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RESEARCH ARTICLE

ON α REGULAR ω–LOCALLY CLOSED SETS İN TOPOLOGİCAL SPACES

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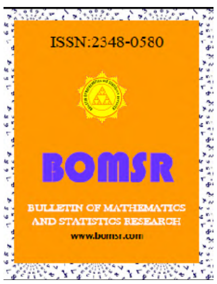


ABSTRACT

In this paper, we introduce three weaker forms of locally closed sets called αrω-LC sets, αrω-LC* set and αrω-LC** sets each of which is weaker than locally closed set and study some of their properties in topologivcal spaces.

Keywords:– αrω-closed sets, αrω-open sets, locally closed sets, αrω-locally closed sets.

2000 AMS Mathematics Subject Classification: 54A05, 54G65

1 . INTRODUCTION

Kuratowski and Sierpinski [2] introduced the notion of locally closed sets in topological spaces. According to Bourbaki [10], a subset of a topological space (X, τ) is locally closed in (X, τ) if it is the intersection of an open set and a closed set in (X, τ). Stone[9] has used the term FG for locally closed set. Ganster and Reilly [7] have introduced locally closed sets, which are weaker forms of both closed and open sets. After that Balachandran et al [6], Gnanambal [15], Arockiarani et al [5], Pusphalatha [1] and Sheik John[8] have introduced α-locally closed, generalized locally closed, semi locally closed, semi generalized locally closed, regular generalized locally closed, strongly locally closed and w-locally closed sets and their continuous maps in topological space respectively. Recently as a generalization of closed sets αrω-closed sets and continuous maps were introduced and studied by R. S. Wali et al [11].

2.Preliminaries: Throughout the paper (X, τ), (Y, σ) and (Z, η)(or simply X,Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), Cl(A), Int(A), αCl(A) and A^c denote the closure of A, the interior of A , the α-closure of A and the compliment of A in X respectively.

We recall the following definitions, which are useful in the sequel.

Definition 2.1 : A subset A of topological space (X, τ) is called a

1. locally closed (briefly LC or lc) set [7] if A=U∩F,where U is open and F is closed in X.
2. rw–closed set [13] if Cl(A) ⊆ U whenever A ⊆ U and U is regular semi-open.
3. αrω–closed set [11] if αCl(A) ⊆ U whenever A ⊆ U and U is αrω-open.
4. αg–locally closed set if A=U∩F, where U is αg–open and F is αg–closed in X.
5. α–locally closed set if A=U∩F, where U is α–open and F is α–closed in X.
6. wg–locally closed set if A=U∩F, where U is wg–open and F wg–closed in X.
7. gp–locally closed set if A=U∩F where U is gp–open and F is gp–closed in X.
8. gpr–locally closed set if A=U∩F where U is gpr–open and F gpr-closed in X.
9. g–locally closed set if A=U∩F where U is g-open and F is g-closed in X.
10. rwg–locally closed set if A=U∩F where U is rwg-open and F is rwg-closed in X.
11. gspr–locally closed set if A=U∩F where U is gspr-open and F is gspr-closed in X.
12. ωα–locally closed set if A=U∩F where U is ωα-open and F is ωα-closed in X.
13. αgr–locally closed set if A=U∩F where U is αgr-open and F αgr-closed in X.
14. gs- locally closed set if A=U∩F where U is gs-open and F is gs-closed in X.
15. w-lc set if A=U∩F where U is w-open and F is w-closed in X.
16. gprw-lc set if A=U∩F where U is gprw-open and F is gprw-closed in X.
17. rw-lc set if A=U∩F where U is rw -open and F is rw -closed in X.
18. rgα-lc set if A=U∩F where U is rgα-open and F is rgα-closed in X

Definition 2.2: $T_{\alpha r\omega}$ space [32] if every αrω-closed set is closed.

Lemma 2.3 [11] :

1) Every closed (resp regular-closed, α-closed) set is αrω-closed set in X.
2) Every αrω-closed set is αg-closed set.
3) Every αrω-closed set is αgr-closed (resp gs-closed, gspr-closed, wg-closed, rwg-closed , gp-closed, gpr-closed) set in X.

3. αrω-locally closed sets in topological spaces.

Definition 3.1: A Subset A of t.s (X, τ) is called αrω-locally closed (briefly αrω-LC) if A=U∩F where U is αrω-open in (X, τ) and F is αrω-closed in (X, τ) .

