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RESEARCH ARTICLE

ESTIMATION OF THE PARAMETERS OF THE ZERO-ONE INFLATED POWER SERIES DISTRIBUTIONS

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ABSTRACT

In many sampling involving non negative integer data, the zeros and ones are observed to be significantly higher than the expected assumed model. Such models are called zero-one inflated models. The class of zero-one inflated generalized power series distributions was recently considered and studied due to its empirical needs and application. Members of the class of power series distributions are so many well-known discrete distributions such as; Poisson, Binomial, negative binomials, as well as most of their modified forms. In this paper, estimation of the parameters of class of zero-one inflated power series distributions was considered using the methods of moments and maximum likelihood estimators. The parameters of the zero-one inflated geometric distributions were estimated by these methods as well as estimating its distribution frequencies.

KEY WORDS:Power Series Distributions, Zero-One Inflated Model, Maximum likelihood Estimator, Moments Estimator, Inflated Geometric Distribution.

1. INTRODUCTION

A well-known class of discrete distributions called power series distributions (PSD) has been studied by many researchers due to its member's empirical applications. Well-known discrete distributions such as, the Poisson, binomial, negative binomial and the logarithmic, are members of this class.

A special class of the PSDs arises in the researchers' literature as a statistical model in application situations involving the frequency of the observed zeros is significantly higher than the predicated frequency by the standard distribution. Hence, as a result of mis-specifying the proper statistical model, leads to the so called the zero-inflated distributions.

Recently, models based on zero-inflated power series distributions (ZIPSD) were studied by many researchers. In particular, Gupta et al (1995) studied the structural properties of the ZIPSDs. Patil and Shirke (2007) provided three asymptotic test for testing the parameters of the ZIPSD.

Bhattacharya et al. (2008) considered the ZIPSD as a mixture of a power series distribution and a degenerate distribution at zero. Finally, Edwin (2014) studied the construction and estimation of the ZIPSDs, as well as, considered some its applications.

Alshkaki (2016), introduced an extension to the ZIPSDs, in which not only the number of frequencies with zeros is inflated, but the number of frequencies with ones are also inflated as well, that is, a zero-one inflated power series distributions (ZOIPSD). He studied its structure properties, as well as its relation to the standard and the zero inflated cases.

In this paper, we introduce in Section 2, the definition of the class of the PSDs with some of its members. Then, in Section 3, we introduce the definition of the ZOIPSDs, and its mean, variance, recurrence relation for the moment, moment generating function, and factorial moment generating function, were given in Section 4. Then in Section 5, we consider the method of moments estimator of its parameters, followed by the method of maximum likelihood estimator for its parameters also in Section 6. Finally, empirical estimation of the parameters of the zero-one inflated geometric distribution as well as fitting its frequencies were presented in Section 7.

2. POWER SERIES DISTRIBUTIONS

Let $g(\theta) = \sum_{x\in T} a_x \theta^x$ be a power series for $\theta \epsilon \Omega = \{\theta; 0 < \theta < \omega\}$, where ω is the radius of convergence of $g(\theta)$, and $a_x \geq 0$ for all $x \in T \subseteq I = \{0,1,2,\dots\}$. Then the discrete random variable (rv) X having probability mass function (pmf);

$$P(X = x) = \frac{a_x \theta^x}{g(\theta)} \qquad x \in T \tag{2.1}$$

is said to have a PSD with range T and parameter space Ω.The function $g(\theta)$ is called the series defining function (sdf). We will denote that by writing $X \sim PSD(\theta, g(\theta))$.

The discrete distributions; Poisson, binomial, negative binomial, and the logarithmic series distributions, are well-known member of the PSD's with sdf's; e^θ, $(1+\theta)^n$, $(1-\theta)^{-k}$, and $-\ln(1-\theta)$, respectively, seeAbdulrazak and Patil (1986, 1994) and Johnson et al (2005) for further details.

