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RESEARCH ARTICLE

# SPECIAL RECTANGLES AND JARASANDHA NUMBERS

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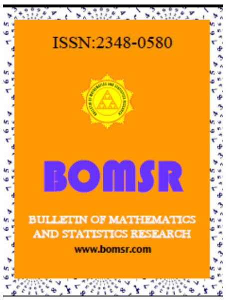


**ABSTRACT**

We search for rectangles with dimensions $x, y$ such that the expression $x^2-3y^2+k^2-A+(k+4y)S-S^2$ is represented by 2-digit & 4-digit Jarasandha numbers. In the above expression $k \geq 0$, $A$ & $S$ denote the area and semi-perimeter of the rectangle respectively. Also, total number of rectangles, each satisfying the above relation is obtained.

**Keywords:** Rectangles, Jarasandha numbers.

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## 1. INTRODUCTION

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. Number theory is one of the largest and oldest branches of mathematics. The main goal of number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In number theory, rectangles have been a matter of interest to various mathematicians. For an extensive variety of fascinating problems, one may refer [1-5]. Apart from the polygonal numbers, we have some more fascinating patterns of numbers namely Jarasandha numbers, nasty numbers and dhuruva numbers. These numbers have been presented in [6-9].

In [10], special pythagorean triangles connected with nasty numbers are obtained. In [11], rectangle in which area is represented as a special polygonal number are given. In [12-13], special pythagorean triangles connected with Jarasandha numbers are obtained. Recently in [14], rectangles in connection between dhuruva numbers of digits 3 & 5 are obtained.

In this communication, we search for rectangles with dimensions $x, y$ such that the expression $x^2-3y^2+k^2-A+(k+4y)S-S^2$ is represented by 2-digit & 4-digit Jarasandha numbers. Also, total number of rectangles, each satisfying the above relation is obtained.

## 2. Notations

$A$ =Area of the rectangle

$S$ = Semi perimeter of the rectangle.

**Definition:**

A rectangle is said to be primitive if $u, v$ are of opposite parity and $\gcd(u,v)=1$, where $x=u+v, y=u-v$ & $u>v>0$.

**3. Jarasandha Numbers**

In our indian epic mahabharatha, we come across a person named 'JARASANDHA'. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and return to life. In fact, he was given life by the two halves of his body. In the field of mathematics, we have numbers exhibiting the same property as Jarasandha.

Consider a number of the form $XC$. This may split as two numbers $X$ and $C$ and if these numbers are added and squared we get the same number $XC$.

(i.e) $XC=(X+C)^2=XC$

**Note:** If $C$ is an n-digit number, then $(X+C)^2=(10^n)(X)+C$

**4. Method of Analysis**

Let $x, y$ be two non-zero distinct positive integers representing the length and breadth of the rectangle. Let $k \geq 0$ be any given integer.

The problem under consideration is to solve the equation,

$x^2-3y^2+k^2-A+(k+4y)S-S^2=$Jarasandha number (1)

Introduction of the linear transformations

$x=u+v \qquad y=u-v$ (2)

in (1) & (2) gives $(u+k)^2-v^2=$Jarasandha number. (3)

**Case 1:**

Consider the 2-digit Jarasandha number 81,

$\therefore (3) \Rightarrow (u+k)^2-v^2=81$

Applying the method of factorization, we have

$u+k=41,15$

$v\quad=40,12$

From the above values, the following results are observed:

**Table 1:**

| $k$ | **Number of rectangles related to 81** | **Observations** |
|---|---|---|
| 0 | 2 | One rectangle is primitive and the other is non-primitive. |
| 1, 2 | 1 | For $k$ =1, the rectangle is non-primitive.<br>For $k$ =2, the rectangle is primitive. |

**Case 2:**

Consider the 4-digit Jarasandha number 2025,

In this case, $(u+k)^2-v^2=2025$,

Applying the method of factorization, we have

$u+k=1013, 339, 205, 117, 75, 53, 51$

$v\quad=1012, 336, 200, 108, 60, 28, 24$

From the above values, the following results are observed:

