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RESEARCH ARTICLE

A NOTE ON I_π- CONTINUOUS FUNCTIONS

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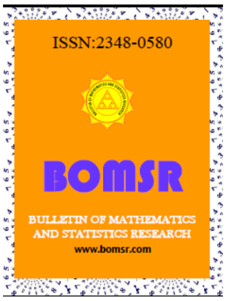

ABSTRACT

In this paper we define the notion of I_π-continuous functions and discuss their properties. We also investigate the relationship between the defined classes of functions and other classes of functions with counter examples.

Keywords: I_π-open set, I_π-closed set, I_π-continuous function, I_π- open function, I_π-closed function

1. INTRODUCTION

The topic of ideals in general topological spaces is treated in the classic text by Kuratowski[12]. This topic has excellent potential for application in other branches of mathematics. Ideals have been frequently used in the fields closely related to topology such as real analysis measure theory and lattice theory. This subject was continued to study by general topologists in recent years [3, 7]. In 1990 Jankovic and Hamlett [10, 11] defined I-open set in ideal topological spaces. Later bd El-Monsef [2] studied I-continuity for functions. Then Hatir and Noiri[8] introduced semi-I-open set and Semi-I-continuity in 2005. The purpose of this paper is to introduce the concept of I_π-open set and I_π-continuous functions and study their properties.

2. Preliminaries

Throughout this paper (X, τ) is a topological space on which no separation axioms are assumed unless explicitly stated. The notation (X, τ, I) will denote the topological space (X, τ) and an ideal I on X with no separation properties assumed. For A ⊆ (X, τ), Cl(A) and Int(A) respectively denote the closure and interior of A with respect to τ.

Definition: 2.1[12]

An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies the following properties: (1) A ∈ I and B ⊆ A Implies B ∈ I.

(2) A ∈ I and B ∈ I implies A∪ B ∈ I.

An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I).

Definition: 2.2[12]

For a subset A of X, $A^*(I) = \{x \in X: U \cap A \notin I$ for each neighbourhood U of $x\}$ is called the local function of A with respect to I and τ. We simply write A^* instead of $A^*(I)$.

Definition: 2.3[12]

It is well known that $Cl^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(I)$ which finar than τ.

Definition: 2.4[10]

A basis β(I, τ) for $\tau^*(I)$ can be described as follows: β(I, τ) = {U— E: U ∈ τ and E ∈ I}.

Definition: 2.5

A subset A of an ideal topological space (X, τ, I) is

(1) ∗-perfect [9], if A =A^*

(2) ∗- closed [10], if $A^* \subseteq A$

(3) ∗- dense-in-itself [9], if $A \subseteq A^*$

(4) *-dense [5], if $Cl^*(A) = X$

(5) τ^*-closed set [10], if $A = Cl^*(A)$

Definition: 2.6[15]

A subset A of a space (X, τ) is said to be regular open set, if A = int(cl(A)).

Definition: 2.7[17]

Finite union of regular open sets in (X, τ) is π-open in (X, τ).The complement of π-open in (X, τ) is π-closed in (X, τ).

Definition: 2.8[1]

Given a space (X, τ, I), a set operator $()^{*\pi}$: P(X) → P(X) is called the π-local function of I with respect to τ is defined as follows: for A ⊆ X, $(A)^{*\pi}(I,\tau) = \{x \in X \mid U_x \cap A \notin I$, for every $U_x \in \pi N(x)\}$, where πN(x)} = {U ∈ πO(x) | x ∈U}. We write π-local function as $A^{*\pi}(I)$ or $A^{*\pi}$ or $A^{*\pi}(I, \tau)$.

Definition: 2.9[2]

A subset A of an ideal topological space (X, τ, I) is said to be I- open if $A \subseteq int(A^*)$.

Definition: 2.10[2]

A subset F ⊆ (X, τ, I) is called I-closed if its complement is I-open.

Definition: 2.11[11]

A function f : (X, τ, I) ⟶ (Y, σ) is said to be I- continuous if $f^{-1}(V)$ is I-open in X for every open set Vof Y.