The set of all αrω-locally closed sets of (X, τ) is denoted by αrω-LC(X, τ) .

Example 3.2: Let X={a, b, c, d} and τ = {X, ϕ ,{a}, {c,d}, {a,c,d}}

1. αrω-C(X, τ) = { X, ϕ, {b}, {a, b}, {a, b, c}, {b, c, d}, {a, b, d} }
2. αrω-LC Set = {X , ϕ , {a}, {b}, {c} , {d}, {a, b} , {c, d}, {a, d} ,{a , c}, a, b, c}, {b, c, d}, {a, b, d}, {a , c, d} }

Remark 3.3: The following are well known

(i) A Subset A of (X, τ) is αrω-LC set iff it's complement X-A is the union of a αrω-open set and a αrω-closed set.

(ii) Every αrω-open (resp. αrω-closed) subset of (X, τ) is a αrω-LC set.

(iii) The Complement of a αrω-LC set need not be a αrω-LC set.

(In Example 3.2 the set {a,c} is αrω-LC set , but complement of {a,c} is {b, d} , which is not αrω-LC set.)

Theorem 3.4: Every locally closed set is a αrω-LC set but not conversely.

Proof: The proof follows from the two defintions [follows from Lemma 2.3] and fact that every closed (resp.open) set is αrω-closed (αrω-open).

Example 3.5: Let X={a,b,c} and τ ={X, ϕ ,{a}} then {a,b} is αrω-LC set but not a locally closed set in (X, τ) .

Theorem 3.6: Every α-locally closed set is a αrω-LC set but not conversely.

Proof: The proof follows from the two defintions [follows from Lemma 2.3] and fact that every α-closed (resp. α-open) set is αrω-closed (αrω-open).

Example 3.7: Let X={a, b, c, d} and τ = {X, ϕ ,{a}, {c,d}, {a,c,d}} then {a,c} is αrω-LC set but not a α-locally closed set in (X, τ) .

Theorem 3.8: Every r-locally closed set is a αrω-LC set but not conversely.

Proof: The proof follows from the two defintions [follows from Lemma 2.3] and fact that every r-closed (resp. r-open) set is αrω-closed (αrω-open).

Example 3.9: Let X={a,b,c} and τ ={X, ϕ ,{a}} then {c} is αrω-LC set but not a r-locally closed set in (X, τ) .

Theorem 3.10: The following holds

i) Every αrω–locally closed set is αg–locally closed set.
ii) Every αrω–locally closed set is wg–locally closed set (resp gs– locally closed set, rwg–locally closed set, gp–locally closed set, gspr–locally closed set, gpr–locally closed set, αgr–locally closed set).

Proof: (i) The proof follows from the definitions and fact that every αrω–closed (resp. αrω-open) set is αg-closed (αg-open) set.

(ii) Similarly we can prove (ii).

Remark 3.11: The converse of the above Theorem need not be true, as seen from the following example.

Example 3.12: Let X={a,b,c} and τ ={X, ϕ ,{a},{b,c}} then {a,b} is αg–locally closed set , wg–locally closed set , gs– locally closed set, rwg– locally closed set, gp–locally closed set, gspr– locally closed set, gpr– locally closed set, αgr– locally closed set but not a αrω-LC set in (X, τ) .

Definition 3.13: A subset A of (X, τ) is called a αrω-LC* set if there exists a αrω-open set G and a closed F of (X, τ) s.t A = G∩F the collection of all αrω-LC* sets of (X, τ) will be denoted by αrω-LC*(X, τ) .

Definition 3.14: A subset B of (X, τ) is called a αrω-LC** set if there exists an open set G and αrω-closed set F of (X, τ) s.t B = G∩F the collection of all αrω-LC** sets of (X, τ) will be denoted by αrω-LC**(X, τ) .

Theorem 3.15:

1. Every locally closed set is a αrω-LC* set.
2. Every locally closed set is a αrω-LC** set.
3. Every αrω-LC* set is αrω-LC set.
4. Every αrω-LC** set is αrω-LC set.

Proof: The proof are obivious from the definitions and the relation between the sets.

However the converses of the above results are not true as seen from the following examples.