3. ZERO-ONE INFLATED PSDs

Let $X \sim PSD(\theta, g(\theta))$as given in (2.1) with $T = \{0, 1, 2, \dots\}$, and let $\alpha \in (0,1)$ be an extra proportion added to the proportion of zero of the rv X, and let $\beta \in (0,1)$ be an extra proportion added to the proportion of ones of the rv X, such that $0 < \alpha + \beta < 1$, then the rv Z defined by;

$$P(Z = z) = \begin{cases} \alpha + (1-\alpha-\beta)\dfrac{a_0}{g(\theta)}, & z = 0 \\ \beta + (1-\alpha-\beta)\dfrac{a_1\,\theta}{g(\theta)} & z = 1 \\ (1-\alpha-\beta)\dfrac{a_z\,\theta^z}{g(\theta)} & z = 2, 3, 4, \dots \end{cases} \tag{3.1}$$

and is zero otherwise, is said to have a ZOIPSD, and will denotethat by writing $Z \sim ZOIPSD(\theta, g(\theta); \alpha, \beta)$.

Note that, if $\beta \to 0$, the (3.1) reduces to the form of the ZIPSD. Similarly, the case with $\alpha \to 0$ and $\beta \to 0$, reduces to the standard case of PSD.

4. STRUCTURE PROPERTIES OF THE ZOIPSDs

Let the rv $Z \sim \mathrm{ZOIPSD}(\theta, g(\theta); \alpha, \beta)$, then Alshkaki (2016) showed the following;

$$E(Z) = \beta + (1-\alpha-\beta)\,\theta \frac{g'(\theta)}{g(\theta)} \tag{4.1}$$

$$Var(Z) = \beta(1-\beta) + (1-\alpha-\beta)\theta\left\{\theta\frac{g''(\theta)}{g(\theta)} + (1-2\beta)\frac{g'(\theta)}{g(\theta)} - (1-\alpha)\theta\left[\frac{g'(\theta)}{g(\theta)}\right]^2\right\},$$

$$\mu'_{r+1} = \beta + \theta\frac{d}{d\theta}\mu'_r + \left(\frac{\mu'_1-\beta}{1-\alpha-\beta}\right)(\mu'_r-\beta) \tag{4.2}$$

where μ'_r, for r=1, 2, 3, …, is the rth moment of the rv Z.

The probability generating function $G_Z(s)$, the moment generating function$M_Z(t)$, and the factorialmoment generating function $M_{[Z]}(t)$, are respectively, given by:

$$G_Z(s) = \alpha + \beta s + (1-\alpha-\beta)\frac{g(\theta s)}{g(\theta)}$$

$$M_Z(t) = \alpha + \beta e^t + (1-\alpha-\beta)\frac{g(\theta e^t)}{g(\theta)}$$

$$M_{[Z]}(t) = \alpha + \beta(1+t) + (1-\alpha-\beta)\frac{g(\theta+\theta t)}{g(\theta)}$$

5. MOMENT ESTIMATORS OF THE PARAMETERS OF THE ZOIPSDs

Let us first find the first three distribution moments about the origin for the $ZOIPSD$. From (4.1) we have,

$$\mu'_1 = \beta + (1-\alpha-\beta)\theta\frac{g'(\theta)}{g(\theta)} \tag{5.1}$$

Therefore,

$$\frac{d}{d\theta}\mu'_1 = \frac{d}{d\theta}\left\{\beta + (1-\alpha-\beta)\,\theta\frac{g'(\theta)}{g(\theta)}\right\}$$

$$= (1-\alpha-\beta)\frac{d}{d\theta}\left\{\theta\frac{g'(\theta)}{g(\theta)}\right\}$$

$$= (1-\alpha-\beta)\left\{\theta\,\frac{g''(\theta)}{g(\theta)} - \theta\left[\frac{g'(\theta)}{g(\theta)}\right]^2 + \frac{g'(\theta)}{g(\theta)}\right\} \tag{5.2}$$

