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RESEARCH ARTICLE

CONSTRUCTION OF CONTROL CHARTS BASED ON SIX SIGMA INITIATIVES FOR MEAN USING RANGE

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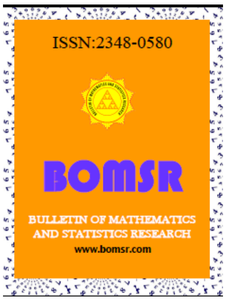

ABSTRACT

A control chart is a statistical device used for the study and control of repetitive process. W.A. Shewhart (1931) of Bell Telephone Laboratories suggested control charts based on the 3 sigma limits. Now the companies in developed and developing countries started applying Six Sigma initiatives in their manufacturing process, which results in lesser number of defects. The companies practicing Six Sigma initiatives are expected to produce 3.4 or less number of defects per million opportunities, a concept suggested by Motorola (1980). If the companies practicing Six Sigma initiatives use the control limits suggested by Shewhart, then no point fall outside the control limits because of the improvement in the quality of the process. In this paper an attempt is made to construct a control chart based on six sigma initiatives for x bar chart using range specially designed for the companies applying Six Sigma initiatives in their organization. Suitable tables are also constructed and presented for the engineers to take quick decisions.

Keywords: Control Chart, Process control, Six Sigma, Six Sigma Quality Level.

1. INTRODUCTION

Radhakrishnan and Balamurugan (2009a, 2009b, 2009c, 2010a, 2010b, 2010c, 2010d, 2010e, 2010f, 2010g, 2011a, 2011b, 2011c, 2011d and 2011e) constructed Control Charts based on Six Sigma Initiatives for Number of Defects, Mean, Average Fraction Defectives, Number of Defectives, Exponentially Weighted Moving Average, Fraction Defectives, Mean using Standard Deviation, Number of Defects - Average Fraction Defectives, Standard Deviation, Average Fraction Defectives per Multiple Units, Standard Deviation with Variable Sample Size, Range, Moving Average, Weightage and Fraction Defectives with varying sample size. The Control Charts originated by W.A. Shewhart (1931) was based on 3 – Sigma control limits. If the same charts are used for the products of the companies which adopt six sigma initiatives in their process, then no point will fall outside the control limits because of the improvement in the quality. So a separate Control Chart is required to monitor the outcomes of the companies, which adopt Six Sigma Initiatives.

2. Concepts and Terminologies

2.1 Upper Specification Limit (USL)

It is the greatest amount in which a process or product is within the acceptable performance limits.

2.2 Lower Specification Limit (LSL)

It is the smallest amount in which a process or product is within the acceptable performance limits.

2.3 Tolerance Level (TL)

It is the difference between USL and LSL, TL = USL-LSL

2.4 Process Capability (C_p)

This is the ratio of tolerance level to six times standard deviation of the process.

$$c_p = \frac{TL}{6\sigma} = \frac{USL - LSL}{6\sigma}$$

2.5 Subgroup Size (n)

In order to make control chart analysis effective, it is essential to pay due regard to the rational selection of the subgroups. It is the choice of the sample size n and the frequency of sampling.

It is also the number of observed values in any given sample or subgroup.

2.6 Quality Control Constant ($A_{6\sigma}$)

To construct the Six Sigma based control limits the Quality Control constants such as are introduced in this thesis.

$$A_{6\sigma} = \frac{4.831}{\sqrt{n}}$$

3. Conditions for Application

- Human involvement should be less in the manufacturing process
- The company adopts six sigma quality initiatives in its processes

4. Construction of six sigma based control chart for mean using range

In this section a procedure to construct a six sigma based control chart for mean using range. The Tables – A and B are also constructed and presented for the engineers to take quick decisions.