Example 3.16: Let X={a,b,c} and τ ={X, ϕ ,{a}}

(i) The set {b} is αrω-LC* set but not a locally closed set in (X, τ) .
(ii) The set {b} is αrω-LC** set but not a locally closed set in (X, τ) .
(iii) The set {a,b} is αrω-LC set but not a αrω-LC* set in (X, τ) .

Example 3.17: Let X={a,b,c} and τ ={X, ϕ ,{a}} then the set {a,b} is αrω-LC set but not a αrω-LC** set in (X, τ) .

Remark 3.18: αrω-LC* sets and αrω-LC** sets are independent of each other as seen from the examples.

Example 3.19: **(i)** Let X={a,b,c,d} and τ {X, ϕ ,{a}, {c,d}, {a,c,d}} then set {a, d} is αrω-LC** set but not a αrω-LC* set in (X, τ) .

(ii) Let X={a,b,c} and τ ={X, ϕ ,{a}} then the set {a,b} is αrω-LC* set but not a αrω-LC** set in (X, τ) .

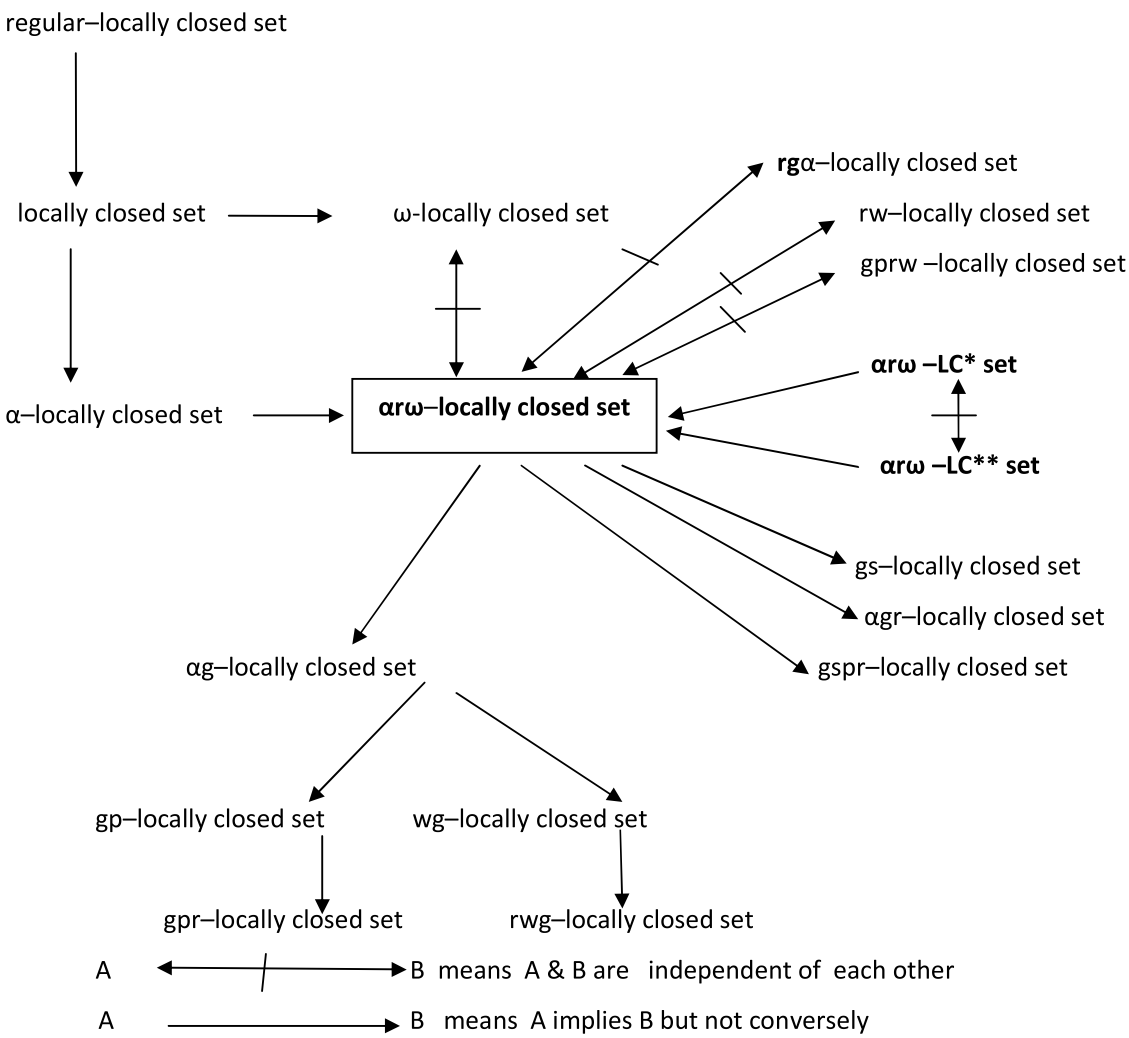
Remark 3.20 The following examples shows that αrω-locally closed sets are independent of ω–locally closed, rw–locally closed, rgα–locally closed, gprw–locally closed sets.