Using (5.1) and (5.2) in (4.2), we have that,

$$\mu'_2 = \beta + (1-\alpha-\beta)\theta^2\frac{g''(\theta)}{g(\theta)} - (1-\alpha-\beta)\theta^2\left[\frac{g'(\theta)}{g(\theta)}\right]^2 + (1-\alpha-\beta)\theta\frac{g'(\theta)}{g(\theta)} + \frac{1}{(1-\alpha-\beta)}\left[(1-\alpha-\beta)\theta\frac{g'(\theta)}{g(\theta)}\right]^2,$$

And hence,

$$\mu'_2 = \beta + (1-\alpha-\beta)\theta\frac{g'(\theta)}{g(\theta)} + (1-\alpha-\beta)\theta^2\frac{g''(\theta)}{g(\theta)} \tag{5.3}$$

Similarly, we have that,

$$\mu'_3 = \beta + (1-\alpha-\beta)\theta\left\{\theta\frac{g''(\theta)}{g(\theta)} - \theta\left[\frac{g'(\theta)}{g(\theta)}\right]^2 + 2\frac{g'(\theta)}{g(\theta)} + \theta^2\frac{g'''(\theta)}{g(\theta)} + \theta^2\left(\frac{g'(\theta)}{g(\theta)}\right)\left(\frac{g''(\theta)}{g(\theta)}\right)\right\} \tag{5.4}$$

Let $z_1, z_2, \ldots, z_n$ be a random sample from $ZOIPSD$as given by (3.12), and let,

$$m_k' = \frac{\sum_{i=1}^{n} z_i^k}{n}, \qquad k = 1,2,3.$$

be their sample moments about the origin, then solving the following simultaneous:

$$m_1' = \beta + (1-\alpha-\beta)\theta\frac{g'(\theta)}{g(\theta)}, \tag{5.5}$$

$$m_2' = \beta + (1-\alpha-\beta)\theta\frac{g'(\theta)}{g(\theta)} + (1-\alpha-\beta)\theta^2\frac{g''(\theta)}{g(\theta)} \tag{5.6}$$

$$m_3' = \beta + (1-\alpha-\beta)\theta\left\{\theta\frac{g''(\theta)}{g(\theta)} - \theta\left[\frac{g'(\theta)}{g(\theta)}\right]^2 + 2\frac{g'(\theta)}{g(\theta)} + \theta^2\frac{g'''(\theta)}{g(\theta)} + \theta^2\left(\frac{g'(\theta)}{g(\theta)}\right)\left(\frac{g''(\theta)}{g(\theta)}\right)\right\} \tag{5.7}$$

for θ, α, and β give us the moments estimators for these parameters.

A numerical computation procedure, say, Newton-Raphson method, is needed to find the parameter estimates of θ, α and β using (5.5), (5.6), and (5.7), as will be considered in Section 7.

6. MAXIMUM LIKELIHOOD ESTIMATORS OF THE PARAMETERS OF THE ZOIPSDs

Let $z_1, z_2, \ldots, z_n$ be a random sample from $ZOIPSD$ as given by (3.1), and let for i=1, 2, … n,

$$\alpha_i = \begin{cases} 1 & if\, z_i = 0, \\ 0 & otherwise \end{cases}$$

and

$$\beta_i = \begin{cases} 1 & if\, z_i = 1, \\ 0 & otherwise \end{cases}$$

Then, (3.1) can be written in the following form for i,=1, 2, … , n, and $z_i \in T$,

$$P(Z_i = z_i) = \left\{\alpha + (1-\alpha-\beta)\frac{a_0}{g(\theta)}\right\}^{\alpha_i}\left\{\beta + (1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}\right\}^{\beta_i}\left\{(1-\alpha-\beta)\frac{a_{z_i}\theta^{z_i}}{g(\theta)}\right\}^{1-\alpha_i-\beta_i}$$

