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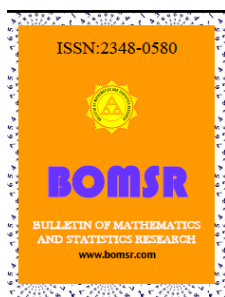


FUZZY NEUTROSOPHIC SUBGROUPS

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ABSTRACT

In this paper we introduce the notion of fuzzy neutrosophic subgroups. Also we obtain the fuzzy neutrosophic subgroups generated by fuzzy neutrosophic set and investigate some of their properties.

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1.INTRODUCTION

Smarandache [13] initiated the concept of neutrosophic set which overcomes the inherent difficulties that existed in fuzzy sets[14] and intuitionistic fuzzy sets [5,6].Following this, the neutrosophic sets are explored to different heights in all fields of science and engineering.I.Arockiarani et al. defined the notion of fuzzy neutrosophic sets [1].In 1989, R.Biswas [8] introduced the concept of intuitionistic fuzzy subgroups and studied some of their properties . In this paper we define fuzzy neutrosophic subgroups and discuss their properties.

2.PRELIMINARIES:

Definition 2.1:[1] A Fuzzy neutrosophic set A on the universe of discourse X is defined as

$$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X \text{ where } T, I, F: X \rightarrow [0,1] \text{ and } 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

Definition 2.2: [1] Let X be a non empty set, and

$$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$$

- (i) $A \subseteq B$ for all x if $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$
- (ii) $A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \min(F_A(x), F_B(x)) \rangle$
- (iii) $A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \max(F_A(x), F_B(x)) \rangle$
- (iv) $A \setminus B(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1 - I_B(x)), \max(F_A(x), T_B(x)) \rangle$

Definition 2.3:[1] A Fuzzy neutrosophic set A over the universe X is said to be null or empty Fuzzy neutrosophic set if $T_A(x) = 0$, $I_A(x) = 0$, $F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

Definition 2.4:[1] A Fuzzy neutrosophic set A over the universe X is said to be absolute fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

Definition 2.5: [1]The complement of a Fuzzy neutrosophic set A is denoted by A^c and is defined as $A^c = \langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x) \rangle$ where $T_{A^c}(x) = F_A(x), I_{A^c}(x) = 1 - I_A(x), F_{A^c}(x) = T_A(x)$

The complement of a Fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.

Definition 2.6:[2]Let X and Y be two non empty sets and $f : X \rightarrow Y$ be a function .

- (i) If $B = \{ \langle y, T_B(y), I_B(y), F_B(y) \rangle : y \in Y \}$ is a fuzzy neutrosophic set in Y then the pre image of B under f , denoted by $f^{-1}(B)$, is the fuzzy neutrosophic set in X defined by $f^{-1}(B) = \{ \langle x, f^{-1}(T_B(x)), f^{-1}(I_B(x)), f^{-1}(F_B(x)) \rangle : x \in X \}$

Where $f^{-1}(T_B(x)) = T_B(f(x))$

- (ii) If $A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$ is a fuzzy neutrosophic set in X then the image of A under f , denoted by $f(A)$, is the fuzzy neutrosophic set in Y defined by $f(A) = \{ \langle y, f(T_A(y)), f(I_A(y)), f_{\sim}(F_A(y)) \rangle : y \in Y \}$

$$f(T_A(y)) = \begin{cases} \text{Sup}_{x \in f^{-1}(y)} T_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 0 & \text{otherwise} \end{cases}$$

where $f(I_A(y)) = \begin{cases} \text{Sup}_{x \in f^{-1}(y)} I_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 0 & \text{otherwise} \end{cases}$

$$f_{\sim}(F_A(y)) = \begin{cases} \text{inf}_{x \in f^{-1}(y)} F_A(x) & \text{if } f^{-1}(y) \neq 0_N \\ 1 & \text{otherwise} \end{cases}$$

And $f_{\sim}(F_A(y)) = (1 - f(1 - F_A))y$

Definition 2.7:[3] Let $(X, .)$ be a group and let A be fuzzy neutrosophic set in X . Then A is called a fuzzy neutrosophic group (in short, FNG) in X if it satisfies the following conditions: (i) $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$ (ii) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x), F_A(x^{-1}) \leq F_A(x)$

Definition 2.8:[3]Let $(X, .)$ be a groupoid and let A and B be two fuzzy neutrosophic sets in X . Then the fuzzy neutrosophic product of A and $B, A \circ B$, is defined as follows: for any $x \in X$,

$$T_{A \circ B}(x) = \begin{cases} \vee_{yz=x} [T_A(y) \wedge T_B(z)] & \text{for each } (y, z) \in X \times X \text{ with } yz = x, \\ 0 & \text{otherwise} \end{cases}$$

$$I_{A \circ B}(x) = \begin{cases} \vee_{yz=x} [I_A(y) \wedge I_B(z)] & \text{for each } (y, z) \in X \times X \text{ with } yz = x, \\ 0 & \text{otherwise} \end{cases}$$

$$F_{A \circ B}(x) = \begin{cases} \wedge_{yz=x} [F_A(y) \vee F_B(z)] & \text{for each } (y, z) \in X \times X \text{ with } yz = x, \\ 1 & \text{otherwise} \end{cases}$$

Definiion 2.9:[4] Let G be a groupoid and let $A \in FNS(G)$. Then A is called a :

- (1) fuzzy neutrosophic left ideal (in short $FNLI$) of G if for any $x, y \in G, A(xy) \geq A(y)$.(i.e.,) $T_A(xy) \geq T_A(y), I_A(xy) \geq I_A(y)$ and $F_A(xy) \leq F_A(y)$
- (2) fuzzy neutrosophic right ideal (in short $FNRI$) of G if for any $x, y \in G, A(xy) \geq A(x)$.(i.e.,) $T_A(xy) \geq T_A(x), I_A(xy) \geq I_A(x)$ and $F_A(xy) \leq F_A(x)$
- (3) fuzzy neutrosophic ideal (in short FNI) of G if it is both a $FNLI$ and $FNRI$

It is clear that A is a FNI of G if and only if for any $x, y \in G, T_A(xy) \geq T_A(x) \vee T_A(y), I_A(xy) \geq I_A(x) \vee I_A(y)$ and $F_A(xy) \leq F_A(x) \wedge F_A(y)$. Moreover ,a FNI (respectively $FNLI, FNRI$) is a $FNSGP$ of G . Note that for any $FNSGP A$ of G we have $T_A(x^n) \geq T_A(x), I_A(x^n) \geq I_A(x)$ and $F_A(x^n) \leq F_A(x)$ for each $x \in G$, where x^n is any composite of x 's.