Example 3.21: Let X={a,b,c} and τ ={X, ϕ ,{a},{b,c}} then {a,b} is ω–locally closed set , rw–locally closed set, rgα–locally closed set, gprw–locally closed set but not a αrω-LC set in (X, τ) .

Example 3.22: Let X={a,b,c,d} and τ ={X, ϕ ,{a},{b},{a,b}{a,b,c}} then {b,c} is αrω-LC set but not a rw–locally closed set, rgα–locally closed set, gprw–locally closed set in (X, τ) .

Example 3.23: Let X={a,b,c} and τ ={X, ϕ ,{a}} then {a,c} is αrω-LC set but not a ω–locally closed set in (X, τ) .

Remark 3.24: From the above discussion and known results we have the following implications in the diagram.



Theorem 3.25: If αrωO(X, τ) = τ then

i) αrω-LC(X, τ) = LC (X, τ) .

ii) αrω-LC(X, τ) = α-LC (X, τ) .

iii) αrω-LC(X, τ) ⊆ GLC (X, τ) .

iv) αrω-LC(X, τ) ⊆ ω-LC (X, τ).

v) αrω-LC(X, τ) ⊆ RW-LC (X, τ).

Proof: (i) For any space (X, τ) , W.K.T LC(X, τ) ⊆ αrω-LC(X, τ) . Since αrωO(X, τ) = τ , that is every αrω-open set is open and every αrω-closed set is closed in (X, τ) , αrω-LC(X, τ) ⊆ LC (X, τ) ; hence αrω-LC(X, τ) = LC (X, τ) .

(ii) For any space (X, τ) , LC(X, τ) ⊆ α-LC(X, τ) ⊆ αrω-LC(X, τ) From (i) it follows that αrω-LC(X, τ) = α-LC(X, τ) .

(iii) For any space (X, τ) , LC(X, τ) ⊆ GLC (X, τ) from (i) αrω-LC(X, τ) = LC (X, τ) and hence αrω-LC(X, τ) ⊆ GLC (X, τ) .

(iv) For any space (X, τ) , LC(X, τ) ⊆ ω-LC (X, τ) from (i) αrω-LC(X, τ) = LC (X, τ) and hence αrω-LC(X, τ) ⊆ ω-LC (X, τ) .

(v) For any space (X, τ) , LC(X, τ) ⊆RW-LC (X, τ) from (i) αrω-LC(X, τ) = LC (X, τ) and hence αrω-LC(X, τ) ⊆ RW-LC (X, τ) .

Theorem 3.26: If αrωO(X, τ) = τ , then αrω-LC*(X, τ) = αrω-LC** (X, τ) = αrω-LC(X, τ) .

Proof: For any space,(X, τ) LC (X, τ) ⊆ αrω-LC*(X, τ) ⊆ αrω-LC(X, τ) and LC (X, τ) ⊆ αrω-LC**(X, τ) ⊆ αrω-LC(X, τ). since αrωO(X, τ) = τ , αrω-LC(X, τ) =LC(X, τ) by theorem 3.25, it follows that LC(X, τ) = αrω-LC*(X, τ) = αrω-LC**(X, τ) = αrω-LC(X, τ) .

Remark 3.27: The converse of the theorem 3.26 need not be true in general as seen from the following example.

Example 3.28: Let X={a, b, c} with the topology τ= { X, ϕ , {a}} then αrω-LC*(X, τ) = αrω-LC**(X, τ) = αrω-LC(X, τ) . However αrωO(X, τ) ={ X, ϕ , {a} , {a,c} , {a,b} } ≠ τ.