Definition: 2.12[6]

A function f : (X, τ, I) ⟶ (Y, σ) is said to be I- irresolute, if $f^{-1}(V)$ is I-open in X for every I-open set Vof Y.

Definition: 2.13[5]

A function f : (X, τ) ⟶ (Y, σ) is said to be π-continuous, if $f^{-1}(V)$ is π-open in X for every open set Vof Y.

3. I_π- open sets

Definition: 3.1

A subset A of an ideal topological space (X, τ, I) is said to be I_π- open if $A \subseteq int(A^{*\pi})$. The complement of I_π-open set is I_π-closed.

Remark: 3.2

Every I-open set is I_π-open, but the converse need not be true.

Example: 3.3

X = {a, b, c, d}

τ = {X, φ, {a}, {a, b}, {a, b, c}}

I = {φ, {a}}

Take A = {b, c, d}. Then A is I_π-open set but not I-open.

Remark: 3.4

(1) I_π-openess and openess are independent concepts.
(2) I_π-openess and π-openess are independent concepts.

Example: 3.5

X = {a, b, c, d}
τ = {X, φ, {c}, {a, b}, {a, b, c}}
I = {φ, {a}}
If we take A = {b, c, d} then A is I_π-open set but not open.

Example: 3.6

X = {a, b, c, d}
τ = {X, φ, {c}, {a, b}, {a, b, c}}
I = {φ, {c}, {d}, {c, d}}
If we take B = {a, b, c} then B is open set but not I_π-open.

Example: 3.7

In example: 3.6 if we take A = {b} then A is I_π-open set but not π-open.

Example: 3.8

X = {a, b, c, d}
τ = {X, φ, {d}, {a, c}, {a, c, d}}
I = {φ, {c}, {d}, {c, d}}
If we take A = {a, c, d} then A is π-open set but not I_π- open.

Remark: 3.9

For a subset A of an ideal topological space (X, τ, I) we have X \ $(\text{int}(A))^{*\pi} \neq \text{int}(X\setminus A)^{*\pi}$ as shown by the following example.

Example: 3.10

X = {a, b, c, d}
τ = {X, φ, {d}, {a, c}, {a, c, d}}
I = {φ, {c}, {d}, {c, d}}
If A = {a,c} then $(\text{int}(A))^{*\pi}$ = {a, b, c} and X \ $(\text{int}(A))^{*\pi}$ = {d} but $\text{int}(X\setminus A)^{*\pi} = \emptyset$. Therefore X \ $(\text{int}(A))^{*\pi} \neq \text{int}(X\setminus A)^{*\pi}$.

Theorem: 3.11

If a subset A of an ideal topological space (X, τ, I) is I_π-closed then A ⊇ $(\text{int}(A))^{*\pi}$

Proof: Obvious

Corollary: 3.12

Let A be subset of an ideal topological space (X, τ, I) such that X \ $(\text{int}(A))^{*\pi} = \text{int}(X\setminus A)^{*\pi}$. Then A is I_π-closed if and only if A ⊇ $(\text{int}(A))^{*\pi}$

Preposition: 3.13

Let (X, τ, I) be an ideal topological spce with Δ an arbitrary index set. Then
(1) If $\{A_\alpha : \alpha \in \Delta\} \subseteq I_\pi O(X)$ then $\cup\{A_\alpha : \alpha \in \Delta\} \in I_\pi O(X)$.
(2) If $A \in I_\pi O(X)$ and B ∈ τ then $(A \cap B) \in I_\pi O(X)$.