We will denote the set of all $FNSGP$ s of G as $FNSGP(G)$.

Definition 2.10:[4]Let (G, \cdot) be a groupoid and let $0_N \neq A \in FNS(G)$. Then A is called a fuzzy neutrosophic subgroupoid in G (in short , $FNSGP$ in G) if $A \circ A \subset A$.

Definition 2.11:[4]Let (G, \cdot) be a groupoid and let $A \in FNS(X)$. Then A is called a fuzzy neutrosophic subgroupoid in G (in short , $FNSGP$ in G) if for any $x, y \in G, T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y)$ and $F_A(xy) \leq F_A(x) \vee F_A(y)$. It is clear that 0_N and 1_N are both $FNSGP$ s of G .

Definition 2.12:[4]Let $A \in FNS(G)$. Then A is said to have the sup property if for any $T \in P(G)$,there exists a $t_0 \in T$ such that $A(t_0) = \bigcup_{t \in T} A(t)$.i.e.,

$$T_A(t_0) = \bigvee_{t \in T} T_A(t), I_A(t_0) = \bigvee_{t \in T} I_A(t), F_A(t_0) = \bigwedge_{t \in T} F_A(t),$$

where $P(G)$ denotes the power set of G .

Definition 2.13:[4]Let A be a fuzzy neutrosophic set in X and let $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$.Then the set $X_A^{(\lambda, \mu, \nu)} = \{x \in X : A(x) \geq C_{(\lambda, \mu, \nu)}(x)\} = \{x \in X : T_A(x) \geq \lambda, I_A \geq \mu, F_A \leq \nu\}$ is called a (λ, μ, ν) – level subset of A .

3. FUZZY NEUTROSOPHIC SUBGROUPS

Definition 3.1:

Let G be a group and let $A \in FNSGP(G)$. Then A is called a fuzzy neutrosophic subgroup (in short , $FNSG$) of G if $A(x^{-1}) \geq A(x)$.(i.e.,) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x)$ and $F_A(x^{-1}) \leq F_A(x)$ for each $x \in G$.

Proposition 3.2:

Let $\{A_\alpha\}_{\alpha \in \beta} \subset FNSG(G)$. Then $\bigcap_{\alpha \in \beta} A_\alpha \in FNSG(G)$.

Proposition 3.3:

Let A and B be any two $FNSG$ s of a group G . Then the following conditions are equivalent:

- (1) $A \circ B \in FNSG(G)$.
- (2) $A \circ B = B \circ A$

Proof: Proof is immediate.

Proposition 3.4: Let $A \in FNSG(G)$. Then $A(x^{-1}) = A(x)$,

(i.e.,) $T_A(x^{-1}) = T_A(x), I_A(x^{-1}) = I_A(x), F_A(x^{-1}) = F_A(x)$ and $A(x) \leq A(e)$,

(i.e.,) $T_A(x) \leq T_A(e), I_A(x) \leq I_A(e), F_A(x) \geq F_A(e)$ for each $x \in G$, where e is the identity element of G .

Proof:

Let $x \in G$. Then $T_A(x) = T_A((x^{-1})^{-1}) \geq T_A(x^{-1})$, for each $x \in G$.

$I_A(x) = I_A((x^{-1})^{-1}) \geq I_A(x^{-1})$, for each $x \in G$.

$F_A(x) = F_A((x^{-1})^{-1}) \leq F_A(x^{-1})$, for each $x \in G$.

Since $A \in FNSG(G), T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x)$ and $F_A(x^{-1}) \leq F_A(x)$ for each $x \in G$.

Hence $T_A(x^{-1}) = T_A(x), I_A(x^{-1}) = I_A(x), F_A(x^{-1}) = F_A(x)$. (i.e.,) $A(x^{-1}) = A(x)$.

On the other hand ,

$T_A(e) = T_A(xx^{-1}) \geq T_A(x) \wedge T_A(x^{-1}) = T_A(x), I_A(e) = I_A(xx^{-1}) \geq I_A(x) \wedge I_A(x^{-1}) = I_A(x),$

$F_A(e) = F_A(xx^{-1}) \leq F_A(x) \vee F_A(x^{-1}) = F_A(x)$

Hence $T_A(x) \leq T_A(e), I_A(x) \leq I_A(e), F_A(x) \geq F_A(e)$ for each $x \in G$. (i.e.,) $A(x) \leq A(e)$.

This completes the proof.

Proposition 3.5: If $A \in FNSG(G)$, then

$G_A = \{x \in G : A(x) = A(e), i.e., T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)\}$ is a subgroup of G .

Proof:

Let $x, y \in G_A$. Then $T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)$ and

$T_A(y) = T_A(e), I_A(y) = I_A(e), F_A(y) = F_A(e)$.

Thus $T_A(xy^{-1}) \geq T_A(x) \wedge T_A(y^{-1})$

$= T_A(x) \wedge T_A(y)$ (by Proposition 3.4)

$= T_A(e) \wedge T_A(e) = T_A(e)$.

Similarly , $I_A(xy^{-1}) \geq I_A(e)$.

$F_A(xy^{-1}) \leq F_A(x) \vee F_A(y^{-1})$

$= F_A(x) \vee F_A(y)$ (by Proposition 3.4)

$= F_A(e) \vee F_A(e) = F_A(e)$.

On the other hand , by proposition 3.4 $T_A(xy^{-1}) \leq T_A(e), I_A(xy^{-1}) \leq I_A(e), F_A(xy^{-1}) \geq F_A(e)$

.So $T_A(xy^{-1}) = T_A(e), I_A(xy^{-1}) = I_A(e), F_A(xy^{-1}) = F_A(e)$. (i.e.,) $A(xy^{-1}) = A(e)$. Thus

$xy^{-1} \in G_A$. Hence G_A is a subgroup of G .