Theorem 3.29: If GO(X, τ) = τ, then GLC(X, τ) ⊆ αrω-LC(X, τ)

Proof: For any space (X, τ) w.k.t LC(X, τ) ⊆ GLC(X, τ) and LC (X, τ) ⊆αrω-LC(X, τ) .....(i) GO(X, τ) = τ, that is every g-open set is open and every g-closed set is closed in (X, τ) and so GLC(X, τ) ⊆LC(X, τ). That is GLC(X, τ) =LC(X, τ) ......(ii) , from (i) and (ii) we have GLC(X, τ) ⊆ αrω-LC(X, τ) .

Theorem 3.30: If ωO(X, τ) = τ, then ω-LC(X, τ) ⊆ αrω-LC(X, τ)

Proof: For any space (X, τ), w.k.t LC(X, τ) ⊆ ω-LC(X, τ) and LC (X, τ) ⊆αrω-LC(X, τ) .....(i) ωO(X, τ) = τ, that is every ω-open set is open and every ω-closed set is closed in (X, τ) and so ω-LC(X, τ) ⊆ LC(X, τ). That is ω-LC(X, τ) =LC(X, τ) ......(ii) from (i) and (ii) we have ω-LC(X, τ) ⊆ αrω-LC(X, τ)

Theorem 3.31: If RWO(X, τ) = τ, then RW-LC(X, τ) ⊆ αrω-LC(X, τ)

Proof: For any space (X, τ) w.k.t LC(X, τ) ⊆ RW-LC(X, τ) and LC (X, τ) ⊆ αrω-LC(X, τ) .....(i) RWO(X, τ) = τ, that is every rw-open set is open and every rw-closed set is closed in (X, τ) and so RW-LC(X, τ) ⊆ LC(X, τ). That is RW-LC(X, τ) = LC(X, τ) ......(ii) from (i) and (ii) we have RW-LC(X, τ) ⊆ αrω-LC(X, τ)

Theorem 3.32: If αrωC(X, τ) ⊆ LC(X, τ) then αrω-LC(X, τ) = αrω-LC*(X, τ)

Proof: Let αrωC(X, τ) ⊆ LC(X, τ) , For any space (X, τ) , w.k.t αrω-LC*(X, τ) ⊆ αrω-LC(X, τ) ...(i) Let A ϵ αrωC(X, τ) , then A = U∩F, where U is αrω-open and F is a αrω-closed ın (X, τ) . Now F ϵ αrω-LC(X, τ) by hypothesis F is locally closed set in (X, τ) , then F= G∩E, where G is an open set and E is a closed set in (X, τ) .

Now, A = U∩F= U ∩ (G∩E)= (U∩G) ∩ E, where U∩G is αrω-open as the intersection of αrω-open sets is αrω-open and E is a closed set in (X, τ) , It follows that A is αrω-LC*(X, τ) .That is A ϵ αrω-LC*(X, τ) and so, αrωC(X, τ) ⊆ αrω-LC*(X, τ) ......(ii).

From (i) and (ii) we have αrω-LC(X, τ) = αrω-LC*(X, τ) .

Remark 3.33: The converse of the theorem 3.32 need not be true in general as seen from the following example.

Example 3.34: Consider X= {a,b,c} and τ = { X, ϕ ,{a}} , then αrω-LC(X, τ) = αrω-LC*(X, τ) =P(X). But αrωC(X, τ) =={ X, ϕ , {b} , {c} , {b,c} } and LC(X, τ) = { X, ϕ ,{a},{b,c}} That is αrωC(X, τ) $\nsubseteq$ LC(X, τ) .

Theorem 3.35: For a subset A of (X, τ) if A ϵ αrω-LC(X, τ) then A = U ∩ (αrω-cl(A)) for some open set U.

Proof : Let, Aϵ αrω-LC(X, τ) then there exist a αrω-open U and a αrω-closed set F s.t. A = U∩F. Since A$\subseteq$ F, αrω-cl(A) $\subseteq$ αrω-cl(F) = F. Now U ∩ (αrω-cl(A)) $\subseteq$ U∩F = A, that is U∩ (αrω-cl(A)) $\subseteq$ A.
and A$\subseteq$ U and A $\subseteq$ αrω-cl(A) implies A $\subseteq$ U ∩ (αrω-cl(A)) and therefore A = U ∩ (αrω-cl(A)) for some αrω-open set U.