Fix the tolerance level (TL) and process capability (C_P) to determine the process standard deviation ($\sigma_{6\sigma}$). Apply the value of $\sigma_{6\sigma}$ in the control limits $\bar{\bar{X}} \pm A_{6\sigma}\sigma_{6\sigma}$, to get the Six Sigma based control limits for Mean using Range. The value of $A_{6\sigma}$ is obtained using $p(\text{z} \le \text{z}_{6\sigma}) = 1 - \alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard normal variate. For a specified TL and C_P of the process, the value of σ (termed as $\sigma_{6\sigma}$) is calculated from $c_p = \frac{TL}{6\sigma}$ presented in Table – A for various combinations of TL and C_P. Further the value of $A_{6\sigma}$ is also obtained using the procedure given above and are presented in Table – B, for different values of n. The six sigma based control limits for mean using range are constructed as

$$\begin{aligned} &\text{UCL}_{6\sigma} &&= \bar{\bar{X}} + A_{6\sigma}\sigma_{6\sigma} \\ &\text{Central Line CL}_{6\sigma} &&= \bar{\bar{X}} \\ &\text{LCL}_{6\sigma} &&= \bar{\bar{X}} - A_{6\sigma}\sigma_{6\sigma} \end{aligned}$$

where $\sigma_{6\sigma}$ is in place of $\frac{\overline{R}}{d_2}$ in 3 – Sigma control limits.

4.1 Example

The example provided by Gupta & Kapoor (2001, Page No. 1.19) is considered here. The following data are the results of overall heights (inches) of fragmentation Bomb bases for samples of five measurements.

Table 1.1: Overall heights of fragmentation Bomb bases

Group No.	Mean $\overline{X}$	Range (R)	Group No.	Mean $\overline{X}$	Range (R)
1	0.8372	0.010	11	0.8380	0.006
2	0.8324	0.009	12	0.8322	0.002
3	0.8318	0.008	13	0.8356	0.013
4	0.8344	0.004	14	0.8322	0.005
5	0.8346	0.005	15	0.8404	0.008
6	0.8332	0.011	16	0.8372	0.011
7	0.8340	0.009	17	0.8282	0.006
8	0.8344	0.003	18	0.8346	0.006
9	0.8308	0.002	19	0.8360	0.004
10	0.8350	0.006	20	0.8374	0.006

$$n = 5, \overline{\overline{X}} = 0.83398 \, and \, \overline{R} = 0.00665$$

*4.1a Construction of control limits (3 – Sigma) for mean using r**ange***

The 3 – Sigma control limits suggested by Shewhart (1931) for Mean using Range are

$$\overline{\overline{X}} \pm A_2 \overline{R}$$

$$UCL_{3\sigma} = \overline{\overline{X}} + A_2\overline{R} = 0.83398 + \left(0.58 \times 0.00665\right) = 0.8378$$

$$\text{Central line } CL_{3\sigma} = \overline{\overline{X}} = 0.83398$$

$$LCL_{3\sigma} = \overline{\overline{X}} - A_2\overline{R} = 0.83398 - \left(0.58 \times 0.00665\right) = 0.8301$$

(from the Table A, $A_2 = 0.58$ for sample size n =5)

From the resulting Figure 3.5.1, it is clear that the process does not exhibit statistical control since the Group numbers 11 and 15 goes above the upper control limit and the Group number 17 goes below the lower control limit.

4.*1b Construction of six sigma based control limits for mean using r**ange***

For a given TL = 0.011 (USL - LSL = 0.013 - 0.002) & C_P = 1.5, it is found from the Table – A (APPENDIX I) that the value of $\sigma_{6\sigma}$ is 0.0012. The value of $A_{6\sigma}$ is obtained from Table – B (APPENDIX II) for n = 5 as 2.16 and the Six Sigma based control limits control limits for Mean using Range are $\overline{\overline{X}} \pm \left(2.16 \times \sigma_{6\sigma}\right)$ with

$$UCL_{6\sigma} = \overline{\overline{X}} + A_{6\sigma}\sigma_{6\sigma} = 0.83398 + \left(2.16 \times 0.0012\right) = 0.8366$$

$$\text{Central line } CL_{6\sigma} = \overline{\overline{X}} = 0.83398$$

$$LCL_{6\sigma} = \overline{\overline{X}} - A_{6\sigma}\sigma_{6\sigma} = 0.83398 - \left(2.16 \times 0.0012\right) = 0.8314$$

From the resulting Figure 4.1.1, it is clear that the process does not exhibit statistical control since the Group numbers 1, 11, 15, 16 and 20 goes above the upper control limit and the Group numbers 9 and 17 goes below the lower control limit.