Proposition 3.6:

Let $A \in FNSG(G)$. If $A(xy^{-1}) = A(e)$. (i.e.,) $T_A(xy^{-1}) = T_A(e), I_A(xy^{-1}) = I_A(e),$

$F_A(xy^{-1}) = F_A(e)$ for any $x, y \in G$, then $A(x) = A(y)$, (i.e.,)

$T_A(x) = T_A(y), I_A(x) = I_A(y), F_A(x) = F_A(y)$.

Proof:

Let $x, y \in G$. Then $T_A(x) = T_A((xy^{-1})y) \geq T_A(xy^{-1}) \wedge T_A(y) = T_A(e) \wedge T_A(y) = T_A(y)$.

On the other hand , by Proposition 3.4 $T_A(x^{-1}) = T_A(x)$, then we have

$$T_A(xy^{-1}) = T_A((yx^{-1})^{-1}) = T_A(yx^{-1}) \text{ and thus}$$

$$T_A(y) = T_A((yx^{-1})x) \geq T_A(yx^{-1}) \wedge T_A(x) = T_A(xy^{-1}) \wedge T_A(x) = T_A(e) \wedge T_A(x) = T_A(x).$$

So $T_A(x) = T_A(y)$. By the similar arguments, we have $I_A(x) = I_A(y), F_A(x) = F_A(y)$.

This completes the proof.

Corollary 3.7 :

Let $A \in FNSG(G)$.If G_A is a normal subgroup of G , then A is constant on each coset of G_A .

Proof:

Let $a \in G$ and let $x \in aG_A$. Then there exists $x' \in G_A$ such that $x = ax'$. Since G_A is normal and $x' \in G_A, xa^{-1} = ax'a^{-1} \in G_A$. Thus $T_A(xa^{-1}) = T_A(e), I_A(xa^{-1}) = I_A(e)$ and $F_A(xa^{-1}) = F_A(e)$.By Proposition 3.6 , $T_A(x) = T_A(a), I_A(x) = I_A(a)$ and $F_A(x) = F_A(a)$. So A is constant on aG_A for each $a \in G$. By the similar arguments , we can see that A is constant on $G_A a$ for each $a \in G$. Hence A is constant on each coset of G_A .

Note:

Let H be a subgroup of G .Then the number of right [respectively left] cosets of H in G is called index of H in G and denoted by $[G : H]$. If G is a finite group ,then there can be only a finite number of distinct right [respectively left] cosets of H .Hence the index $[G : H]$ is finite .If G is an infinite group ,then $[G : H]$ may be either finite or infinite.

Corollary 3.8 :

Let $A \in FNSG(G)$ and let G_A be normal .If G_A has a finite index, then A has the sup -property.

Proof:

Let $T \subset G$.Since G_A has finite index ,let the index $[G : G_A] = n$, say

$$A = \{a_1G_A, \dots, a_nG_A\}, \text{ where } a_i \in G (i=1, \dots, n) \text{ and } a_iG_A \cap a_jG_A = \emptyset \text{ for any } i \neq j. \text{ Let } t \in T. \text{ Since}$$

$$G = \bigcup_{i=1}^n a_iG_i, \text{ there exists } i \in \{1, 2, \dots, n\} \text{ such that } t \in a_iG_A. \text{ Since } G_A \text{ is normal, by corollary}$$

3.7, $T_A(t) = T_A(a_i), I_A(t) = I_A(a_i), F_A(t) = F_A(a_i)$ on a_iG_A , say $T_A(t) = \alpha_i, I_A(t) = \beta_i, F_A(t) = \gamma_i$,where $\alpha_i, \beta_i, \gamma_i \in I$ and $\alpha_i + \beta_i + \gamma_i \leq 3$. Thus there exists a $t_0 \in T$ such that

$$T_A(t_0) = \bigvee_{t \in T} T_A(t) = \bigvee_{i=1}^n \alpha_i, I_A(t_0) = \bigvee_{t \in T} I_A(t) = \bigvee_{i=1}^n \beta_i, F_A(t_0) = \bigwedge_{t \in T} F_A(t) = \bigwedge_{i=1}^n \gamma_i. \text{ Hence } A \text{ has the}$$

sup- property.

Proposition 3.9 :

$A \in FNSG(G)$ if and only if $T_A(xy^{-1}) \geq T_A(x) \wedge T_A(y), I_A(xy^{-1}) \geq I_A(x) \wedge I_A(y),$

$$F_A(xy^{-1}) \leq F_A(x) \vee F_A(y) \text{ for any } x, y \in G.$$

Proof:

Proof follows from Definition 3.1 and Proposition 3.4.

Proposition 3.10 :

A group G cannot be the union of two proper $FNSGs$.

Proof:

Let A and B be proper $FNSG$ s of a group G such that $A \cup B = 1_N, A \neq 1_N$ and $B \neq 1_N$.

$A \cup B = 1_N \Rightarrow T_A \vee T_B = 1, I_A \vee I_B = 1, F_A \wedge F_B = 0$. Then $T_A = 1$ or $T_B = 1, I_A = 1$ or $I_B = 1, F_A = 0$ or $F_B = 0$. Since $A \neq 1_N$ and $B \neq 1_N, T_A \neq 1$ or $I_A \neq 1$ or $F_A \neq 0$ and $T_B \neq 1$ or $I_B \neq 1$ or $F_B \neq 0$. In either cases, this is a contradiction. This completes the proof.

Proposition 3.11:

If A is a $FNSGP$ of a group G , then A is a $FNSG$ of G .

Proof:

Let $x \in G$. Since G is finite, x has the finite order say n . Then $x^n = e$, where e is the identity of G . Thus $x^{-1} = x^{n-1}$. Since A is a $FNSGP$ of a group $G, T_A(x^{-1}) = T_A(x^{n-1}) = T_A(x^{n-2}x) \geq T_A(x)$

$$I_A(x^{-1}) = I_A(x^{n-1}) = I_A(x^{n-2}x) \geq I_A(x)$$

$$F_A(x^{-1}) = F_A(x^{n-1}) = F_A(x^{n-2}x) \leq F_A(x)$$

Hence A is a $FNSG$ of G .