Remark 3.36: The converse of the theorem 3.35 need not be true in general as seen from the following example.

Example 3.37: Consider X={a, b, c, d} and τ = {X, ϕ ,{a}, {c,d}, {a,c,d}}

1. αrω-C(X, τ) = { X, ϕ, {b}, {a, b}, {a, b, c}, {b, c, d}, {a, b, d} }
2. αrω-LC Set = {X , ϕ , {a}, {b}, {c} , {d}, {a, b} , {c, d}, {a, d} ,{a , c}, a, b, c}, {b, c, d}, {a, b, d}, {a , c, d} }

Take A= {b,d}, αrω-cl(A)= {b,d} now, A = X ∩ (αrω-cl(A)) for some αrω-open set X but {b,d} $\notin$ αrω-LC (X, τ) .

Theorem 3.38: For a subset A of (X, τ) , the following are equivalent.

(i) A ϵ αrω-LC*(X, τ) .
(ii) A= U ∩ (cl(A) for some αrω-open set U.
(iii) cl(A)-A is αrω-closed.
(iv) A ∪ (cl(A)^c is αrω-open.

Proof : (i)implies (ii) Let Aϵ αrω-LC*(X, τ) then there exists a αrω-open set U and a closed set F s.t A = U∩F. Since A $\subseteq$ F, cl(A) $\subseteq$ cl(F)= F. Now U∩cl(A) $\subseteq$ U∩F = A that is U∩ cl(A)= A. Conversely A $\subseteq$ U, and A $\subseteq$ cl(A) implies A $\subseteq$ U∩ cl(A) and therefore A = U∩cl(A) for some αrω-open set U.
(ii) implies (i) since U is a αrω-open set and cl(A) is a closed set, A= U ∩ (cl(A) ϵ αrω-LC*(X, τ) .
(iii) implies (iv) let F=cl(A)-A, then F is αrω-closed by the assumption and X-F=X-[cl(A)-A] = X ∩ [cl(A)-A]^c = A ∪ (X-cl(A)) = A ∪ (cl(A))^c. But X- F is αrω-open. This shows that A ∪ (cl(A))^c is αrω-open.
(iv) implies (iii) Let U = A ∪ (cl(A)^c then U is αrω-open ,this implies X-U is αrω-closed and X-U = X – (A∪cl(A)^c) = cl(A) ∩ (X-A) = cl(A)-A is αrω-closed.
(iv) impies (ii) Let U = A ∪ (cl(A)^c then U is αrω-open .hence we prove that A =U∩ $(\mathrm{cl}(A)$cfor some αrω-open set U.

Now A = U∩ $(\mathrm{cl}(A)$ = [A∪(cl(A)^c] ∩ cl(A) = A∩ [cl(A)] ∪ cl(A)^c∩ cl (A) = A ∪ ϕ = A.

Therefore A = U∩ $(\mathrm{cl}(A)$ for some αrω-open set U.

(ii) implies (iv) Let A = U ∩ (cl(A) for some αrω-open set then we P.T A ∪ (cl(A)^c is αrω-open . Now A ∪ (cl(A)^c = (U∩ $(\mathrm{cl}(A))$ ∪ [cl(A))^c = U∩ $(\mathrm{cl}(A))$ ∪ [cl(A))^c =U ∩ X = U, which is αrω-open. Thus A = (cl(A))^c is αrω-open.

Theorem 3.39: For a subset A of (X, τ) , the following are equivalent.

(i) A ϵ αrω-LC(X, τ) .
(ii) A= U ∩ (cl(A) for some αrω-open set U.
(iii) cl(A)-A is αrω-closed.
(iv) A ∪ (cl(A)^c is αrω-open.

Proof: Similar to **Theorem 3.38**

Theorem 3.40: For a subset A of (X, τ) if A∈αrω-LC**(X, τ) ,then there exists an open set U s.t A = U ∩ αrω-cl(A).

Proof: Let A∈αrω-LC**(X, τ) , then there exist an open set U and a αrω-closed set F s.t A = U ∩ F.
Since A ⊆ U and A ⊆ αrω-cl(A) we have A ⊆ αrω-cl(A).
And since A ⊆ F and αrω-cl(A) ⊆ αrω-cl(F) = F, as F is αrω-closed. Thus U ∩ αrω-cl(A) ⊆ U ∩ F = A.
That is U ∩ αrω-cl(A) ⊆ A ; hence A= U ∩ αrω-cl(A) ,for some open set U.