Proof:

1) Let $\{A_\alpha : \alpha \in \Delta\}$ be I_π-open. Then $A_\alpha \subseteq \text{int}(A_\alpha^{*\pi})$ for α ∈ Δ.Thus $\bigcup A_\alpha \subseteq \bigcup(\text{int}(A_\alpha^{*\pi})) \subseteq \text{int}(\bigcup A_\alpha^{*\pi}) \subseteq \text{int}(\bigcup A_\alpha)^{*\pi}$ for every α ∈ Δ. Hence $\bigcup\{A_\alpha : \alpha \in \Delta\} \in I_\pi O(X)$.
2) Assume that A is I_π- open and B ∈ τ. Then $A \subseteq \text{int}(A^{*\pi})$ and $B \subseteq \text{int}(B)$. We have to prove that (A ∩ B) is I_π- open. $(A \cap B) \subseteq \text{int}(A^{*\pi}) \cap \text{int}(B) \subseteq \text{int}(A^{*\pi} \cap B) \subseteq \text{int}(A \cap B)^{*\pi}$.

Theorem: 3.14

If $A \in I_\pi O(X)$ and $B \in I_\pi O(Y)$ then $A \times B \in I_\pi O(X \times Y)$, if $A^{*\pi} \times B^{*\pi} = (A \times B)^{*\pi}$, where X × Y is the product space.

Proof:

Suppose A and B are I_π- open sets. Then A⊆ int($A^{*\pi}$) and B⊆ int($B^{*\pi}$). Therefore A × B ⊆ int($A^{*\pi}$) × int($B^{*\pi}$) = int($A^{*\pi} \times B^{*\pi}$) = int$(A \times B)^{*\pi}$. Therefore A × B ∈ I_πO(X × Y).

Theorem: 3.15

If (X, τ, I) is an ideal space, A ∈ τ and B ∈ I_πO(X, τ) then there exists an open subset G of X such that A ∩ G = φ implies A ∩ B = φ.

Proof:

Let A be an open set and B be an I_π-open set. Since B ∈ I_πO(X, τ) then B ⊆ int($B^{*\pi}$). Let G = int($B^{*\pi}$) be an open set such that B ⊆ G, but A ∩ G = φ. Then G ⊆ X∖ A implies that cl(G) ⊆ X∖ A.Therefore B ⊆ X∖ A. Hence A ∩ B = φ.

Theorem: 3.16

Let {X_α : α ∈ Δ} be a family of spaces, X = ∏ X_α be the product space and A = $\prod_{\alpha=1}^{n} A_\alpha \times \prod_{\alpha\neq\beta} X_\beta$ be a non empty subset of X where n is a positive integer if and only if A ∈ I_πO(X).

Proof:

Necessity: Suppose A_α ∈ I_πO(X_α) for each ($1 \le \alpha \le n$). Since A = $\prod_{\alpha=1}^{n} A_\alpha \times \prod_{\alpha\neq\beta} X_\beta$ ⊆ int($A^{*\pi}$). Then A ∈ I_πO(X).

Sufficiency: Assume that A ∈ I_πO(X). Then A ⊆ int($A^{*\pi}$) = int$(\prod_{\alpha=1}^{n} A_\alpha \times \prod_{\alpha\neq\beta} X_\beta)^{*\pi}$. Since A ≠ φ and A ∈ I_πO(X) then int($A^{*\pi}$) ≠ φ. Hence int($A_\alpha^{*\pi}$) ≠ φ for each ($1 \le \alpha \le n$).Therefore A_α ⊆ int($A_\alpha^{*\pi}$). This implies that A_α ∈ I_πO(X_α) for each ($1 \le \alpha \le n$).

4. I_π- continuous functions

Definition: 4.1

A function f : (X, τ, I) ⟶ (Y, σ) is said to be I_π- continuous if every V∈ σ, $f^{-1}(V)$ ∈ I_πO(X, τ).

Definition: 4.2

A function f : (X, τ, I) ⟶ (Y, σ, J) is said to be I_π- irresolute, if $f^{-1}(V)$ is I_π- open in X for every I_π- open set Vof Y.

Remark: 4.3

Every I- continuous function is I_π- continuous, but the converse need not be true.

Example: 4.4

Let X = Y = {a, b, c}, τ = {X, φ, {a}}, I = {φ, {a}} on X and σ = {Y, φ, {a}, {b}, {a, b}}.Then the function f : (X, τ, I) ⟶ (Y, σ) is defined as f(a) = b, f(b) = a and f(c) = c is I_π-continuous but not I-continuous, because {a, b} ∈ σ but $f^{-1}(\{a,b\})$ = {a, b} ∉ IO(X).

