,R) be a crisp approximation space and X is a rough neutrosophic set on (U,R). We consider rough neutrosophic relation in Definition 3.1 in the case $U \equiv V, R_U \equiv R_V \equiv R$, and $X \equiv Y$. From here onwards, the rough neutrosophic relation R is called rough neutrosophic relation on (U,R) based on the rough neutrosophic set X .

Definition 6.1: The rough neutrosophic relation R is said to be reflexive rough neutrosophic relation if $T_R(x,x) = 1$, $I_R(x,x) = 1, F_R(x,x) = 0$ for all $(x,x) \in U \times U$..

Proposition 6.2: Let R_1, R_2 be two rough neutrosophic relation on U based X . If R_1, R_2 are the reflexive rough neutrosophic relations then also $R_1 \wedge R_2$, $R_1 \vee R_2$, $R_1 \otimes R_2$, $R_1 \oplus R_2$, $R_1 \circ R_2$ also.

Proof:

If R_1, R_2 are the reflexive rough neutrosophic relations then $T_{R_1}(x,x), T_{R_2}(x,x) = 1$ for all $(x,x) \in U \times U, T_X(x) > 0$. We have

$T_{R_1 \wedge R_2}(x,x) = \min\{T_{R_1}(x,x), T_{R_2}(x,x)\} = 1$ for all $(x,x) \in U \times U, T_X(x) > 0$ and $R_1 \wedge R_2$ is reflexive rough neutrosophic relation.

$T_{R_1 \vee R_2}(x,x) = \max\{T_{R_1}(x,x), T_{R_2}(x,x)\} = 1$ for all $(x,x) \in U \times U, T_X(x) > 0$ and $R_1 \vee R_2$ is reflexive rough neutrosophic relation.

$T_{R_1 \otimes R_2}(x,x) = T_{R_1}(x,x), T_{R_2}(x,x) = 1$ for all $(x,x) \in U \times U, T_X(x) > 0$ and $R_1 \otimes R_2$ is reflexive rough neutrosophic relation.

$T_{R_1 \oplus R_2}(x,x) = T_{R_1}(x,x) + T_{R_2}(x,x) - T_{R_1}(x,x) \bullet T_{R_2}(x,x) = 1$ for all $(x,x) \in U \times U, T_X(x) > 0$ and $R_1 \oplus R_2$ is reflexive rough neutrosophic relation.

$T_{R_1 \circ R_2}(x,x) = \max_{y \in U}\{\min[T_{R_1}(x,x), T_{R_2}(x,x)]\} = 1$ for all $(x,x) \in U \times U, T_X(x) > 0$ and $R_1 \circ R_2$ is reflexive rough neutrosophic relation.

Similarly we can prove for indeterminacy and falsity functions

Definition 6.3:

The rough neutrosophic relation R is said to be α-reflexive rough neutrosophic relation where $\alpha = \min_{[x_i]_R \subset U} \alpha_i$ if $T_R(x_i, y)\alpha_i$ for all $(x_i, y) \in U \times U, y \in [x_i]_R$ and $T_X(x) > 0$.

Proposition 6.4: Let R_1, R_2 be two rough neutrosophic relation on based . If R_1, R_2 are the α-reflexive rough neutrosophic relations then $R_1 \wedge R_2$, $R_1 \vee R_2$ also.

Proof:

If R_1, R_2 are the α-reflexive rough neutrosophic relations then $T_{R_1}(x_i, y) = T_{R_2}(x_i, y) = \alpha$ where $\alpha = \min_{[x_i]_R \subset U} \alpha_i$ if $T_{R_1}(x_i, y) = T_{R_2}(x_i, y) = \alpha_i$ for all $(x_i, y) \in U \times U, y \in [x_i]_R$ and $T_X(x) > 0.$

We have

$T_{R_1 \wedge R_2}(x_i, y) = \min\{T_{R_1}(x_i, y), T_{R_2}(x_i, y)\} = \alpha$

Where $\alpha = \min_{[x_i]_R \subset U} \alpha_i$ if $T_{R_1}(x_i, y) = T_{R_2}(x_i, y) = \alpha_i$ for all $(x_i, y) \in U \times U, y \in [x_i]_R$ and $T_X(x) > 0$ for all $(x,x) \in U \times U$, $T_X(x) > 0$ and $R_1 \wedge R_2$ α-reflexive rough neutrosophic relation.