Remark 3.41: The converse of the theorem 3.40 need not be true in general as seen from the following example.

Example 3.42: Consider X={a, b, c, d} and τ = {X, ϕ ,{a}, {c,d}, {a,c,d}}

1. αrω-C(X, τ) = { X, ϕ, {b}, {a, b}, {a, b, c}, {b, c, d}, {a, b, d} }
2. αrω-LC Set = {X , ϕ , {a}, {b}, {c} , {d}, {a, b} , {c, d}, {a, d} ,{a , c}, a, b, c}, {b, c, d}, {a, b, d}, {a , c, d} }

Take A= {b,d}, αrω-cl(A)= {b,d} now, A = X ∩ (αrω-cl(A)) for some αrω-open set X but {b,d} ∈ αrω-LC**(X, τ) .

Theorem 3.43: For A and B in (X, τ) the following are true.
(i) if A∈αrω-LC*(X, τ) and B∈αrω-LC*(X, τ) ,then A∩B∈αrω-LC*(X, τ) .
(ii) if A∈αrω-LC**(X, τ) and B is open,then A∩B∈αrω-LC**(X, τ) .
(iii) if A∈αrω-LC(X, τ) and B is αrω-open,then A∩B∈αrω-LC(X, τ) .
(iv)) if A∈αrω-LC*(X, τ) and B is αrω-open,then A∩B∈αrω-LC*(X, τ) .
(v)) if A∈αrω-LC*(X, τ) and B is closed,then A∩B∈αrω-LC*(X, τ) .

Proof:(i) Let A , B ∈αrω-LC*(X, τ) ,it follows from theorem 3.38 that there exist αrω-open sets P and Q s.t A =P∩ cl(A) and B=Q ∩ cl(B).
Therefore A∩B=P∩ cl(A) ∩ Q ∩ cl(B)=P ∩ Q ∩[cl(A) ∩ cl(B)] where P ∩ Q is αrω-open and cl(A) ∩ cl(B) is closed. This shows that A∩B ∈ αrω-LC*(X, τ) .
(ii)Let A ∈ αrω-LC**(X, τ) and B is open . Then there exist an open set P and αrω-closed set F s.t A = P ∩ F. Now, A∩B = P∩ F ∩ B = (P∩B) ∩ F, Where (P∩B) is open and F is αrω-closed . This implies A∩B∈αrω-LC**(X, τ) .
(iii) Let A∈αrω-LC(X, τ) and B is αrω-open then there exists a αrω-open set P and a αrω-closed set F s.t A = P∩ F. Now, A∩B = P ∩ F ∩ B = (P∩B) ∩ F, Where (P∩B) is αrω-open and F is αrω-closed .This shows that A∩B∈αrω-LC(X, τ) .
(iv) Let A∈αrω-LC*(X, τ) and B is αrω-open then there exists a αrω-open set P and a αrω-closed set F s.t A = P∩ F. Now, A ∩ B = (P ∩ F) ∩ B = (P ∩ B) ∩ F, Where (P ∩ B) is αrω-open and F is closed. This implies that A ∩ B ∈ αrω-LC*(X, τ) .
(v) A ∈ αrω-LC*(X, τ) and B is closed. Then there exist an αrω-open set P and a closed set F s.t A = P ∩ F. Now, A∩B = (P∩ F) ∩ B = P∩(F ∩ B), Where (F∩B) is closed and P is αrω-open .This implies A∩B ∈ αrω-LC*(X, τ) .

Definition 3.44

A topological space (X, τ) is called αrω-submaximal if every dense subset is αrω-open.

Theorem 3.45 If (X, τ) is submaximal space then it is αrω-submaximal space but converse need not be true in general.

Proof: Let (X, τ) be submaximal space and A be a dense subset of (X, τ). Then A is open . But every open set is αrω-open and so A is αrω-open . Therefore (X, τ) is a αrω-submaximal space.

Example 3.46 Let X = {a, b, c} and τ = { ϕ , {a}, X}. Then Topological space (X, τ) is αrω-submaximal but set A={a,b} is dense in (X, τ) but not open therefore (X, τ) is not submaximal.