Remark: 4.5

The concept of continuity and I_π- continuity are independent.

Example: 4.6

Let X = Y = {a, b, c}, τ = {X, φ, {a}, {c}, {a, b}, {a,c}}, I = {φ, {b}, {c}, {b, c}} on X and σ = {Y, φ, {a}, {c}, {a, c}}.Then the identity function f : (X, τ, I) ⟶ (Y, σ) is continuous but not I_π- continuous, because {c} ∈ σ but $f^{-1}(\{c\})$ = {c} ∉ I_πO(X)

Example: 4.7

Let X = Y = {a, b, c}, τ = {X, φ, {a}}, I = {φ, {b}} on X and σ = {Y, φ, {a}, {b}, {a, b}}.Then the function f : (X, τ, I) ⟶ (Y, σ) is defined as f(a) = a = f(b) and f(c) = c is I_π- continuous but not continuous, because {a} ∈ σ but $f^{-1}(\{a\})$ = {a, b} is not open in X.

Theorem: 4.8

For a function f : (X, τ, I) ⟶ (Y, σ) the following are equivalent:

1) f is I_π- continuous

2) For each $x \in$ X and V ∈ σ containing f(x), there exists W ∈ I_πO(X) containing x such that f(W) ⊆ V
3) For each $x \in$ X and V ∈ σ containing f(x), $(f^{-1}(V))^{*\pi}$ is a neighbourhood of x.

Proof:

(1) ⟹ (2): Assume that f is I_π- continuous function. Let $x \in$ X and V ∈ σ such that f(x) ∈ V. Then W = f^{-1}(V) is I_π-open set. Clearly $x \in$ W and f(W) ⊆ V.

(2) ⟹ (3): Since $x \in$ X and V ∈ σ containing f(x), then by(2) there exists W ∈ I_πO(X) containing x such that f(W) ⊆ V. Thus $x \in W \subseteq int(W^{*\pi}) \subseteq int(f^{-1}(V))^{*\pi} \subseteq (f^{-1}(V))^{*\pi}$. Hence $(f^{-1}(V))^{*\pi}$ is a neighbourhood of x.

(3) ⟹ (1): Obvious

Definition: 4.9

A subset A of an ideal topological space (X, τ, I) is

(1) I_π-perfect, if A =$A^{*\pi}$

(2) I_π- dense-in-itself , if A ⊆ $A^{*\pi}$

Theorem: 4.10

For a function f : (X, τ, I) ⟶ (Y, σ) the following are equivalent:

(1) f is I_π- continuous

(2) The inverse image of each closed set in Y is I_π- closed

(3) $int(f^{-1}(M))^{*\pi} \subseteq f^{-1}(M^{*\pi})$ for each I_π-dense-in-itself subset M ⊆Y

(4) $f((int(U))^{*\pi}) \subseteq (f(U))^{*\pi}$ for each U ⊆ X and for each I_π-perfect subset of Y

Proof:

(1) ⟹ (2): Let F ⊆ Y be closed, then Y\F is open. Then by (1) f^{-1}(Y\F) = X\ f^{-1}(F) is I_π- open. Thus f^{-1}(F) is I_π- closed.

(2) ⟹ (3): Let M ⊆ Y. Since $M^{*\pi}$ is closed, then by (2) $f^{-1}(M^{*\pi})$ is I_π- closed. Thus $f^{-1}(M^{*\pi}) \supseteq (int(f^{-1}(M^{*\pi})))^{*\pi}$, since M is I_π-dense-in-itself. Then $f^{-1}(M^{*\pi}) \supseteq (int(f^{-1}(M))^{*\pi})^{*\pi} \supseteq int(f^{-1}(M))^{*\pi}$. Hence $int(f^{-1}(M))^{*\pi} \subseteq f^{-1}(M^{*\pi})$.