$T_{R_1 \vee R_2}(x_i, y) = \max\{T_{R_1}(x_i, y), T_{R_2}(x_i, y)\} = \alpha$

Where $\alpha = \min_{[x_i]_R \subset U} \alpha_i$ if $T_{R_1}(x_i, y) = T_{R_2}(x_i, y) = \alpha_i$ for all $(x_i, y) \in U \times U, y \in [x_i]_R$ and $T_X(x) > 0$ for all $(x,x) \in U \times U$, $T_X(x) > 0$ and $R_1 \vee R_2$ α-reflexive rough neutrosophic relation.

Similarly we can prove for indeterminacy and falsity functions

Definition 6.5: The rough neutrosophic relation R is said to be symmetric rough neutrosophic relation if $T_R(x,y) = T_R(x,y)$ for all $(x_i, y) \in U \times U$.

Note: If R is a symmetric rough neutrosophic relation then matrix M(R) is a symmetric matrix.

Proposition 6.6: Let R_1, R_2 be two rough neutrosophic relation on U based X . If R_1, R_2 are the symmetric rough neutrosophic relations then also $R_1 \wedge R_2$, $R_1 \vee R_2$, $R_1 \otimes R_2$, $R_1 \oplus R_2$, $R_1 \circ R_2$ also.

Definition 6.7: The rough neutrosophic relation R is said to be transitive rough neutrosophic relation if $R \circ R \subseteq R$.

7. SIMILARITY ROUGH NEUTROSOPHIC RELATION

Definition 7.1: The rough neutrosophic relation R on U based on the rough neutrosophic set X is called a similarity rough neutrosophic relation if it has the reflexive, symmetric, transitive.

Definition 7.2: The rough neutrosophic relation R on U based on the rough neutrosophic set X is called a α- similarity rough neutrosophic relation if it has the α-reflexive, symmetric, transitive.

Now, we consider an illustration example.

Example 7.3: We consider the decision system in Table 1. In which is the collection of condition attributes and is the decision attribute.

Table 1

U	A	B	C	D
U_1	(0.1,0.3,0.6)	(0.2,0.5,0.3)	(0.3,0.4,0.5)	Y
U_2	(0.1,0.4,0.5)	(0.3,0.4,0.3)	(0.4,0.5,0.2)	N
U_3	(0.2,0.4,0.4)	(0.3,0.5,0.2)	(0.5,0.8,0.6)	Y
U_4	(0.2,0.7,0.1)	(0.3,0.3,0.4)	(0.5,0.3,0.3)	Y
U_5	(0.6,0.4,0.1)	(0.4,0.2,0.4)	(0.3,0.9,0.1)	N

Let R= IND(A, B, C) is an equivalence relation on U. (U, R) is a crisp approximation space.

Let U={ U_1, U_2, U_3, U_4, U_5} and (U, R) is a crisp approximation space and

U/R={ U_1 }{U_2 }{U_3, U_4 }{U_5}}

We consider

$$F=\frac{(0.5,0.4,0.1)}{U_1}+\frac{(0.2,0.3,0.4)}{U_2}+\frac{(1,1,0)}{U_3}+\frac{(1,1,0)}{U_4}+\frac{(1,1,0)}{U_5}$$ is a neutrosphic set in U.

We compute

$$\underline{R_V}(F)=\frac{(0.7,0.6,0.1)}{U_1}+\frac{(1,1,0)}{U_2}+\frac{(1,1,0)}{U_3}+\frac{(0.7,0.6,0.1)}{U_4}+\frac{(1,1,0)}{U_5}$$

$$\overline{R_V}(F)=\frac{(0.7,0.6,0.1)}{U_1}+\frac{(1,1,0)}{U_2}+\frac{(1,1,0)}{U_3}+\frac{(0.7,0.6,0.1)}{U_4}+\frac{(1,1,0)}{U_5}$$

We , $\overline{R_V}(F)=\underline{R_V}(F)$, so is not a rough neutrosophic set on U.