Theorem 3.47 A topological space (X, τ) αrω-submaximal if and only if
P(X) = αrω-LC*(X, τ).

Proof:

Necessity: Let A∊P(X) and U = A ∪ (X-cl(A)). Then it follows cl(U)= cl(A ∪ (X-cl(A))) =cl(A) ∪ (X-cl(A)) = X. Since (X, τ) is αrω-sub maximal, U is αrω-open, so A∊ αrωLC*(X, τ) from the Theorem 3.38 Hence P(X) = αrωLC*(X, τ).

Sufficiency: Let A be dense sub set of (X, τ). Then by assumption and Theorem 3.38 (iv) that A ∪ (X-cl(A)) = A holds, A∊αrωLC*(X, τ) and A is αrω-open. Hence (X, τ) αrω-sub maximal.

Theorem 3.48: If (X, τ) $T_{\alpha r\omega}$-space then αrω-LC(X, τ) = LC(X, τ).

Proof: Straight Forward.

Theorem 3.49: Let (X, τ) and (Y, σ) be toplogical spaces.

i) If A∊ αrω-LC(X, τ) and B ∊ αrω-LC(Y, σ) then A × B ∊ αrω-LC (X × Y , τ ×σ).
ii) If A∊ αrω-LC*(X, τ) and B ∊ αrω-LC*(Y, σ) then A × B ∊ αrω-LC*(X × Y , τ ×σ).
iii) If A∊ αrω-LC**(X, τ) and B∊ αrω-LC**(Y, σ) then A× B ∊ αrω-LC**(X × Y ,τ × σ).

Proof : i) If A αrω-LC(X, τ) and B αrω-LC(Y, σ). Then there exist αrω-open sets U and V of (X, τ) and (Y,σ) and αrω-clos d sets G and F of X and Y respectively such that A= U ∩G and B= V ∩F. Then A × B= (U × V) ∩(G × F) holds.

Hence A × B αrω-LC(X × Y , τ × σ).

ii) and iii) Similarly the follow from the definition.

References

[1]. A. Pushpalatha (2000), Studies on generalizations of mappings in topological spaces, Ph.D., thesis, Bharathiar University, Coimbatore.

[2]. C. Kuratowski and W. Sierpinski(1921), Sur les differences deux ensembles fermes, Tohoku Math .Jl., 20 22-25

[3]. H. P. Perk and J. K.Perk, on locally δg *closed sets*, $l\delta gc$-continuous functions,solitions and fractals 19 (2004), 995-1002.

[4]. I. Arockiarani and K. Balachandran , on θ-g locally closed set (pre print).

[5]. I. Arockiarani and K.Balachandran and M. Ganster, Regular generalized locally closed sets and rgl-continuous functions , Indian J. Pure . Appl. Math , 28(1997), 661-669.

[6]. K. Balachandran,P.sundaram, and H.Maki, g-lc and glc-continuous function, indian j pure appp Math,27 (1966),235-244.

[7]. M. Ganster and I.L.Reilly, locally closed sets ad lc-contınuous function internret, j, math, sci, 12(1989),417-424.

[8]. M. Sheik john, a study on generalization of closed sets on continuous maps in topological & bitopological spaces, Ph.D thesis, Bharathiar university,Coimbatore (2002).

[9]. M. Stone , Absolutly FG spaces , proc,Amer.math, soc 80(1980) 515-520.

[10]. N. Bourkbaki, General topology ,part I adison-wesley,reading mass.,1966

[11]. R. S. Wali &, Mandalgeri P. S , On α Regular ω-closed sets in topological spaces, Int. J ofMath Archive. 2014; 5(10):68-76

[12]. R. S. Wali & P.S. Mandalgeri, On α Regular ω-open sets in topological spaces, J. of comp & Math Sci., Vol 5(6), 2014, pp 490-499

[13]. S. S. Benchalli and R.S Wali on rw-Closed sets is Topological Spaces, Bull, Malays, Math, sci, soc30 (2007), 99-110

[14]. Y. Gnanambal , On generalized pre regular closed sets in topological spaces, Indian J. Pure. Appl. Math., 28(3)(1997), 351-360

[15]. Y. Gnanambal , On generalized pre-regular Closed sets and generalized of locally closed sets in Topological Spaces , Ph.D thesis, Bharathiar university,Coimbatore (1998)