(3) ⟹ (4): Let U ⊆ X and W = f(U) then by (3) $f^{-1}(W^{*\pi}) \supseteq int(f^{-1}(W))^{*\pi} \supseteq (int\ (U))^{*\pi}$. Hence $f((int(U))^{*\pi} \subseteq W^{*\pi} = (f(U))^{*\pi}$.

(4) ⟹ (1):

Let V ∈ σ, W = Y\V and U = f^{-1}(W) then f(U) ⊆ W and by (4) $f((int(U))^{*\pi}) \subseteq (f(U))^{*\pi} \subseteq W^{*\pi}$ = W. Thus $f^{-1}(W) \supseteq (int(U))^{*\pi} = (int(f^{-1}(W)))^{*\pi}$. Therefore f^{-1}(W) = f^{-1}(Y\V) is I_π- closed. Hence f^{-1}(V) is I_π-open in X and f is I_π- continuous.

Theorem: 4.11

The function f : (X, τ, I) ⟶ (Y, σ) is I_π- continuous if and only if the graph function g : X ⟶ X× Y is I_π-continuous.

Proof:

Necessity: Let f be I_π-continuous. Now let $x \in$ X and let V be any open set in X × Y containing g(x) = (x, f(x)). Then there exists a basic open set U × W such that g(x) ∈ U × W ⊆ V. Since f is I_π-continuous, there exists a I_π- open set A in X such that $x \in$ A ⊆ X and f(A) ⊆ W. Since A ∩ U is I_π-open set in X and A ∩ U ⊆ U, g(A ∩ U) ⊆ U × W ⊆ V. Hence g is I_π- continuous.

Sufficiency: Let g: X ⟶ X× Y be I_π- continuous and let V be a open set containing f(x). Then X × V is open in X × Y. Since g is I_π- continuous, there exists I_π- open set W such that g(W) ⊆ X × V. This implies that f(W) ⊆ V. Hence f is I_π-continuous.

Theorem: 4.12

Let f: (X, τ, I) ⟶ (Y, σ) be an I_π- continuous and U ∈ τ. Then the restriction f|U is an I_π- continuous.

Proof:

Let V ∈ σ. Then f^{-1}(V) ⊆ int(f^{-1}(V))$^{*\pi}$. Then U ∩ f^{-1}(V) ⊆ U ∩ int(f^{-1}(V))$^{*\pi}$. Thus $(f|U)^{-1}$(V) ⊆ U ∩ int(f^{-1}(V))$^{*\pi}$. Since U ∈ τ^{π}, Then $(f|U)^{-1}$(V) = int[U ∩ (f^{-1}(V))$^{*\pi}$] ⊆ int[U ∩ f^{-1}(V)]$^{*\pi}$ = int[$(f|U)^{-1}$(V)]$^{*\pi}$. Therefore f|U is I_π- continuous.

Theorem: 4.13

Let f: (X, τ, I) ⟶ (Y, σ, J) be a function and {U_α: α ∈ Δ } be a π-open cover of X. If the restriction f|U is I_π- continuous for each α ∈ Δ , then f is I_π- continuous.

Proof: Similar to Theorem: 4.11

Theorem: 4.14

Let f: (X, τ, I) ⟶ (Y, σ) be I_π- continuous and f^{-1}(V$^{*\pi}$) ⊆ [f^{-1}(V)]$^{*\pi}$ for each V ⊆ Y. Then the inverse image of each I_π- open set is I_π-open.

Proof: Obvious

Remark: 4.15

The composition of two I_π- continuous functions need not be I_π- continuous as shown in the following example.

Example: 4.16

Let X = Z = {a, b, c} and Y = {a, b, c, d} with topologies τ = {X, φ, {a}, {b}, {a, b}}, σ = {Y, φ, {a}, {c}, {a, c}} and μ = {Z, φ, {c}, {b, c}}. Let I = {φ, {c}} be an ideal on X and J = {φ, {a}} be an ideal on Y. Let f: (X, τ, I) ⟶ (Y, σ) be identity function and g: (Y, σ, J) ⟶ (Z, μ) be defined as g(a) = a, g(b) = b = g(d) and g(c) = {c}. Then f and g are I_π- continuous and the composition function g ∘ f is not I_π- continuous, because {c}∈ μ but (g ∘ f)$^{-1}$({c}) = {c} is not I_π-open set in X.

