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RESEARCH ARTICLE

ANTI S-FUZZY SUBSEMIRINGS OF A SEMIRING

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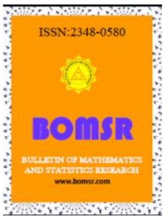

ABSTRACT

In this paper, we made an attempt to study the algebraic nature of an anti S-fuzzy subsemiring of a semiring.

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KEY WORDS: Fuzzy set, anti S-fuzzy subsemiring, pseudo anti S-fuzzy coset.

INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (*R ; + ; .*). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (*R ; + ; .*) share the same properties as a ring except that (*R ; +*) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. After the introduction of fuzzy sets by L.A.Zadeh[11], several researchers explored on the generalization of the concept of fuzzy sets. The notion of anti fuzzy left h-ideals in hemiring was introduced by Akram.M and K.H.Dar [1]. The notion of homomorphism and anti-homomorphism of fuzzy and anti-fuzzy ideal of a ring was introduced by N.Palaniappan & K.Arjunan[6]. In this paper, we introduce the some Theorems in anti S-fuzzy subsemiring of a semiring.

1.PRELIMINARIES

1.1 Definition: A S-norm is a binary operation S: $[0, 1]\times[0, 1] \to [0, 1]$ satisfying the following requirements;

(i) 0 S x = x, 1 S x = 1 (boundary condition)

(ii) x S y = y S x (commutativity)

(iii) x S (y S z) = (x S y) S z (associativity)

(iv) if $x \leq y$ and $w \leq z$, then $x \text{ S } w \leq y \text{ S } z$ (monotonicity).

1.2 Definition: Let X be a non-empty set. A **fuzzy subset** A of X is a function A : $X \to [0, 1]$.

1.3 Definition: Let (R , + , .) be a semiring. A fuzzy subset A of R is said to be an anti S-fuzzy subsemiring(anti fuzzy subsemiring with respect to S-norm) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) \le S(\ \mu_A(x), \mu_A(y)\ )$,

(ii) $\mu_A(xy) \le S(\ \mu_A(x), \mu_A(y)\ )$, for all x and y in R.

1.4 Definition: Let A and B be fuzzy subsets of sets G and H, respectively. The anti-product of A and B, denoted by A×B, is defined as $A\times B = \{\langle\ (x, y\ ), \mu_{A\times B}(x,y)\ \rangle$ / for all x in G and y in H }, where $\mu_{A\times B}(x, y) = \max\{\ \mu_A(x), \mu_B(y)\ \}$.

1.5 Definition: Let A be a fuzzy subset in a set S, the anti-strongest fuzzy relation on S, that is a fuzzy relation on A is V given by $\mu_V(x, y) = \max\{\ \mu_A(x), \mu_A(y)\ \}$, for all x and y in S.

1.6 Definition: Let (R, +, .) and (R^l, +, .) be any two semirings. Let $f : R \to R^l$ be any function and A be an anti S-fuzzy subsemiring in R, V be an anti S-fuzzy subsemiring in f (R) = R^l, defined by $\mu_V(y) = \inf_{x\in f^{-1}(y)} \mu_A(x)$, for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.7 Definition: Let A be an anti S-fuzzy subsemiring of a semiring (R, +, .) and a in R. Then the pseudo anti S-fuzzy coset $(aA)^p$ is defined by $(\ (a\mu_A)^p\ )(x) = p(a)\mu_A(x)$, for every x in R and for some p in P.

2. PROPERTIES OF ANTI S-FUZZY SUBSEMIRING OF A SEMIRING

2.1 Theorem: Union of any two anti S-fuzzy subsemiring of a semiring R is an anti S-fuzzy subsemiring of R.

Proof: Let A and B be any two anti S-fuzzy subsemirings of a semiring R and x and y in R. Let $A = \{\ (\ x, \mu_A(x)\ )\ /\ x\in R\}$ and $B = \{(\ x, \mu_B(x)\ )\ /\ x\in R\}$ and also let $C = A\cup B = \{\ (\ x, \mu_C(x)\ )\ /\ x\in R\}$, where $\max\{\ \mu_A(x), \mu_B(x)\ \} = \mu_C(x)$. Now, $\mu_C(x+y) \le \max\{\ S(\ \mu_A(x), \mu_A(y)\ ), S(\ \mu_B(x), \mu_B(y)\ )\ \} \le S(\ \mu_C(x), \mu_C(y)\ )$. Therefore, $\mu_C(x+y) \le S(\ \mu_C(x), \mu_C(y)\ )$, for all x and y in R. And, $\mu_C(xy) \le \max\{\ S(\ \mu_A(x), \mu_A(y)\ ), S(\ \mu_B(x), \mu_B(y)\ )\ \} \le S\ (\ \mu_C(x), \mu_C(y)\ )$. Therefore, $\mu_C(xy) \le S\ (\mu_C(x), \mu_C(y)\ )$, for all x and y in R. Therefore C is an anti S-fuzzy subsemiring of a semiring R.

