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RESEARCH ARTICLE

FLUCTUATING FLOW OF A VISCO-ELASTIC FLUID PAST AN INFINITE PLANE, POROUS WALL WITH CONSTANT SUCTION IN SLIP FLOW REGIME

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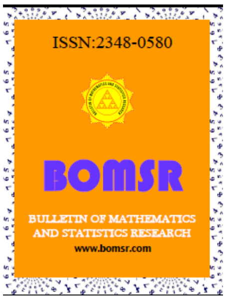

ABSTRACT

We consider the Rivlin Bricksen fluid flow past an infinite plane porous wall with constant suction in slip flow conditions. It is found that, the velocity profile is affected by the rarefaction parameter h_1 in the slip flow, but this is not same in case of no slip flow condition. If $h_1$increases the velocity profile asymptotically approaches the main stream. The main stream fluctuations cause the fluctuations in the skin friction. The skin friction has a phase lead over the main stream fluctuations as in the no slip flow. If h_1 is moderately high, the skin friction fluctuates in the same phase with the main stream. When rarefaction parameter $h_1 \to \infty$, certain member of the class of transient velocity profiles are of the separation type.

1.1 INTRODUCTION

In recent years, the study of flows on the presence of porous walls has been gained importance in technology as well as in biophysics. Some of the applications of these problems are transpiration cooling, boundary layer control and gaseous diffusion. A considerable contribution has been done by Berman A.S [1,2,3], who introduced an appropriate stream function, for the flow of an incompressible viscous fluid, in a channel with porous walls. Extension work have been made by Yuan [4], Sellers [5] and Morduchow [6]. All of them are concerned with the flow of Newtonian fluids in an analogous with porous walls are based on approximate methods for obtaining solutions and injection at the wall of the pipe and rectangular channels. Lighthill [7] has studied the effects of fluctuations of the main stream velocity on the flow of an incompressible fluid past two dimensional bodies. Stuart [8] has obtained the exact solution of the Navier-Stokes equations for such an oscillatory flow over an infinite plane porous wall with constant suction. Many researchers extended the Stuart works for Ravin-Ericksen Visco-elastic fluid [10]. In this paper we studied how Siddappa's results in [9], get modified when his no slip boundary conditions are replaced by the velocity slip conditions.

1.2 Formulation of the problem

Let x denote the distance along the two dimensional infinite, plane porous wall, the equation to the wall being $y = 0, y$- distance normal to it, u and v corresponding components of velocity, t the time, p the pressure, ρ -the density, α- the coefficient of Kinematic viscosity, β- the coefficient of Kinematic viscoelasticity, γ- the coefficient of Kinematic cross-viscosity. Consider the incompressible Rivilson-Ericksen fluid flow past an infinite, plane, porous wall which is independent of x, then the equations of motion becomes:

$$\frac{\partial u}{\partial t} - |v_w|\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \alpha\frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial^2(\frac{\partial u}{\partial t} - v_w\frac{\partial u}{\partial y})}{\partial y^2} \tag{1}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + 2(2\beta + \gamma)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$v = -|v_w| \tag{3}$$

= a suction velocity at the wall

By integrating (2), we get

$$P(x, y, t) = \rho(2\beta + \gamma)\left(\frac{\partial u}{\partial y}\right)^2 g(x, t) \tag{4}$$

where $g(x, t)$ is a function added as a result of integration. Further by consideration (1) as $y \to \infty, u \to U(t)$, and the partial derivatives of u with respect to $y \to 0$, we find that

$$\frac{dU}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5}$$

The equation of our interest is

$$\frac{\partial u}{\partial t} - |v_w|\frac{\partial u}{\partial y} = \frac{dU}{dt} + \alpha\frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial^2(\frac{\partial u}{\partial t} - v_w\frac{\partial u}{\partial y})}{\partial y^2} \tag{6}$$

1.3 Boundary Conditions:

The first order velocity slip condition is

i. $u = \left(\frac{2-f_1}{f_1}\right)L\left(\frac{\partial u}{\partial y}\right) = L_1\left(\frac{\partial u}{\partial y}\right)$ at $y = 0$ (7)

ii. $u = U(t)$ as $y \to \infty$

iii. $\frac{u}{y} = 0$ as $y \to \infty$

where U(t) is the velocity at large distance from the wall and the condition (iii) arises because of the symmetry about the axis of the pipe through the point at infinite. Here f_1 is Maxwell's reflection coefficient $L = \mu(\frac{\pi}{2p\rho})^{\frac{1}{}}$ is the mean free path and is constant for an incompressible fluid and $L_1 = (\frac{2-f_1}{f_1})L$.

1.4 Solution of the problem:

We look for a periodic velocity of the form

$$u = U(t) = U_0\left(1 + \epsilon e^{iwt}\right) \text{ as } y \to \infty \tag{8}$$

$$u(y, t) = U_0[\emptyset_0(y) + \epsilon\, \emptyset_1(y)]e^{iwt} \tag{9}$$

Substituting the expressions (8) and (9) into (6) and equation non-harmonic and harmonic terms in the equation, we have

$$-|v_w|\frac{d\emptyset_0}{dy} = \alpha\frac{d\emptyset_0}{dy^2} - \beta|v_w|\frac{d\emptyset_0}{dy^3} \tag{10}$$

and

$$iw\emptyset_1 - |v_w|\frac{d\emptyset_1}{dy} = iw + \alpha\frac{d^2\emptyset_1}{dy^2} + \beta iw\frac{d^2\emptyset_1}{dy^2} - \beta|v_w|\frac{d^3\emptyset_1}{dy^3} \tag{11}$$