Theorem: 4.17

The following hold for the function f: (X, τ, I) ⟶ (Y, σ) and g: (Y, σ, J) ⟶ (Z, μ)

(1) if f is I_π- continuous and g is continuous then g ∘ f is I_π- continuous.

(2) if f is I_π- irresolute and g is I_π- continuous then g ∘ f is I_π- continuous.

(3) If f is surjection, f^{-1}(B$^{*\pi}$) ⊆ [f^{-1}(B)]$^{*\pi}$ for each B ⊆ Y and both f and g are I_π- continuous, then g ∘ f is also I_π- continuous.

Proof:

(1) Let H be a open subset of Z. Since g is continuous, g^{-1}(H) is open in Y. Since f is I_π- continuous, f^{-1}(g^{-1}(H)) = $(g \circ f)^{-1}$ is I_π-open in X. Thus g ∘ f is I_π- continuous.

(2) Let H be a open subset of Z. Since g is I_π- continuous, g^{-1}(H) is I_π- open in Y. Since f is I_π-irresolute, f^{-1}(g^{-1}(H)) = $(g \circ f)^{-1}$ is I_π-open in X. Thus g ∘ f is I_π- continuous.

(3) Follows from the Theorem: 4.15

Definition: 4.18

A function f: (X, τ) ⟶ (Y, σ, J) is called I_π-open (I_π- closed) , if for each U ∈ τ (U is closed), f(U) ∈ I_πO(Y)(f(U) is I_π- closed).

Remark: 4.19

I_π- open function and π-open function are independent of each other as shown in the following examples.

Examples: 4.20

Let X = Y = {a, b, c}, τ = {X, φ, {a, {a, b}, {a, c}}, σ = {Y, φ, {a}, {a, b}} and J = {φ, {a}, {b}, {a, b}} on Y. Then the identity function f : (X, τ) ⟶ (Y, σ, J) is I_π- open function but not π-open, because {a, c} ∈ τ but $f(\{a, c\})$ = {a, c} ∉ I_πO(Y).

Example: 4.21

Let X = Y = {a, b, c, d}, τ = {X, φ, {a, b},}{a, b, d}}, σ = {Y, φ, {a}, {b}, {a, b}, {a, b, c}} and J = {φ, {b}} on Y. Then the function f : (X, τ) ⟶ (Y, σ, J) is defined as f(a) = b, f(b) = a and f(c) = c is π-open function but not I_π- open, because {a, b} ∈ τ but $f(\{a, b\})$ = {a, b} is not I_π- open.

Theorem: 4.22

A function f: (X, τ) ⟶ (Y, σ, J) is I_π- open, if and only if for each x ∈ X and each neighbourhood U of x, there exists an I_π- open set W ⊆ Y containing f(x) such that W ⊆ f(U).

Proof:

Suppose that f is I_π- open function. For x ∈ X and each neighbourhood U of x, there exists V ∈ τ such that x ∈ V ⊆ U. Since f is I_π- open, W = f(V) ∈ I_πO(Y) and f(x) ∈ W ⊆ f(U).

Conversely let U be a open set of X. For each x ∈ U, there exists W ∈ I_πO(Y) such that f(x) ∈ W ⊆ f(U). Therefore we obtain f(U) = ∪{W: x ∈ U} and f(U) ∈ I_πO(Y). This shows that f is I_π- open function.

Theorem: 4.23

Let f: (X, τ) ⟶ (Y, σ, J) be an I_π- open function, if W ⊆ Y and F ⊆ X is a closed set containing f^{-1}(W) then there exists an I_π- closed set H ⊆ Y containing W such that f^{-1}(H) ⊆ F.