Proposition 3.12:

Let A be a $FNSG$ of a group G and let $x \in G$. Then $A(xy) = A(y)$, (i.e.,) $T_A(xy) = T_A(x), I_A(xy) = I_A(x), F_A(xy) = F_A(x)$ for each $y \in G$ if and only if $A(x) = A(e)$. (i.e.,) $T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)$, where e is the identity of G .

Proof:

Suppose $A(xy) = A(y)$ for each $y \in G$. Then clearly $A(x) = A(e)$.

Conversely, suppose $A(x) = A(e)$. Then by Proposition 3.4,

$T_A(y) \leq T_A(x), I_A(y) \leq I_A(x), F_A(y) \leq F_A(x)$ for each $y \in G$. Since A is a $FNSG$ of G , then $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y), F_A(xy) \leq F_A(x) \vee F_A(y)$. Thus

$T_A(xy) \geq T_A(y), I_A(xy) \geq I_A(y), F_A(xy) \leq F_A(y)$ for each $y \in G$.

On the other hand, by Proposition 3.4,

$$T_A(y) = T_A(x^{-1}xy) \geq T_A(x) \wedge T_A(xy), I_A(y) \geq I_A(x) \wedge I_A(xy), F_A(y) \leq F_A(x) \vee F_A(xy).$$

Since $T_A(x) \geq T_A(y), I_A(x) \geq I_A(y), F_A(x) \leq F_A(y)$ for each $y \in G$.

$$T_A(x) \wedge T_A(xy) = T_A(xy), I_A(x) \wedge I_A(xy) = I_A(xy), F_A(x) \vee F_A(xy) = F_A(xy). \text{ So}$$

$T_A(y) \geq T_A(xy), I_A(y) \geq I_A(xy), F_A(y) \leq F_A(xy)$ for each $y \in G$. Hence

$$T_A(xy) = T_A(y), I_A(xy) = I_A(y), F_A(xy) = F_A(y) \text{ for each } y \in G.$$

Proposition 3.13:

Let $f : G \rightarrow G'$ be a group homomorphism, let $A \in FNSG(G)$ and let $B \in FNSG(G')$. Then the following hold:

- (i) If A has the sup- property, then $f(A) \in FNG(G')$.
- (ii) $f^{-1}(B) \in FNSG(G)$.

Proof:

- (i) By Proposition 5.4 in [4], (i.e.,) Let $f : G \rightarrow G''$ be a groupoid homomorphism and let $A \in FNS(G)$ have the sup property.

(1) If $A \in FNSGP(G)$, then $f(A) \in FNSGP(G'')$.

(2) If A is a $FNI(FNLI, FNRI)$ of G , then $f(A)$ is a $FNI(FNLI, FNRI)$ of G'' .

Since $f(A) \in FNSGP(G)$, it is enough to show that

$$T_{f(A)}(y^{-1}) \geq T_{f(A)}(y), I_{f(A)}(y^{-1}) \geq I_{f(A)}(y), F_{f(A)}(y^{-1}) \leq F_{f(A)}(y) \text{ for each } y \in f(G).$$

Let $y \in f(G)$. Then $\phi \neq f^{-1}(y) \subset G$. Since A has the sup- property, there exists $x_0 \in f^{-1}(y)$

$$\text{such that } T_A(x_0) = \bigvee_{t \in f^{-1}(y)} T_A(t), I_A(x_0) = \bigvee_{t \in f^{-1}(y)} I_A(t), F_A(x_0) = \bigwedge_{t \in f^{-1}(y)} F_A(t).$$

$$\text{Thus } T_{f(A)}(y^{-1}) = f(T_A)(y^{-1}) = \bigvee_{t \in f^{-1}(y^{-1})} T_A(t) \geq T_A(x_0^{-1}) \geq T_A(x_0) = T_{f(A)}(y).$$

$$I_{f(A)}(y^{-1}) = f(I_A)(y^{-1}) = \bigvee_{t \in f^{-1}(y^{-1})} I_A(t) \geq I_A(x_0^{-1}) \geq I_A(x_0) = I_{f(A)}(y).$$

$$F_{f(A)}(y^{-1}) = f(F_A)(y^{-1}) = \bigwedge_{t \in f^{-1}(y^{-1})} F_A(t) \leq F_A(x_0^{-1}) \leq F_A(x_0) = F_{f(A)}(y).$$

Hence $f(A) \in FNSG(G)$.

(ii) By Proposition 5.1 in [],(i.e.,) Let $f : G \rightarrow G''$ be a groupoid homomorphism and let $B \in FNS(G'')$

(1) If $B \in FNSGP(G'')$, then $f^{-1}(B) \in FNSGP(G)$.

(2) If B is a $FNI(FNLI, FNRI)$ of G'' then $f^{-1}(B)$ is a $FNI(FNLI, FNRI)$ of G . Since $f^{-1}(B) \in FNSGP(G)$, it is enough to show that $f^{-1}(B)(x^{-1}) \geq f^{-1}(B)(x)$ for each $x \in G$.

Let $x \in G$. Then

$$T_{f^{-1}(B)}(x^{-1}) = f^{-1}(T_B)(x^{-1}) = T_B(f(x^{-1})) = T_B(((f(x))^{-1})) \geq T_B(f(x)) = T_{f^{-1}(B)}(x),$$

$$I_{f^{-1}(B)}(x^{-1}) = f^{-1}(I_B)(x^{-1}) = I_B(f(x^{-1})) = I_B(((f(x))^{-1})) \geq I_B(f(x)) = I_{f^{-1}(B)}(x), \text{ and}$$

$$F_{f^{-1}(B)}(x^{-1}) = f^{-1}(F_B)(x^{-1}) = F_B(f(x^{-1})) = F_B(((f(x))^{-1})) \leq F_B(f(x)) = F_{f^{-1}(B)}(x).$$

Hence $f^{-1}(B) \in FNSG(G)$.