Hence, the likelihood function L = L($\theta, \alpha, \beta; z_1, z_2, \ldots, z_n$)will be,

$$L = \prod_{i=1}^{n}\left\{\alpha + (1-\alpha-\beta)\frac{a_0}{g(\theta)}\right\}^{\alpha_i}\left\{\beta + (1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}\right\}^{\beta_i}\left\{(1-\alpha-\beta)\frac{a_{z_i}\theta^{z_i}}{g(\theta)}\right\}^{1-\alpha_i-\beta_i}$$

$$= \left\{\alpha + (1-\alpha-\beta)\frac{a_0}{g(\theta)}\right\}^{n_0}\left\{\beta + (1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}\right\}^{n_1}\prod_{i=1}^{n}\left\{(1-\alpha-\beta)\frac{a_{z_i}\theta^{z_i}}{g(\theta)}\right\}^{c_i}$$

where$c_i = 1 - \alpha_i - \beta_i$, $n_0 = \sum_{i=1}^{n}\alpha_i$ and $n_1 = \sum_{i=1}^{n}\beta_i$. Note that n_0 and n_1 represents, respectively, the number of zeros and the number of ones in the sample. Therefore,

$$\log L = n_0 \log\left\{\alpha + (1-\alpha-\beta)\frac{a_0}{g(\theta)}\right\} + n_1\log\left\{\beta + (1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}\right\} + (n-n_0-n_1)\log(1-\alpha-\beta) + \sum_{i=1}^{n} c_i \log a_{z_i} + \sum_{i=1}^{n} c_i z_i \log\theta + (n-n_0-n_1)\log(g(\theta))$$

It follows that,

$$\frac{\partial}{\partial\alpha}logL=\frac{n_0\left[1-\frac{a_0}{g(\theta)}\right]}{\alpha+(1-\alpha-\beta)\frac{a_0}{g(\theta)}}-\frac{n_1\frac{a_1\theta}{g(\theta)}}{\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}}-\frac{(n-n_0-n_1)}{1-\alpha-\beta} \quad (6.1)$$

And hence,

$$\frac{\partial^2}{\partial\alpha^2}logL=-\frac{n_0\left[1-\frac{a_0}{g(\theta)}\right]^2}{\left[\alpha+(1-\alpha-\beta)\frac{a_0}{g(\theta)}\right]^2}-\frac{n_1\left[\frac{a_1\theta}{g(\theta)}\right]^2}{\left[\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}\right]^2}-\frac{(n-n_0-n_1)}{(1-\alpha-\beta)^2}$$

Therefore, $\frac{\partial^2}{\partial\alpha^2}logL<0$, which indicates that L has a local maximum at α. Similarly,

$$\frac{\partial}{\partial\beta}logL=-\frac{n_0\left[\frac{a_0}{g(\theta)}\right]}{\alpha+(1-\alpha-\beta)\frac{a_0}{g(\theta)}}+\frac{n_1\left[1-\frac{a_1\theta}{g(\theta)}\right]}{\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}}-\frac{(n-n_0-n_1)}{1-\alpha-\beta} \quad (6.2)$$

$$\frac{\partial^2}{\partial\beta^2}logL=-\frac{n_0\left[\frac{a_0}{g(\theta)}\right]^2}{\left[\alpha+(1-\alpha-\beta)\frac{a_0}{g(\theta)}\right]^2}-\frac{n_1\left[1-\frac{a_1\theta}{g(\theta)}\right]^2}{\left[\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}\right]^2}-\frac{(n-n_0-n_1)}{(1-\alpha-\beta)^2}$$

And hence, $\frac{\partial^2}{\partial\beta^2}logL<0$, which indicates that L has a local maximum at β. And finally,

$$\frac{\partial}{\partial\theta}logL=-\frac{n_0(1-\alpha-\beta)a_0\frac{g'(\theta)}{[g(\theta)]^2}}{\alpha+(1-\alpha-\beta)\frac{a_0}{g(\theta)}}+\frac{\sum_{i=1}^{n}c_i z_i}{\theta}-(n-n_0-n_1)\frac{g'(\theta)}{g(\theta)}$$
$$-\frac{n_1(1-\alpha-\beta)a_1\theta\frac{g'(\theta)}{[g(\theta)]^2}}{\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}}+\frac{n_1(1-\alpha-\beta)\frac{a_1}{g(\theta)}}{\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}} \quad (6.3)$$

