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RESEARCH ARTICLE

# COMMON FIXED POINT THEOREMS FOR OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS IN INTUITIONISTIC FUZZY METRIC SPACES

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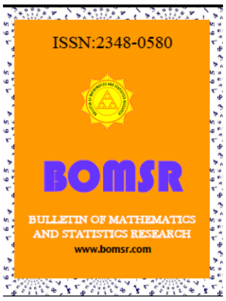


**ABSTRACT**
The present paper deal with common fixed point theorem in intuitionistic fuzzy metric space by employing the notion of occasionally weakly compatible mappings. Our result generalizes the recent result of Jain and Jauhari [7] and other existing results in intuitionistic fuzzy metric space.

**Keywords.** Common fixed points, Intuitionistic fuzzy metric space, compatible maps occasionally weakly compatible mappings and weak compatible mappings.

**Mathematics Subject classification**: Primary 47H10 and Secondary 54H25.



## 1. INTRODUCTION

Zadeh [18] investigation of the notion of fuzzy set has led to rich growth of fuzzy Mathematics. Many authors as Singh and chouhan [15], Jain et al[6], Verma and Chandel [17] have studied the concept of fuzzy metric space. Atanassov [3] introduced and studies the concept of intuitionistic Fuzzy sets. The notion of Intuitionistic Fuzzy metric space due to George and Veeramani [5].Further, using the idea of Intuitionistic Fuzzy metric set, Alaca et. Al [1] defined the notion of Intuitionistic Fuzzy Metric space. Park[13] introduce a notion of Intuitionistic Fuzzy Metric space with the help of continuous t- norms and continuous t- conorms, as a generalization of fuzzy metric space due to Kramosil and Michalek[10], further Coker [4], Turkoglu et.al. [16] and references mentioned there in have been expansively developed the theory of Intuitionistic Fuzzy set and applications. Turkoglu et. al. [16] introduced the notion of Cauchy sequences in intuitionistic fuzzy metric space. They generalized the Jungck's [9] common fixed point theorem in intuitionistic fuzzy metric space and proved the intuitionistic fuzzy version of Pant's theorem [12] by giving the definition of weakly commuting and R-weakly commuting mapping in intuitionistic fuzzy metric space.

## 2. Basic Definitions and Preliminaries.

We begin by briefly recalling some definitions and notations from fixed point theory literature that we will use in sequel, the concept of triangular norms (t - norm) and triangular conforms (t- conorms ) were originally introduced by Schweizer and Skalar[14].

**Definition 2.1 [14].** A binary operation *:[0,1]x[0,1]→[0,1] is called a *t-norm* * satisfies the following conditions:

i. * is commutative and associative,

ii. * is continuous,

iii. a * 1 =a for all a ∈ [0, 1],

iv. a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0,1].

Examples of *t-norm:* a * b = ab and a * b=min{a, b}.

**Definition 2.2[1].** A binary operation ◊:[0,1]x[0,1]→[0,1] is said to be continuous t-co norm if it satisfied the following conditions:

i. ◊ is associative and commutative,

ii. ◊ is continuous,

iii. a ◊ 0 = a for all a ∈ [0,1],

iv. a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for each a, b, c, d ∈ [0,1]

Examples of t-conorm :a ◊ b = min(a+b , 1) and a ◊ b = max(a, b)

**Remark 2.1.[14]** The concept of triangular norms (*t-norm) and* triangular conorms (*t-conorm)* are knows as axiomatic skeletons that we use for characterizing fuzzy intersections and union respectively.

**Definition 2.3 [1].** A 5- tuple (X, M, N, *, ◊) is called intuitionistic fuzzy metric space if X is an arbitrary non empty set, * is a continuous *t-norm, ◊* continuous *t-conorm* and M, N are fuzzy sets on $X^2$ x [0,∞] satisfying the following conditions:For each x, y, z, ∈X and t, s > 0

(IFM-1) M(x , y, t) + N(x, y, t) ≤ 1,

(IFM-2) M(x, y, 0) = 0, for all x, y in X,

(IFM-3) M(x, y, t) = 1 for all x, y in X and t > 0 if and only if x=y,

(IFM-4) $M(x,y,t) = M(y,x,t)$, for all x, y in X and t > 0,

(IFM-5) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$,

(IFM-6) $M(x,y,.)\colon [0,\infty] \to [0,1]$ is left continuous,

(IFM-7) $\lim_{t\to\infty} M(x,y,t) = 1$,

(IFM-8) $N(x,y,0) = 1$, for all x, y in X,

(IFM-9) $N(x,y,t) = 0$, for all x, y in X and t > 0 if and only if x = y,

(IFM-10) $N(x,y,t) = N(y,x,t)$, for all x, y in X and t>0,

(IFM-11) $N(x,y,t) \lozenge N(y,z,s) \geq N(x,z,t+s)$,

(IFM-12) $N(x,y,.)\colon [0,\infty) \to [0,1]$ is right continuous,

(IFM-13) $\lim_{t\to\infty} N(x,y,t) = 0$, for all x, y in X and $t > 0$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The function M(x, y, t) and N(x, y, t) denote the degree of nearness and degree of non- nearness between x and y with respect to t, respectively.