Proposition 3.14:

Let G_p be the cyclic group of prime order p . Then $A \in FNSG(G_p)$ if and only if $A(x) = A(1) \leq A(0)$, (i.e.,) $T_A(x) = T_A(1) \leq T_A(0)$, $I_A(x) = I_A(1) \leq I_A(0)$, $F_A(x) = F_A(1) \geq F_A(0)$ for each $0 \neq x \in G_p$.

Proof:

Suppose $A \in FNSG(G_p)$ and let $0 \neq x \in G_p$. Then

$$T_A(xy) \geq T_A(x) \wedge T_A(xy), I_A(xy) \geq I_A(x) \wedge I_A(xy), F_A(xy) \leq F_A(x) \vee F_A(xy) \text{ for any } x, y \in G_p$$

. Since G_p is the cyclic group of prime order p , $G_p = \{0, 1, 2, \dots, p-1\}$. Since x is the sum of 1's and

1 is the sum of x 's, $T_A(x) \geq T_A(1) \geq T_A(x)$, $I_A(x) \geq I_A(1) \geq I_A(x)$ and $F_A(x) \leq I_A(1) \leq I_A(x)$

. Thus $T_A(x) = T_A(1)$, $I_A(x) = I_A(1)$, $F_A(x) = F_A(1)$. Since 0 is the identity element of G_p ,

$$T_A(x) \leq T_A(0), I_A(x) \leq I_A(0), F_A(x) \geq F_A(0). \text{ Hence the necessary conditions hold.}$$

Conversely, suppose the necessary conditions hold, and let $x, y \in G_p$. Then we have four cases : (i)

$x \neq 0, y \neq 0$ and $x = y$ (ii) $x \neq 0, y = 0$ (iii) $x = 0, y \neq 0$ (iv) $x \neq 0, y \neq 0$ and $x \neq y$.

Case (i) Suppose $x \neq 0, y \neq 0$ and $x = y$. Then by the hypothesis, $T_A(x) = T_A(y) = T_A(1) \leq T_A(0)$,

$I_A(x) = I_A(y) = I_A(1) \leq I_A(0)$ and $F_A(x) = F_A(y) = F_A(1) \geq F_A(0)$. So

$$T_A(x - y) = T_A(0) \geq T_A(x) \wedge T_A(y), I_A(x - y) = I_A(0) \geq I_A(x) \wedge I_A(y) ,$$

$$F_A(x - y) = F_A(0) \leq F_A(x) \vee F_A(y).$$

Case (ii) Suppose $x \neq 0$ and $y = 0$. Since $x - y \neq 0$, by the hypothesis,

$$T_A(x - y) = T_A(x) = T_A(1) \leq T_A(0) = T_A(y) , I_A(x - y) = I_A(x) = I_A(1) \leq I_A(0) = I_A(y) \text{ and}$$

$$F_A(x - y) = F_A(x) = F_A(1) \geq F_A(0) = F_A(y) .\text{So}$$

$$T_A(x - y) \geq T_A(x) \wedge T_A(y), I_A(x - y) \geq I_A(x) \wedge I_A(y), F_A(x - y) \leq F_A(x) \vee F_A(y) .$$

Case (iii) is the same as Case (ii).

Case (iv) Suppose $x \neq 0, y \neq 0$ and $x \neq y$. Since $x - y \neq 0$, by the hypothesis,

$$T_A(x - y) = T_A(x) = T_A(y) = T_A(1) \leq T_A(0), I_A(x - y) = I_A(x) = I_A(y) = I_A(1) \leq I_A(0) ,$$

$$F_A(x - y) = F_A(x) = F_A(y) = F_A(1) \geq F_A(0) .\text{So}$$

$$T_A(x - y) \geq T_A(x) \wedge T_A(y), I_A(x - y) \geq I_A(x) \wedge I_A(y), F_A(x - y) \leq F_A(x) \vee F_A(y) .\text{In all,}$$

$$T_A(x - y) \geq T_A(x) \wedge T_A(y), I_A(x - y) \geq I_A(x) \wedge I_A(y), F_A(x - y) \leq F_A(x) \vee F_A(y) .\text{Hence by}$$

Proposition 3.5, $A \in FNSG(G_p)$.

Proposition 3.15:

The *FNI* (*FNLI*, *FNRI*) in a group G are just the constant mappings.

Proof:

Suppose A is a constant mapping and let $x, y \in G$. Then $T_A(xy) = T_A(x) = T_A(y) ,$

$$I_A(xy) = I_A(x) = I_A(y), F_A(xy) = F_A(x) = F_A(y) .\text{So } A \text{ is a } FNI \text{ of } G .$$

Now suppose A is a *FNLI* of G . Then $T_A(xy) \geq T_A(y), I_A(xy) \geq I_A(y), F_A(xy) \leq F_A(y)$ for any $x, y \in G$. In particular, $T_A(x) \geq T_A(e), I_A(x) \geq I_A(e), F_A(x) \leq F_A(e)$ for each $x \in G$.

Moreover, $T_A(e) = T_A(x^{-1}x) \geq T_A(x), I_A(e) = I_A(x^{-1}x) \geq I_A(x), F_A(e) = F_A(x^{-1}x) \leq F_A(x)$

For each $x \in G$. So $T_A(x) = T_A(e), I_A(x) = I_A(e), F_A(x) = F_A(e)$ for each $x \in G$. Hence A is a constant mapping.

Proposition 3.16:

Let A be a *FNSG* of a group G . Then for each $(\lambda, \mu, \nu) \in I^3$ with $(\lambda, \mu, \nu) \leq A(e)$,

(i.e.,) $\lambda \leq T_A(e), \mu \leq I_A(e), \nu \geq F_A(e)$, $G_A^{(\lambda, \mu, \nu)}$ is a subgroup of G , where e is the identity of G .