And,

$$\frac{\partial^2}{\partial\theta^2}logL=-\frac{n_0(1-\alpha-\beta)a_0}{\alpha+(1-\alpha-\beta)\frac{a_0}{g(\theta)}}\left\{\frac{(1-\alpha-\beta)a_0\left(\frac{g'(\theta)}{[g(\theta)]^2}\right)^2}{\alpha+(1-\alpha-\beta)\frac{a_0}{g(\theta)}}+\frac{g''(\theta)}{[g(\theta)]^2}-2\frac{\left[g'(\theta)\right]^2}{[g(\theta)]^3}\right\}-\frac{\sum_{i=1}^{n}c_i z_i}{\theta^2}$$
$$-(n-n_0-n_1)\left\{\frac{g''(\theta)}{g(\theta)}-\left(\frac{g'(\theta)}{g(\theta)}\right)^2\right\}$$
$$-\frac{n_1(1-\alpha-\beta)a_1}{\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}}\left\{\frac{(1-\alpha-\beta)a_1\theta^2\left(\frac{g'(\theta)}{[g(\theta)]^2}\right)^2}{\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}}\right.$$
$$\left.-\frac{(1-\alpha-\beta)a_1\theta}{\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}}\frac{g'(\theta)}{[g(\theta)]^3}-2\theta\frac{\left[g'(\theta)\right]^2}{[g(\theta)]^3}+\theta\frac{g''(\theta)}{[g(\theta)]^2}+\frac{g'(\theta)}{[g(\theta)]^2}\right\}$$
$$-\frac{n_1(1-\alpha-\beta)a_1}{\left[\beta+(1-\alpha-\beta)\frac{a_1\theta}{g(\theta)}\right]^2}\left\{\frac{\beta g'(\theta)+(1-\alpha-\beta)a_1}{[g(\theta)]^2}\right\}$$

Therefore, the local maximum of θ has to be explicitly examined.

Now, let $\frac{\partial}{\partial\alpha}logL = 0$,

$$p_0 = \alpha + (1-\alpha-\beta)\frac{a_0}{g(\theta)}, \tag{6.4}$$

and

$$p_1 = \beta + (1-\alpha-\beta)\frac{a_1\theta}{g(\theta)} \tag{6.5}$$

Then, (6.1) reduces to;

$$0 = \frac{n_0}{p_0} - \frac{n_1}{p_0}\frac{a_0}{g(\theta)} - \frac{n_1}{p_1}\frac{a_1\theta}{g(\theta)} - \frac{n-n_0-n_1}{1-\alpha-\beta}$$

Or equivalently,

$$\frac{n-n_0-n_1}{1-\alpha-\beta} = \frac{n_0}{p_0} - \frac{n_1}{p_0}\frac{a_0}{g(\theta)} - \frac{n_1}{p_1}\frac{a_1\theta}{g(\theta)} \tag{6.6}$$

And hence,

$$\beta = 1-\alpha-A(\theta) \tag{6.7}$$

where,

$$A(\theta) = \frac{n-n_0-n_1}{\frac{1}{p_0}\left[n_0 - n_1\frac{a_0}{g(\theta)}\right] - \frac{1}{p_1}n_1\frac{a_1\theta}{g(\theta)}} \tag{6.8}$$

Similarly, let $\frac{\partial}{\partial\beta}logL = 0$, then, (6.2) reduces to;

$$\frac{n-n_0-n_1}{1-\alpha-\beta} = -\frac{n_0}{p_1}\frac{a_0}{g(\theta)} + \frac{n_1}{p_1} - \frac{n_1}{p_1}\frac{a_1\theta}{g(\theta)} \tag{6.9}$$

Therefore, using (6.6) with (6.9), we have that;

$$-\frac{n_0}{p_1}\frac{a_0}{g(\theta)} + \frac{n_1}{p_1} - \frac{n_1}{p_1}\frac{a_1\theta}{g(\theta)} = \frac{n_0}{p_0} - \frac{n_1}{p_0}\frac{a_0}{g(\theta)} - \frac{n_1}{p_1}\frac{a_1\theta}{g(\theta)}$$