We consider

$$F=\frac{(0.5,0.4,0.1)}{U_1}+\frac{(0.2,0.3,0.4)}{U_2}+\frac{(1,1,0)}{U_3}+\frac{(0.5,0.5,1)}{U_4}+\frac{(1,1,0)}{U_5}$$ is a neutrosphic set in U.

We compute

$$\underline{R_V}(F)=\frac{(0.7,0.6,0.1)}{U_1}+\frac{(1,1,0)}{U_2}+\frac{(1,1,0)}{U_3}+\frac{(0.5,0.5,1)}{U_4}+\frac{(1,1,0)}{U_5}$$

$$\overline{R_V}(F)=\frac{(0.7,0.6,0.1)}{U_1}+\frac{(1,1,0)}{U_2}+\frac{(1,1,0)}{U_3}+\frac{(1,1,0)}{U_4}+\frac{(1,1,0)}{U_5}$$

It is easy that , $\overline{R_V}(F)\neq\underline{R_V}(F)$, so is a rough neutrosophic set on U.

We can put $T_{R_F}(u_5,u_5)=1$; $T_{R_F}(u_i,u_j)=1$, $I_{R_F}(u_5,u_5)=1$; $I_{R_F}(u_i,u_j)=1$ and $F_{R_F}(u_5,u_5)=0$; $F_{R_F}(u_i,u_j)=0$ if and i,j= 1,2 and $0>T_{R_F}(u_i,u_j), I_{R_F}(u_i,u_j), F_{R_F}(u_i,u_j)>1$ in the other cases.

$$M(R_F) = \begin{bmatrix} (1,1,0) & (1,1,0) & r_{13} & r_{14} & r_{15} \\ (1,1,0) & (1,1,0) & r_{23} & r_{24} & r_{25} \\ r_{31} & r_{32} & r_{33} & r_{34} & r_{35} \\ r_{41} & r_{42} & r_{43} & r_{44} & r_{45} \\ r_{51} & r_{52} & r_{53} & r_{54} & (1,1,0) \end{bmatrix}$$

We have R_F is a rough neutrosophic relation on U based on F.

Consider $= F_2 = \frac{(1,1,0)}{U_1} + \frac{(0.5,0.5,0.5)}{U_2} + \frac{(1,1,0)}{U_3} + \frac{(1,1,0)}{U_4} + \frac{(1,1,0)}{U_5}$

$$\underline{R_V}(F_2) = \frac{(1,1,0)}{U_1} + \frac{(1,1,0)}{U_2} + \frac{(0.5,0.5,0.5)}{U_3} + \frac{(0.5,0.5,0.5)}{U_4} + \frac{(1,1,0)}{U_5}$$

$$\overline{R_V}(F_2) = \frac{(1,1,0)}{U_1} + \frac{(1,1,0)}{U_2} + \frac{(1,1,0)}{U_3} + \frac{(1,1,0)}{U_4} + \frac{(1,1,0)}{U_5}$$

$$= M(R_{F_2}) = \begin{bmatrix} (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (0,0,1) & (0.5,0.5,0.5) & (0.5,0.5,0.5) & 0 \\ (0,0,1) & (0,0,1) & (0.5,0.5,0.5) & (0.5,0.5,0.5) & 0 \\ (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}$$

R_{F_2} is clearly the $\frac{1}{2}$ - reflexive, symmetric rough neutrosophic relation.

Also $M(R_{F_2} \circ R_{F_2}) = \begin{bmatrix} (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \\ (0,0,1) & (0,0,1) & (0.5,0.5,0.5) & (0.5,0.5,0.5) & 0 \\ (0,0,1) & (0,0,1) & (0.5,0.5,0.5) & (0.5,0.5,0.5) & 0 \\ (1,1,0) & (1,1,0) & (0,0,1) & (0,0,1) & (1,1,0) \end{bmatrix}.$

It is obvious that $M(R_{F_2} \circ R_{F_2}) \le M(R_{F_2})$ then $M(R_{F_2})$ is the transitive rough neutrosophic relation; so that, is a $M(R_{F_2})$ $\frac{1}{2}$ - similarity relation.

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