Proof:

Clearly, $G_A^{(\lambda, \mu, \nu)} \neq \phi$. Let $x, y \in G_A^{(\lambda, \mu, \nu)}$. Then $A(x) \geq (\lambda, \mu, \nu)$ and $A(y) \geq (\lambda, \mu, \nu)$.(i.e.,)

$T_A(x) \geq \lambda, I_A(x) \geq \mu, F_A(x) \leq \nu$ and $T_A(y) \geq \lambda, I_A(y) \geq \mu, F_A(y) \leq \nu$. Since $A \in FNSG(G)$,

$T_A(xy) \geq T_A(x) \wedge T_A(y) \geq \lambda, I_A(xy) \geq I_A(x) \wedge I_A(y) \geq \mu, F_A(xy) \leq F_A(x) \vee F_A(y) \leq \nu$. Thus

$A(xy) \geq (\lambda, \mu, \nu)$. So $xy \in G_A^{(\lambda, \mu, \nu)}$. On the other hand ,

$T_A(x^{-1}) \geq T_A(x) \geq \lambda, I_A(x^{-1}) \geq I_A(x) \geq \mu, F_A(x^{-1}) \leq F_A(x) \leq \nu$. Thus $A(x^{-1}) \geq (\lambda, \mu, \nu)$. So

$x^{-1} \in G_A^{(\lambda, \mu, \nu)}$. Hence $G_A^{(\lambda, \mu, \nu)}$ is a subgroup of G .

Proposition 3.17:

Let A be a fuzzy neutrosophic set in a group G such that $G_A^{(\lambda, \mu, \nu)}$ is a subgroup of G for each

$(\lambda, \mu, \nu) \in I^3$ with $(\lambda, \mu, \nu) \leq A(e)$. Then A is a *FNSG* of a group G .

Proof:

For any $x, y \in G$, let $A(x) = (t_1, s_1, r_1)$ and let $A(y) = (t_2, s_2, r_2)$. Then clearly, $x \in G_A^{(t_1, s_1, r_1)}$ and $y \in G_A^{(t_2, s_2, r_2)}$. Suppose $t_1 < t_2$, $s_1 < s_2$ and $r_1 > r_2$. Then $G_A^{(t_2, s_2, r_2)} \subset G_A^{(t_1, s_1, r_1)}$. Thus $y \in G_A^{(t_1, s_1, r_1)}$. Since $G_A^{(t_1, s_1, r_1)}$ is a subgroup of G , $xy \in G_A^{(t_1, s_1, r_1)}$. Then $A(xy) = (t_1, s_1, r_1)$.

(i.e.,) $T_A(xy) \geq t_1, I_A(xy) \geq s_1, F_A(xy) \leq r_1$.

So $T_A(xy) \geq T_A(x) \wedge T_A(y), I_A(xy) \geq I_A(x) \wedge I_A(y), F_A(xy) \leq F_A(x) \vee F_A(y)$. For each $x \in G$, let $A(x) = (\lambda, \mu, \nu)$. Then $x \in G_A^{(\lambda, \mu, \nu)}$. Since $G_A^{(\lambda, \mu, \nu)}$ is a subgroup of G , $x^{-1} \in G_A^{(\lambda, \mu, \nu)}$. So $A(x^{-1}) \geq (\lambda, \mu, \nu)$. (i.e.,) $T_A(x^{-1}) \geq T_A(x), I_A(x^{-1}) \geq I_A(x), F_A(x^{-1}) \leq F_A(x)$. Hence A is a *FNSG* of a group G .

Proposition 3.18:

Let A be a fuzzy neutrosophic set in X and let $(\lambda_1, \mu_1, \nu_1), (\lambda_2, \mu_2, \nu_2) \in \text{Im}(A)$. If

$$\lambda_1 < \lambda_2, \mu_1 < \mu_2, \nu_1 > \nu_2 \text{ then } A^{(\lambda_1, \mu_1, \nu_1)} \supset A^{(\lambda_2, \mu_2, \nu_2)}.$$

Result 3.19: Let A be a fuzzy neutrosophic set in a group G . Then A is a *FNSG* of G if and only if $A^{(\lambda, \mu, \nu)}$ is a subgroup of G for each $(\lambda, \mu, \nu) \in \text{Im}(A)$.

Definition 3.20:

Let A be a *FNSG* of group G and let $(\lambda, \mu, \nu) \in \text{Im}(A)$. Then the subgroup $A^{(\lambda, \mu, \nu)}$ is called a (λ, μ, ν) – level subgroup of A .

Lemma 3.21:

Let A be any fuzzy neutrosophic set in X . Then $T_A(x) = \vee \{ \lambda : x \in A^{(\lambda, \mu, \nu)} \}$, $I_A(x) = \vee \{ \mu : x \in A^{(\lambda, \mu, \nu)} \}$, $F_A(x) = \wedge \{ \nu : x \in A^{(\lambda, \mu, \nu)} \}$ where $x \in X$ and $(\lambda, \mu, \nu) \in I^3$ with $\lambda + \mu + \nu \leq 3$.

Proof: Let $\alpha = \vee \{ \lambda : x \in A^{(\lambda, \mu, \nu)} \}$, $\beta = \vee \{ \mu : x \in A^{(\lambda, \mu, \nu)} \}$, $\gamma = \wedge \{ \nu : x \in A^{(\lambda, \mu, \nu)} \}$ and let $\varepsilon > 0$ be arbitrary. Then $\alpha - \varepsilon < \vee \{ \lambda : x \in A^{(\lambda, \mu, \nu)} \}$, $\beta - \varepsilon < \vee \{ \mu : x \in A^{(\lambda, \mu, \nu)} \}$, $\gamma + \varepsilon > \wedge \{ \nu : x \in A^{(\lambda, \mu, \nu)} \}$

. Thus there exist $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$ such that $x \in A^{(\lambda, \mu, \nu)}$,

$\alpha - \varepsilon < \lambda, \beta - \varepsilon < \mu, \gamma + \varepsilon > \nu$. Since $x \in A^{(\lambda, \mu, \nu)}$, $T_A(x) \geq \lambda, I_A(x) \geq \mu, F_A(x) \leq \nu$. Thus

$T_A(x) > \alpha - \varepsilon, I_A(x) > \beta - \varepsilon, F_A(x) < \gamma + \varepsilon$. Since $\varepsilon > 0$ is arbitrary,

$T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma$.