2.2 Theorem: If A and B are any two anti S-fuzzy subsemirings of the semirings R_1 and R_2 respectively, then anti-product A×B is an anti S-fuzzy subsemiring of $R_1\times R_2$.

Proof: Let A and B be two anti S-fuzzy subsemirings of the semirings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1\times R_2$. Now, $\mu_{A\times B}[\ (x_1, y_1) + (x_2, y_2)\ ] \le \max\{\ S(\ \mu_A(x_1), \mu_A(x_2)\ ), S(\ \mu_B(y_1), \mu_B(y_2)\ )\ \} \le S\ (\ \mu_{A\times B}(x_1, y_1), \mu_{AxB}(x_2, y_2)\ )$. Therefore, $\mu_{A\times B}[(x_1, y_1) + (x_2, y_2)\ ] \le S(\ \mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\ )$. Also, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] \le \max\{\ S(\ \mu_A(x_1), \mu_A(x_2)\ ), S(\ \mu_B(y_1), \mu_B(y_2)\ )\ \} \le S(\ \mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\ )$. Therefore, $\mu_{A\times B}[(x_1, y_1)(x_2, y_2)] \le S(\ \mu_{AxB}(x_1, y_1), \mu_{AxB}(x_2, y_2)\ )$. Hence A×B is an anti S-fuzzy subsemiring of semiring of $R_1\times R_2$.

2.3 Theorem: Let A be a fuzzy subset of a semiring R and V be the anti-strongest fuzzy relation of R. If A is an anti S-fuzzy subsemiring of R, then V is an anti S-fuzzy subsemiring of R×R.

Proof: Suppose that A is an anti S-fuzzy subsemiring of a semiring R. Then for any $x = (x_1, x_2\ )$ and $y = (y_1, y_2\ )$ are in R×R. We have, $\mu_V(x+y) = \max\{\mu_A(x_1+ y_1), \ \mu_A(x_2+y_2)\ ) \le \max\{\ S(\ \mu_A(x_1), \mu_A(y_1)\ ), S(\ \mu_A(x_2), \mu_A(y_2)\ )\ \} \le S(\ \mu_V(x_1, x_2), \ \mu_V(y_1, y_2)\ ) = S(\ \mu_V(x), \mu_V(y)\ )$. Therefore, $\mu_V(x+y) \le S(\ \mu_V(x), \mu_V(y)\ )$, for all x and y in R×R. And, $\mu_V(xy) = \max\{\mu_A(x_1y_1), \mu_A(x_2y_2)\ \} \le \max\{\ S(\ \mu_A(x_1), \mu_A(y_1)\ ), S(\ \mu_A(x_2), \mu_A(y_2)\ )\ \} \le S(\ \mu_V(x_1, x_2), \mu_V(y_1, y_2)\ ) = S(\ \mu_V(x), \mu_V(y)\ )$. Therefore, $\mu_V(xy) \le S(\ \mu_V(x), \mu_V(y)\ )$, for all x and y in R×R. This proves that V is an anti S-fuzzy subsemiring of R×R.

2.4 Theorem: If A is an anti S-fuzzy subsemiring of a semiring (R, +, .), then $H = \{x\ /\ x\in R: \mu_A(x) = 0\ \}$ is either empty or is a subsemiring of R.

Proof: It is trivial.

2.5 Theorem: Let A be an anti S-fuzzy subsemiring of a semiring (R, +, .). If $\mu_A(x+y) = 1$, then either $\mu_A(x) = 1$ or $\mu_A(y) = 1$, for all x and y in R.

Proof: It is trivial.

2.6 Theorem: Let A be an anti S-fuzzy subsemiring of a semiring (R, +, .), then the pseudo anti S-fuzzy coset $(aA)^p$ is an anti S-fuzzy subsemiring of a semiring R, for every a in R.

Proof: Let A be an anti S-fuzzy subsemiring of a semiring R. For every x and y in R, we have, $((a\mu_A)^p)(x+y) \le p(a)S(\mu_A(x), \mu_A(y)) \le S(p(a)\mu_A(x), p(a)\mu_A(y)) = S(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore, $((a\mu_A)^p)(x+y) \le S((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Now, $((a\mu_A)^p)(xy) \le p(a)S(\mu_A(x), \mu_A(y)) \le S(p(a)\mu_A(x), p(a)\mu_A(y)) = S(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Therefore, $((a\mu_A)^p)(xy) \le S(((a\mu_A)^p)(x), ((a\mu_A)^p)(y))$. Hence $(aA)^p$ is an anti S-fuzzy subsemiring of a semiring R.

2.7 Theorem: Let (R, + , .) and (R^l, +, .) be any two semirings. The homomorphic image of an anti S-fuzzy subsemiring of R is an anti S-fuzzy subsemiring of R^l.