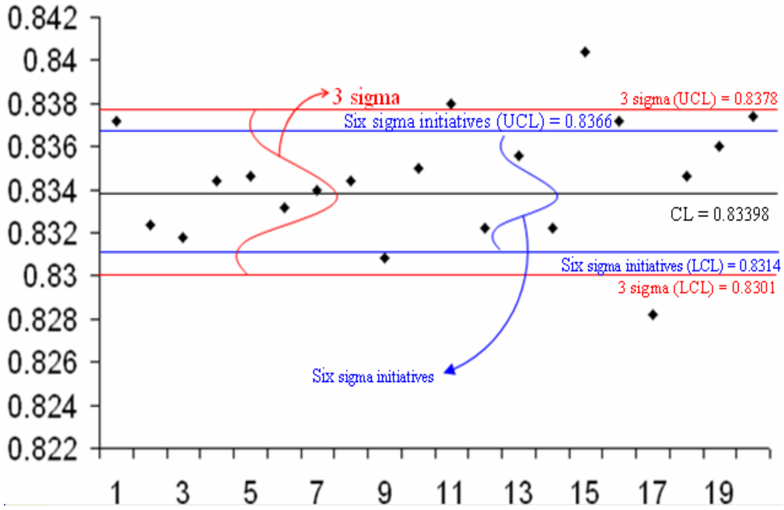

Figure 4.1.1: Comparison of the process: 3σ limits and six sigma based control limits

5. Conclusion

In this paper, a procedure is given to construct a control chart based on six sigma initiatives for the mean using range with an example. It is found that the process was not in control even when Six Sigma initiatives are adopted. It is very clear from the comparison that when the process is centered with reduced variation many points fall outside the control limits than the 3 sigma control limits, which indicate that the process is not in the level it was expected. So a correction in the process is very much required to reduce the variations. The charts suggested in this paper will be very useful for the companies practicing Six Sigma initiatives in their process. These charts will replace the existing Shewhart (1931) control charts in future when all the companies started implementing Six Sigma Initiatives in their organization.

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Table A: $\sigma_{6\sigma}$ Values for a specified C_p and TL

TL / C_p	0.0001	…….	1	2	…….	50	100	200
1	0	…….	0.17	0.33	…….	8.33	16.67	33.33
1.1	0	…….	0.15	0.30	…….	7.58	15.15	30.30
1.2	0	…….	0.14	0.28	…….	6.94	13.89	27.78
1.3	0	…….	0.13	0.27	…….	6.41	12.82	25.64
1.4	0	…….	0.12	0.24	…….	5.95	11.90	23.81
1.5	0	…….	0.11	0.22	…….	5.56	11.11	22.22
1.6	0	…….	0.10	0.21	…….	5.21	10.42	20.83
1.7	0	…….	0.10	0.20	…….	4.90	9.80	19.61
1.8	0	…….	0.09	0.19	…….	4.63	9.26	18.52
1.9	0	…….	0.09	0.18	…….	4.39	8.77	17.54
2	0	…….	0.08	0.17	…….	4.17	8.33	16.67
2.1	0	…….	0.08	0.16	…….	3.97	7.94	15.87
2.2	0	…….	0.08	0.15	…….	3.79	7.58	15.15
2.3	0	…….	0.07	0.14	…….	3.62	7.25	14.49
2.4	0	…….	0.07	0.14	…….	3.47	6.94	13.89
2.5	0	…….	0.07	0.13	…….	3.33	6.67	13.33

Table B: $A_{6\sigma}$ Values for various sample size

Sample size (n)	$A_{6\sigma}$
2	3.42
3	2.79
. . .	. . .
100	1.83
101	1.71
9	1.61
10	1.53
. . .	. . .
790	0.17
800	0.17
1000	0.15