We now show that $T_A(x) \leq \alpha, I_A(x) \leq \beta, F_A(x) \geq \gamma$. Suppose $T_A(x) = t_1, I_A(x) = t_2, F_A(x) = t_3$

. Then $t_1 + t_2 + t_3 \leq 3$. Thus $x \in A^{(t_1, t_2, t_3)}$. So $t_1 \in \{ \lambda : x \in A^{(\lambda, \mu, \nu)} \}$, $t_2 \in \{ \mu : x \in A^{(\lambda, \mu, \nu)} \}$,

$t_3 \in \{ \nu : x \in A^{(\lambda, \mu, \nu)} \}$. Thus, $t_1 \leq \vee \{ \lambda : x \in A^{(\lambda, \mu, \nu)} \}$, $t_2 \leq \vee \{ \mu : x \in A^{(\lambda, \mu, \nu)} \}$, $t_3 \geq \wedge \{ \nu : x \in A^{(\lambda, \mu, \nu)} \}$

. (i.e.,) $T_A(x) \leq \alpha, I_A(x) \leq \beta, F_A(x) \geq \gamma$.

This completes the proof.

We shall denote by $\langle A \rangle$ the *FNSG* generated by the fuzzy neutrosophic set A in G . We shall use the same notation $\langle A^{(\lambda, \mu, \nu)} \rangle$ for the ordinary subgroup of the group generated by the level subset $A^{(\lambda, \mu, \nu)}$.

Theorem 3.22:

Let G be a group and let $A \in \text{FNS}(G)$. Let $A^* \in \text{FNS}(G)$ be defined as follows :for each $x \in G, T_{A^*}(x) = \vee \{ \lambda : x \in A^{(\lambda, \mu, \nu)} \}$, $I_{A^*}(x) = \vee \{ \mu : x \in A^{(\lambda, \mu, \nu)} \}$, $F_{A^*}(x) = \wedge \{ \nu : x \in A^{(\lambda, \mu, \nu)} \}$, where

$\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$. Then A^* is a *FNSG* of G such that $A^* = \bigcap \{B \in \text{FNSG}(G) : A \subset B\}$. In this case, A^* is called the fuzzy neutrosophic subgroup generated by A in G and denoted by (A) .

Proof:

Let $(t_1, t_2, t_3) \in \text{Im}(A^*)$ and let $\alpha = t_1 - \frac{1}{n}, \beta = t_2 - \frac{1}{n}, \gamma = t_3 + \frac{1}{n}$ where n is any sufficiently large positive number. Let $x \in G$. Suppose $x \in A^{*(t_1, t_2, t_3)}$. Then $T_{A^*}(x) \geq t_1, I_{A^*}(x) \geq t_2, F_{A^*}(x) \leq t_3$.

Thus there exist $\lambda, \mu, \nu \in I$ with $\lambda + \mu + \nu \leq 3$ such that $\lambda > \alpha, \mu > \beta, \nu < \gamma$ and $x \in A^{(\lambda, \mu, \nu)}$.

$(\alpha, \beta, \gamma) < (\lambda, \mu, \nu)$ and $\alpha + \beta + \gamma \leq 3, A^{(\lambda, \mu, \nu)} \subset A^{(\alpha, \beta, \gamma)}$. So $x \in A^{(\alpha, \beta, \gamma)}$. (i.e.,) $x \in (A^{(\alpha, \beta, \gamma)})$

Now suppose $x \in (A^{(\alpha, \beta, \gamma)})$. Then

$\alpha \in \{\lambda : x \in (A^{(\lambda, \mu, \nu)})\}, \beta \in \{\mu : x \in (A^{(\lambda, \mu, \nu)})\}, \gamma \in \{\nu : x \in (A^{(\lambda, \mu, \nu)})\}$. Thus $\alpha \leq \bigvee \{\lambda : x \in (A^{(\lambda, \mu, \nu)})\},$

$\beta \leq \bigvee \{\mu : x \in (A^{(\lambda, \mu, \nu)})\}, \gamma \geq \bigwedge \{\nu : x \in (A^{(\lambda, \mu, \nu)})\}$. So

$$t_1 - \frac{1}{n} \leq T_{A^*}(x), t_2 - \frac{1}{n} \leq I_{A^*}(x), t_3 + \frac{1}{n} \geq F_{A^*}(x).$$

(i.e.,) $t_1 \leq T_{A^*}(x), t_2 \leq I_{A^*}(x), t_3 \geq F_{A^*}(x)$.

Hence $x \in A^{*(t_1, t_2, t_3)}$. (i.e.,) $(A^{(\alpha, \beta, \gamma)}) \subset A^{*(t_1, t_2, t_3)}$.

Hence $A^{*(t_1, t_2, t_3)} = (A^{(\alpha, \beta, \gamma)})$. Since $(A^{(\alpha, \beta, \gamma)})$ is a subgroup of $G, A^{*(t_1, t_2, t_3)}$ is a subgroup of G . By Result 3.19, A^* is a *FNSG* of G .

Now, we show that $A \subset A^*$. Let $x \in G$. Then by Lemma 3.21, $T_A(x) = \bigvee \{\lambda : x \in (A^{(\lambda, \mu, \nu)})\},$

$I_A(x) = \bigvee \{\mu : x \in (A^{(\lambda, \mu, \nu)})\}, F_A(x) = \bigwedge \{\nu : x \in (A^{(\lambda, \mu, \nu)})\}$. Thus

$T_A(x) \leq \bigvee \{\lambda : x \in (A^{(\lambda, \mu, \nu)})\}, I_A(x) \leq \bigvee \{\mu : x \in (A^{(\lambda, \mu, \nu)})\}, F_A(x) \geq \bigwedge \{\nu : x \in (A^{(\lambda, \mu, \nu)})\}$. So $A \subset A^*$.