Which reduces to;

$$\frac{1}{p_1}\left[n_1 - n_0\frac{a_0}{g(\theta)}\right] = \frac{1}{p_0}(n_0-n_1)$$

Or equivalently,

$$p_0 = \frac{(n_0-n_1)p_1}{\left[n_1 - n_0\frac{a_0}{g(\theta)}\right]}$$

That is; with the use of (6.4);

$$\alpha + (1-\alpha-\beta)\frac{a_0}{g(\theta)} = \frac{(n_0-n_1)p_1}{\left[n_1 - n_0\frac{a_0}{g(\theta)}\right]}$$

And hence,

$$\beta = \frac{g(\theta)}{a_0}\left[\alpha + (1-\alpha)\frac{a_0}{g(\theta)} - B(\theta)\right] \tag{6.10}$$

Where,

$$B(\theta) = \frac{(n_0-n_1)p_1}{\left[n_1 - n_0\frac{a_0}{g(\theta)}\right]} \tag{6.11}$$

Therefore, (6.10) and (6.7) give,

$$\frac{g(\theta)}{a_0}\left[\alpha+(1-\alpha)\frac{a_0}{g(\theta)}-B(\theta)\right]=1-\alpha-A(\theta)$$

From which we find that,

$$\alpha = B(\theta) - \frac{a_0}{g(\theta)}A(\theta) \tag{6.12}$$

And hence,

$$\beta = 1 - B(\theta) - A(\theta)\left[1-\frac{a_0}{g(\theta)}\right] \tag{6.13}$$

Now, setting $\frac{\partial}{\partial\theta}logL = 0$, then, (6.3) reduces to;

$$\frac{\sum_{i=1}^{n} c_i\, z_i}{\theta} - \frac{n_0}{p_0}(1-\alpha-\beta)a_0\frac{g'(\theta)}{[g(\theta)]^2} - (n-n_0-n_1)\frac{g'(\theta)}{g(\theta)} = \frac{n_1}{p_1}(1-\alpha-\beta)a_1\theta\frac{g'(\theta)}{[g(\theta)]^2} - \frac{n_1}{p_1}(1-\alpha-\beta)\frac{a_1}{g(\theta)} \tag{6.14}$$

From (6.4) and (6.5), we have;

$$p_0 - \alpha = (1-\alpha-\beta)\frac{a_0}{g(\theta)}, \tag{6.15}$$

and

$$p_1 - \beta = (1-\alpha-\beta)\frac{a_1\theta}{g(\theta)} \tag{6.16}$$

Hence, (6.14) can written with the use of (6.15) and (6.16) as,

$$\frac{\sum_{i=1}^{n} c_i\, z_i}{\theta} - \frac{n_0}{p_0}(p_0-\alpha)\frac{g'(\theta)}{g(\theta)} - (n-n_0-n_1)\frac{g'(\theta)}{g(\theta)} = \frac{n_1}{p_1}(p_1-\beta)\frac{g'(\theta)}{g(\theta)} - \frac{n_1}{p_1}(p_1-\beta)\frac{1}{\theta}$$

From which we find, with $m = \sum_{i=1}^{n} c_i\, z_i$, that

$$\theta = \frac{m + \frac{n_1}{p_1}(p_1-\beta)}{\frac{n_1}{p_1}(p_1-\beta) + \frac{n_0}{p_0}(p_0-\ \alpha) + (n-n_0-n_1)}\frac{g(\theta)}{g'(\theta)} \tag{6.17}$$

Therefore, we can use any numerical procedure, say, Newton Rapson method to find $\hat{\theta}$ using (6.17) with α and β given by (6.12) and (6.13), respectively, and estimate p_0 and p_1 initially by their sample estimates, the proportion of zeros and the proportion of ones in the sample, respectively, that is; $\widehat{p_0} = \frac{\sum_{i=1}^{n}\alpha_i}{n}$ and $\widehat{p_1} = \frac{\sum_{i=1}^{n}\beta_i}{n}$.