Proof:

Suppose that f is I_π- open function. Let W be any subset of Y and F be a closed subset of X containing f^{-1}(W). Then X∖ F is open and since f is I_π- open, f(X∖ F) is I_π- open. Hence H = Y ∖ f(X∖ F) is I_π- closed. Then f^{-1}(W) ⊆ F such that W ⊆ H. Moreover we obtain f^{-1}(H) ⊆ F.

Theorem: 4.24

Let f: (X, τ) ⟶ (Y, σ, J) be an I_π- closed function, if W ⊆ Y and F ⊆ X is a open set containing f^{-1}(W) then there exists an I_π- open set H ⊆ Y containing W such that f^{-1}(H) ⊆ F.

Proof: Similar to Theorem: 4.23

Theorem: 4.25

If f: (X, τ) ⟶ (Y, σ, J) is I_π- open, then $f^{-1}(\text{int (B)})^{*\pi} \subseteq f^{-1}((\text{B}))^{*\pi}$ such that f^{-1}(B) is I_π-dense-in-itself for every B ⊆ Y.

Proof:

Follows from Theorem: 4.22

Theorem: 4.26

Let {X_α : α ∈ Δ} be any family of ideal topological spaces. If f: (X, τ, I) ⟶($\prod_{\alpha\in\Delta} X_\alpha$, σ) is an I_π- continuous then $P_\alpha \circ f : X \longrightarrow X_\alpha$ is I_π- continuous for each α ∈ Δ where P_α is the projection of $\prod X_\alpha$ onto X_α.

Proof:

Let f be an I_π- continuous and P_α is be a projection. We prove that $P_\alpha \circ f : X \longrightarrow X_\alpha$ is I_π- continuous for each α ∈ Δ. Consider a fixed α_0 ∈ Δ. Let G_{α_0}be an open set of X_{α_0}. Then $P_{\alpha_0}^{-1}(G_{\alpha_0})$ is an open set in $\prod_{\alpha=\alpha_0} X_\alpha$. Since f is I_π- continuous, $f^{-1}(P_{\alpha_0}^{-1}(G_{\alpha_0}))$ = $(P_{\alpha_0} \circ f)^{-1}(G_{\alpha_0})$ is I_π-open in X. Thus $P_\alpha \circ f$ is an I_π- continuous function.

Theorem: 4.27

For any bijective function f: (X, τ) ⟶ (Y, σ, J) the following are equivalent:

(1) f^{-1} : (Y, σ, J) ⟶ (X, τ) is I_π- continuous

(2) f is I_π- open

(3) f is I_π-closed

Proof:

(1) ⟹ (2)

Let F be a open subset in X. Since f^{-1} is I_π- continuous, then $(f^{-1})^{-1}$(F) = f (F) is I_π-open in Y. Then f is I_π-open.

(2) ⟹ (3)

Let F be a closed subset in X. Then X∖ F is open set. Since f is I_π-open function, f(X∖ F) = X∖ f(F) is I_π-closed set. Then f(F) is I_π-open set. Thus f is I_π-- closed.

(3) ⟹ (1)

Let F be a open subset in X. Then X\ f(F) is closed set. Since f is I_π- closed, then f(X\ F) = X\ f(F) is I_π- closed set. Thus f(F) = $(f^{-1})^{-1}$(F) is I_π-open. Therefore f^{-1} is I_π- continuous.

Theorem: 4.28

If f: (X, τ) ⟶ (Y, σ, J) is I_π-open for each A ⊆ X, f($A^{*\pi}$) ⊆ $[f(A)]^{*\pi}$, then the image of each I_π- open set is I_π-open.

Theorem: 4.29

Let f: (X, τ, I) ⟶ (Y, σ, J) and g: (Y, σ, J) ⟶ (Z, μ, K) be two functions, where I, J and K are ideals on X, Y and Z respectively. Then

(1) if f is open and g is I_π-open then g∘f is I_π-open.

(2) if g∘f is open and g is I_π- continuous injective then f is I_π-open.

(3) If f and g are I_π-open, f is surjective and g($V^{*\pi}$) ⊆ $[g(V)]^{*\pi}$, for each V ⊆ Y then g∘f is I_π-open.

Proof: Obvious

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