Finally, let B be any *FNSG* of G such that $A \subset B$. We show that $A^* \subset B$. Let $x \in G$ and

$A^*(x) = (t_1, t_2, t_3)$. Then $A^{*(t_1, t_2, t_3)} = (A^{(\alpha, \beta, \gamma)})$, where $\alpha = t_1 - \frac{1}{n}, \beta = t_2 - \frac{1}{n}, \gamma = t_3 + \frac{1}{n}$, and n is

any sufficiently large positive integer. Thus $x \in (A^{(\alpha, \beta, \gamma)})$. So $x = a_1 a_2 \dots a_m$, where a_i or a_i^{-1} belongs to $A^{(\alpha, \beta, \gamma)}$ ($i = 1, \dots, m$).

On the other hand, $T_B(x) = T_B(a_1 a_2 \dots a_m)$

$$\geq T_B(a_1) \wedge T_B(a_2) \wedge T_B(a_3) \dots \wedge T_B(a_m)$$

$$\geq T_A(a_1) \wedge T_A(a_2) \wedge \dots \wedge T_A(a_m) \geq \alpha = t_1 - \frac{1}{n}.$$

Similarly $I_B(x) = I_B(a_1 a_2 \dots a_m)$

$$\geq I_B(a_1) \wedge I_B(a_2) \wedge I_B(a_3) \dots \wedge I_B(a_m)$$

$$\geq I_A(a_1) \wedge I_A(a_2) \wedge \dots \wedge I_A(a_m) \geq \beta = t_2 - \frac{1}{n} \text{ and}$$

$$F_B(x) = F_B(a_1 a_2 \dots a_m)$$

$$\leq F_B(a_1) \vee F_B(a_2) \vee F_B(a_3) \dots \vee F_B(a_m)$$

$$\leq F_A(a_1) \vee F_A(a_2) \vee \dots \vee F_A(a_m) \leq \gamma = t_3 + \frac{1}{n}.$$

Since n is sufficiently large positive integer, $T_B(x) \geq t_1, I_B(x) \geq t_2, F_B(x) \leq t_3$. So $A^* \subset B$. Hence $A^* = \bigcap \{B \in FNSG(G) : A \subset B\}$. This completes the proof.

Lemma 3.23:

Let G be a finite group. Suppose there exists a $FNSG$ A of G satisfying the following conditions: for any $x, y \in G$,

(i) $A(x) = A(y) \Rightarrow (x) = (y)$

(ii) $T_A(x) > T_A(y), I_A(x) > I_A(y), F_A(x) < F_A(y) \Rightarrow (x) \subset (y)$. Then G is cyclic.

Proof:

Suppose A is constant on G . Then $A(x) = A(y)$ for any $x, y \in G$. By the condition (i), $(x) = (y)$. So $G = (x)$. Now suppose A is not constant on G . Let

$$\text{Im}(A) = \{(t_0, s_0, r_0), (t_1, s_1, r_1), \dots, (t_n, s_n, r_n)\}, \text{ where}$$

$$t_0 > t_1 > \dots > t_n, s_0 > s_1 > \dots > s_n, r_0 < r_1 < \dots < r_n. \text{ Then by Proposition 3.18 and Result 3.19, we}$$

obtain the chain of level subgroups of $A : A^{(t_0, s_0, r_0)} \subset A^{(t_1, s_1, r_1)} \subset \dots \subset A^{(t_n, s_n, r_n)} = G$.

Let $x \in G - A^{(t_{n-1}, s_{n-1}, r_{n-1})}$. We show that $G = (x)$. Let $g \in G - A^{(t_{n-1}, s_{n-1}, r_{n-1})}$. Since

$$t_0 > t_1 > \dots > t_n, s_0 > s_1 > \dots > s_n, r_0 < r_1 < \dots < r_n, A(g) = A(x) = A^{(t_{n-1}, s_{n-1}, r_{n-1})}. \text{ By the condition}$$

(i), $(g) = (x)$. Thus $G - A^{(t_{n-1}, s_{n-1}, r_{n-1})} \subset (x)$. Now let $g \in A^{(t_{n-1}, s_{n-1}, r_{n-1})}$. Then

$$T_A(g) \geq t_{n-1} > t_n = T_A(x), I_A(g) \geq s_{n-1} > s_n = I_A(x), F_A(g) \leq r_{n-1} < r_n = F_A(x). \text{ By the condition (ii),}$$

$$(g) \subset (x). \text{ Thus } A^{(t_{n-1}, s_{n-1}, r_{n-1})} \subset (x). \text{ So } G = (x). \text{ Hence in either case, } G \text{ is cyclic.}$$

Lemma 3.24:

Let G be a cyclic group of order p^n , where p is prime. Then there exists a $FNSG$ A of G satisfying the following conditions: for any $x, y \in G$,

(i) $A(x) = A(y) \Rightarrow (x) = (y)$

(ii) $T_A(x) > T_A(y), I_A(x) > I_A(y), F_A(x) < F_A(y) \Rightarrow (x) \subset (y)$.

Proof:

Consider the following chain of subgroups of G :

$$(e) = G_0 \subset G_1 \subset \dots \subset G_{n-1} \subset G_n = G, \text{ where } G_i \text{ is the subgroup of } G \text{ generated by an element of}$$

order $p^i, i = 0, 1, \dots, n$ and e is the identity of G . We define a complex mapping

$$A = (T_A, I_A, F_A) : G \rightarrow I^3 \text{ as follows: for each } x \in G, A(e) = (t_0, s_0, r_0) \text{ and}$$

$$A(x) = (t_i, s_i, r_i) \text{ if } x \in G_i - G_{i-1} \text{ for any } i = 1, 2, \dots, n, \text{ where } t_i, s_i, r_i \in I \text{ such that } t_i + s_i + r_i \leq 3,$$

$t_0 > t_1 > \dots > t_n, s_0 > s_1 > \dots > s_n, r_0 < r_1 < \dots < r_n$. Then we can easily check that A is a $FNSG$ of G satisfying the conditions (i) and (ii).

From Lemma 3.23 and Lemma 3.24 we obtain the following :

Theorem 3.25:

Let G be a group of order p^n . Then G is cyclic if and only if there exists a $FNSG$ A of G such that for any $x, y \in G$,

(i) $A(x) = A(y) \Rightarrow (x) = (y)$

(ii) $T_A(x) > T_A(y), I_A(x) > I_A(y), F_A(x) < F_A(y) \Rightarrow (x) \subset (y)$.

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