Proof: Let $f : R \to R^l$ be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x)f(y), for all x and y in R. Let V = f(A), where A is an anti S-fuzzy subsemiring of R. Now, for f(x), f(y) in R^l, $\mu_v(f(x)+f(y)) \le \mu_A(x+y) \le S(\mu_A(x), \mu_A(y))$, which implies that $\mu_v(f(x)+f(y)) \le S(\mu_v(f(x)), \mu_v(f(y)))$. Again, $\mu_v(f(x)f(y)) \le \mu_A(xy) \le S(\mu_A(x), \mu_A(y))$, which implies that $\mu_v(f(x)f(y)) \le S(\mu_v(f(x)), \mu_v(f(y)))$. Hence V is an anti S-fuzzy subsemiring of R^l.

2.8 Theorem: Let (R, +, .) and (R^l, +, .) be any two semirings. The homomorphic preimage of an anti S-fuzzy subsemiring of R^l is an anti S-fuzzy subsemiring of R.

Proof: Let V = f(A), where V is an anti S-fuzzy subsemiring of R^l. Let x and y in R. Then, $\mu_A(x+y) = \mu_v(f(x+y)) \le S(\mu_v(f(x)), \mu_v(f(y))) = S(\mu_A(x), \mu_A(y))$, which implies that $\mu_A(x+y) \le S(\mu_A(x), \mu_A(y))$. Again, $\mu_A(xy) = \mu_v(f(xy)) \le S(\mu_v(f(x)), \mu_v(f(y))) = S(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(xy) \le S(\mu_A(x), \mu_A(y))$. Hence A is an anti S-fuzzy subsemiring of R.

2.9 Theorem: Let (R, +, .) and (R^l, +, .) be any two semirings. The anti-homomorphic image of an anti S-fuzzy subsemiring of R is an anti S-fuzzy subsemiring of R^l.

Proof: Let $f : R \to R^l$ be an anti-homomorphism. Then, f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x), for all x and y in R. Let V = f(A), where A is an anti S-fuzzy subsemiring of R. Now, for f(x), f(y) in R^l, $\mu_v(f(x)+f(y)) \le \mu_A(y+x) \le S(\mu_A(y), \mu_A(x)) = S(\mu_A(x), \mu_A(y))$ which implies that $\mu_v(f(x)+f(y)) \le S(\mu_v(f(x)), \mu_v(f(y)))$. Again, $\mu_v(f(x)f(y)) \le \mu_A(yx) \le S(\mu_A(y), \mu_A(x)) = S(\mu_A(x), \mu_A(y))$ which implies that $\mu_v(f(x)f(y)) \le S(\mu_v(f(x)), \mu_v(f(y)))$. Hence V is an anti S-fuzzy subsemiring of R^l.

2.10 Theorem: Let (R, + , .) and (R^l, +, .) be any two semirings. The anti-homomorphic preimage of an anti S-fuzzy subsemiring of R^l is an anti S-fuzzy subsemiring of R.

Proof: Let V = f(A), where V is an anti S-fuzzy subsemiring of R^l. Let x and y in R.

Then, $\mu_A(x+y) = \mu_v(f(x+y)) \le S(\mu_v(f(y)), \mu_v(f(x))) = S(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(x+y) \le S(\mu_A(x), \mu_A(y))$. Again, $\mu_A(xy) = \mu_v(f(xy)) \le S(\mu_v(f(y)), \mu_v(f(x))) = S(\mu_A(x), \mu_A(y))$ which implies that $\mu_A(xy) \le S(\mu_A(x), \mu_A(y))$. Hence A is an anti S-fuzzy subsemiring of R.

REFERENCES

[1]. Akram . M and K.H.Dar, On anti fuzzy left h- ideals in hemirings, International Mathematical Forum, 2(46); 2295 – 2304, 2007.

[2]. Anitha. N and Arjunan. K, Homomorphism in Intuitionistic fuzzy subhemirings of a hemiring, International J.of.Math. Sci.& Engg. Appls.(IJMSEA), Vol.4 (V); 165 – 172, 2010.

[3]. Anthony.J.M. and H Sherwood, fuzzy groups Redefined, Journal of mathematical analysis and applications, 69; 124 -130, 1979.

[4]. Asok Kumer Ray, On product of fuzzy subgroups, fuzzy sets and sysrems, 105; 181-183, 1999.

[5]. Biswas. R, Fuzzy subgroups and anti-fuzzy subgroups, fuzzy sets and systems, 35; 121-124, 1990.

[6]. Palaniappan. N & K. Arjunan, The homomorphism, anti homomorphism of a fuzzy and an anti-fuzzy ideals of a ring, Varahmihir Journal of Mathematical Sciences, 6(1); 181-006, 2008.

[7]. Palaniappan. N & K. Arjunan, Operation on fuzzy and anti fuzzy ideals , Antartica J. Math ., 4(1); 59-64, 2007.

[8]. Palaniappan. N & K.Arjunan, Some properties of intuitionistic fuzzy subgroups, Acta Ciencia Indica, Vol.XXXIII (2); 321-328, 2007.

[9]. Rajesh Kumar, fuzzy Algebra, University of Delhi Publication Division, Volume 1, 1993.

[10]. Vasantha kandasamy.W.B, Smarandache fuzzy algebra, American research press, Rehoboth, 2003.

[11]. Zadeh . L . A, Fuzzy sets, Information and control, 8; 338-353, 1965.