**Table 2:**

| $k$ | **Number of rectangles related to 2025** | **Observations** |
|---|---|---|
| 0 | 7 | 2 rectangles are primitive and the remaining 5 are non-primitive. |
| 1, 2 | 6 | For $k$ =1, all the rectangles are non-primitive.<br>For $k$ =2, all the rectangles are primitive. |
| 3, 4 | 5 | For $k$ =3, all the rectangles are non-primitive.<br>For $k$ =4, One rectangle is non-primitive and the remaining 4 are primitive. |
| 5-8 | 4 | For $k$ =5,7 all the rectangles are non-primitive.<br>For $k$ =6, One rectangle is primitive and the remaining 3 are non-primitive.<br>For $k$ =8, all the rectangles are primitive. |
| 9-14 | 3 | For $k$ =9,11,13 all the rectangles are non-primitive.<br>For $k$ =10, One rectangle is non-primitive and the remaining 2 are primitive.<br>For $k$ =12, One rectangle is primitive and the remaining 2 are non-primitive.<br>For $k$ =14, all the rectangles are primitive. |
| 15-24 | 2 | For $k$ =15,17,18,19,21,23 all the rectangles are non-primitive.<br>For $k$ =16,20,22 all the rectangles are primitive.<br>For $k$ =24, One rectangle is primitive and other is non-primitive. |
| 25, 26 | 1 | For $k$ =25, the rectangle is non-primitive.<br>For $k$ =26, the rectangle is primitive. |

**Case 3:**

Consider the 4-digit Jarasandha number 3025,

For this choice, we have,

$$u+k = 1513, 305, 143, 73$$

$$v = 1512, 300, 132, 48$$

From the above values, the following results are observed:

**Table 3:**

| $k$ | **Number of rectangles related to 3025** | **Observations** |
|---|---|---|
| 0 | 4 | 2 rectangles are primitive and the remaining 2 are non-primitive. |
| 1-4 | 3 | For $k$ =1,3 all the rectangles are non-primitive.<br>For $k$ =2, 1 rectangle is primitive and the remaining 2 are non-primitive.<br>For $k$ =4, 1 rectangle is non-primitive and the remaining 2 are primitive. |

| | | |
|---|---|---|
| 5-10 | 2 | For $k$ =5,7,9 all the rectangles are non-primitive.<br>For $k$ =6, all the rectangles are primitive.<br>For $k$ =8,10 One rectangle is non-primitive and the other is primitive. |
| 11-24 | 1 | For $k$ =11,13,15,16,17,19,21,22,23 the rectangle is non-primitive.<br>For $k$ =12,14,18,20,24 the rectangle is primitive. |

**Case 4:**

Consider the 4-digit Jarasandha number 9801,

Applying the method of factorization, we have

$$u+k = 4901, 1635, 549, 451, 195, 165, 101$$

$$v \quad = 4900, 1632, 540, 440, 168, 132, 20$$

From the above values, the following results are observed:

**Table 4:**

| $k$ | **Number of rectangles related to 9801** | **Observations** |
|---|---|---|
| 0 | 7 | 2 rectangles are primitive and the remaining 5 are non-primitive. |
| 1, 2 | 6 | For $k$ =1, all the rectangles are non-primitive.<br>For $k$ =2, all the rectangles are primitive. |
| 3-8 | 5 | For $k$ =3,5,6,7 all the rectangles are non-primitive.<br>For $k$ =4, One rectangle is non-primitive and the remaining 4 are primitive.<br>For $k$ =8, all the rectangles are primitive. |
| 9, 10 | 4 | For $k$ =9, all the rectangles are non-primitive.<br>For $k$ =10, all the rectangles are primitive. |
| 11-26 | 3 | For $k$ =11,13,15,17,19,21,23,25 all the rectangles are non-primitive.<br>For $k$ =12,18,24 One rectangle is primitive and the remaining 2 are non-primitive.<br>For $k$ =14, all the rectangles are primitive.<br>For $k$ =16,20,22,26 One rectangle is non-primitive and the remaining 2 are primitive. |
| 27-32 | 2 | For $k$ =27,29,31 all the rectangles are non-primitive.<br>For $k$ =28,32 all the rectangles are primitive.<br>For $k$ =30, One rectangle is primitive and other is non-primitive. |
| 33-80 | 1 | For $k$ =33,35,36,37,39,41,43,45,46,47,49,51,53,55,56,57,59,61,63,65,66,67,69,71, 73,75,76,77,79 the rectangle is non-primitive.<br>For $k$ =34,38,40,42,44,48,50,52,54,58,60,62,64,68,70,72,74,78,80 the rectangle is primitive. |

## 5. Remarkable Observations

For all the cases, we have

1. $S^2 - 4A$ & $Sx - A$ are perfect squares.
2. $\frac{3}{2}S^2 - 6A$ is a Nasty number.
3. For the odd values of $k$, all the rectangles are non-primitive.

**6. Conclusion**

To conclude, one may search for the connections between the rectangles and Jarasandha numbers of higher order and other number patterns.

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