7. EMPIRICAL ESTIMATION: THE ZERO-ONE INFLATED GEOMETRIC DISTRIBUTION

Table (1) gives the number of households according to the number of rural out-migrants per household in the three types of villages; semi-urban, Remote, and growth center, that was used and studied by Sharma (1987). Edwin (2014) used the zero inflated geometric distribution (ZIGD) to estimate the parameters and its frequencies for this data using both the ME and the MLE.

Table (1): Number of households according to the number of rural out-migrants per household in the three types of villages.

Number of Migrants	Number of Households		
	Semi-Urban	Remote	Growth Centre
0	1042	872	978
1	95	176	154
2	19	59	47
3	10	18	18
4+	5	10	10
Total	1171	1135	1208

Table (2) shows the ML estimates of the zero-inflated geometric distribution (Edwin, 2014) and the zero-one inflated geometric distribution (ZOIGD).

Table (2): The ML estimates of the ZIGD and the ZOIGDfor the number of households according to the number of rural out-migrants per household in the three types of villages.

Number of Migrants	Number of Households								
	Semi-Urban			Remote			Growth Centre		
	Actual Obs.	Exp. ZIGD	Exp. ZOIGD	Actual Obs.	Exp. ZIGD	Exp. ZOIGD	Actual Obs.	Exp. ZIGD	Exp. ZOIGD
0	1042	1049	1042	872	871	875	978	982	976
1	95	79	94	176	178	168	154	146	159
2	19	28	20	59	58	64	47	52	45
3	10	10	8	18	19	19	18	18	17
4+	5	5	7	10	9	9	11	10	11
Total	1171	1171	1171	1135	1135	1135	1208	1208	1208
Model Parameters	θ	0.3517	0.4171	θ	0.3253	0.3026	θ	0.3548	0.3834
	α	0.7041	0.7911	α	0.2837	0.1539	α	0.4716	0.5531
	β	-------	0.0388	β	-------	0.0385	β	-------	0.0336
	χ^2	6.1801	1.1321	χ^2	0.20467	0.9457	χ^2	1.03546	0.3090
	df	4	4	df	4	4	df	4	4
	p-value	0.1861	0.8892	p-value	0.9952	0.9180	p-value	0.9044	0.9892

Table (3) shows the MLE estimates of the ZIGD (Edwin, 2014) and the ZOIGD, from which we can see that the MLE method is much better than the ME method to fit the given data, as well as, it best fits than the zero-inflated cases.

Table (3): The MLE estimates of the ZIGD and the ZOIGD for the number of households according to the number of rural out-migrants per household in the three types of villages.

Number of Migrants	Number of Households								
	Semi-Urban			Remote			Growth Centre		
	Actual Obs.	Exp. ZIGD	Exp. ZOIGD	Actual Obs.	Exp. ZIGD	Exp. ZOIGD	Actual Obs.	Exp. ZIGD	Exp. ZOIGD
0	1042	1042	1042	872	872	872	978	978	978
1	95	89	95	176	177	176	154	151	154
2	19	28	20	59	58	59	47	52	48
3	10	9	8	18	19	19	18	18	18
4+	5	4	7	10	9	9	11	9	9
Total	1171	1171	1171	1135	1135	1135	1208	1208	1208
Model Parameters	θ	0.3138	0.4237	θ	0.3291	0.3256	θ	0.3447	0.3719
	α	0.6490	0.7967	α	0.2959	0.2806	α	0.4477	0.5239
	β	-------	0.0417	β	-------	0.0037	β	-------	0.0213
	χ^2	3.6585	1.1214	χ^2	0.1866	0.1637	χ^2	0.9818	0.4653
	df	4	4	df	4	4	df	4	4
	p-value	0.4542	0.8909	p-value	0.9959	0.9968	p-value	0.9121	0.9768

8. CONCLUSIONS

We consider estimation of the parameters of the zero-one inflated generalized power series distributions by the method of moments estimator and maximum likelihood estimator. The method of maximum likelihood estimator is shown to have butter estimates on real data representing the number of households according to the number of rural out-migrants per household in the three types of villages; semi-urban, Remote, and growth center, in a zero-one inflated geometric distribution model, as well as, it best fits than the zero-inflated cases.

9. REFERENCES

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