with the boundary conditions

$$\left.\begin{aligned}\emptyset_0 &= L_1\frac{d\emptyset_0}{dy}\\ \emptyset_1 &= L_1\frac{d\emptyset_1}{dy}\end{aligned}\right\}\text{at } y=0 \text{ and } \left.\begin{aligned}\emptyset_0 = \emptyset_1 = 1\\ \frac{d\emptyset_0}{dy}=\frac{d\emptyset_1}{dy}=0\end{aligned}\right\}\text{at } y\to\infty \tag{12}$$

putting $\eta = \frac{|v_w|y}{\alpha}$, equations (10) and (11) and the boundary conditions (12) reduce to

$$k\frac{d^3\emptyset_0}{d\eta^3}+\frac{d^2\emptyset_0}{d\eta^2}+\frac{d\emptyset_0}{d\eta}=0 \tag{13}$$

$$k\frac{d\emptyset_1}{d\eta^3}+(1-ik\lambda)\frac{d^2\emptyset_1}{d\eta^2}+\frac{d\emptyset_1}{d\eta}-i\lambda\emptyset_1=-i\lambda \tag{14}$$

and

$$\left.\begin{aligned}\emptyset_0 &= h_1\frac{d\emptyset_0}{d\eta}\\ \emptyset_1 &= h_1\frac{d\emptyset_1}{d\eta}\end{aligned}\right\}\text{at } \eta=0 \text{ and } \left.\begin{aligned}\emptyset_0 = \emptyset_1 = 1\\ \frac{d\emptyset_0}{dy}=\frac{d\emptyset_1}{dy}=0\end{aligned}\right\}\text{at } \eta\to\infty \tag{15}$$

where

$\lambda = \frac{w\alpha}{|v_w|^2}$; $k=-\beta\frac{|v_w|^2}{\alpha^2}$; $h_1 = \frac{L_1|v_w|}{\alpha}$ =Rarefaction parameter.

The solution of (13) to the conditions (15) is

$$\emptyset_0 = 1-\frac{e^{-a\eta}}{1+ah_1} \tag{16}$$

where $a=\frac{1-\sqrt{1-4k}}{2k}$ for $k\le 0$ and the other solution is

$$\emptyset_0 = 1-\frac{e^{-b\eta}}{1+bh_1}$$

where $b=\frac{1-\sqrt{1-4k}}{2k}$ for $0<k\le 1/4$.

The solution of (14) subjected to the conditions (15) is

$$\emptyset_1 = 1-\frac{e^{-h\eta}}{1+hh_1} \tag{17}$$

Where $h = h_r + ih_1$ is given by the equation

$$kh^3-(1-ik\lambda)h^2+h+i\lambda=0 \tag{18}$$

This is solved numerically.The total velocity component parallel to the wall is given by

$$u=U_0\left[1-\frac{e^{-a\eta}}{1+ah_1}+\epsilon\left(1-\frac{e^{-h\eta}}{1+hh_1}\right)e^{iwt}\right] \tag{19}$$

Thus, the velocity profile is affected by the rarefaction parameter h_1 in the slip flow. This is not the case in no slip flow since $h_1=0$.

If h_1 increases then the velocity (19) becomes $u=U_0[1+\epsilon e^{iwt}]$, which is the main stream velocity. This is not the case in the no slip flow.

In slip flow, because of the boundary conditions, the shear stress at the wall ζ_0 is proportional to the slip velocity at the wall and is given by

$$\begin{aligned}\zeta_0 &= \rho\alpha\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\rho\alpha}{L_1}(u)_{y=0}\\ &= \rho|v_w|U_0\left[1-\frac{a}{1+ah_1}+\epsilon\left(\frac{h}{1+hh_1}\right)e^{iwt}\right]\\ &= \rho|v_w|U_0\left[1-\frac{a}{1+ah_1}+\epsilon|H|e^{i(wt+\theta)}\right]\end{aligned} \tag{20}$$

Where

$$|H| = \left|\frac{h}{1+hh_1}\right| = \left|\frac{h_r+ih_1}{1+(h_r+ih_1)h_1}\right| = \frac{[h_1^2+\{h_r+h_1(h_1^2+h_r^2)\}^2]^{\frac{1}{2}}}{(1+h_rh_1)^2+h_1^2h_r^2} \tag{21}$$

and $$\theta=\tan^{-1}\left\{\frac{h_1}{h_r+h_1(h_1^2+h_r^2)}\right\} \tag{22}$$

It is clear from (20) that the main stream fluctuating cause the fluctuations in the skin friction. From (21) and (22), for moderately high h_1, $|H|$ and θ decrease irrespective of

λ (small/large). Thus, the skin friction ζ_0 decreases and ultimately tends to zero as h_1 tends to infinite. Hence certain members of the class of the transient velocity profiles are of the separation type. From the equation (20), (22) the akin friction has a phase lead according as θ is positive or negative over the main stream fluctuation (for certain value of the viscoelastic parameter) as in the no slip flow. This becomes insignificant, if h_1 is moderately high and the skin friction fluctuates in same phase with the main stream.

Results and Discussion:

Here we consider the Rivlin Bricksen fluid flow past an infinite plane porous wall with constant suction in slip flow conditions. It is found that, the velocity profile is affected by the rarefaction parameter h_1 in the slip flow, but this is not same in case of no slip flow condition. If h_1 increases the velocity profile asymptotically approaches the main stream. The main stream fluctuations cause the fluctuations in the skin friction. The skin friction has a phase lead over the main stream fluctuations as in the no slip flow. If h_1 is moderately high, the skin friction fluctuates in the same phase with the main stream. When rarefaction parameter $h_1 \rightarrow \infty$, certain member of the class of transient velocity profiles are of the separation type.

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