**Remark 2.2.[16].** An intuitionistic fuzzy metric space with continuous t-norm * and continuous *t-conorm ◊* defined by a * a ≥ a, and (1-a) ◊ (1-a) ≤ (1-a)for all a∈ [0,1]. Then for all x, y∈ X, M(x, y, *) is non decreasing and N(x, y, ◊) is non increasing.

**Lemma 2.1.[16].** Let (X, M, N, *, ◊) intuitionistic fuzzy metric space, If there exists k ∈ (0, 1 ) such that for all x, y ∈ X, M(x, y, kt ) ≥ M(x, y, t ) and , N(x, y, kt ) ≤ N(x, y, t ) for all t > 0, then x = y.

**Definition 2.4.[13].** A sequence $\{x_n\}$ in intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be cauchy sequence if and only if for each ε > 0, t > 0, there exists $n_0$ ∈ N such that M($x_n$, $x_m$, $t$) > 1-ε and N($x_n$, $x_m$, $t$) < ε for all n, m $\geq n_0$.

The sequence $\{x_n\}$ converge to a point x in X if and only if for each ε > 0, t > 0,

there exists $n_0$ ∈ N such that M($x_n$, $x, t$)> 1-ε , N($x_n$, $x, t$) < ε for all n $\geq n_0$.

An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if every Cauchy sequence in it converges to a point in it.

**Definition 2.5[11].** Two self mappings A and S of an intuitionistic fuzzy metric space (X, M, N, *, ◊) are said to be compatible if and only if

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$ for all t > 0, whenever $\{x_n\}$ is a sequence in X such that $Sx_n, Ax_n \to p \text{ or some } p \text{ in } X \text{ as } n \to \infty$ .

**Definition 2.6.[11].** Two self mappings A and B of an intuitionistic fuzzy metric space (X, M, N, *, ◊) are said to be weak compatible if they commute at their coincidence point x, i. e. Ax = Bx implies ABx = BAx for some x in X.

Al Thagafi and Shahzad [2] introduced the notion of occasionally weakly compatible mapping which is more general than the concept of weakly compatible maps.

**Definition 2.7.** Two self mappings A and S of an intuitionistic fuzzy metric space (X, M, N, *, ◊) are said to be occasionally weakly compatible (owc) if and only if there is a point x in X which is coincidence point of A and S at which A and S commute.

**Lemma 2.2.[9].** Let A and B be occasionally weakly compatible (owc) self maps on X .If A and B have a unique point of coincidence, w = Ax = Bx, then w is unique fixed point of A and B.

**Lemma 2.3.[3].** Let $\{x_n\}$ be a sequence in intuitionistic fuzzy metric space (X, M, N, *, ◊). If there exists a number k ∈ (0, 1) such that $M(x_{n+2}, x_{n+1}, kt) \geq M(x_{n+1}, x_n, t)$ and $N(x_{n+2}, x_{n+1}, kt) \leq N(x_{n+1}, x_n, t)$ for all t > 0, and n ∈ N. Then $\{x_n\}$ is cauchy sequence in X.

**Lemma 2.4[3].** In intuitionistic fuzzy metric space (X, M, N, *, ◊) *t-norm sat*isfying r * r ≥ r for all r ∈ [0, 1] is the minimum *t-norm* *i.e.* a*b=min{a, b} for all a, b ∈ [0, 1] and t- conorm satisfying r ◊ r$\leq r$ for all r ∈ [0, 1] is the maximum *t-norm* *i.e.* a ◊ b=max{a, b} for all a, b ∈ [0, 1].

## 3. MAIN RESULT

In this section, a fixed point theorem for six self maps using the concept of occasionally weakly compatible maps has been established which generalizes the results of Jain and Jauhari [7] from fuzzy metric space to intuitionistic fuzzy metric space. This result extends and generalizes many fixed point results in intuitionistic fuzzy metric space.

**Theorem 3.1.** Let (X, M, N, *, ◊) be a complete intuitionistic fuzzy metric space with continuous *t-norm *and* continuous *t-conorm ◊ and t * t ≥ t and t ◊ t ≥ t* for all t ∈ [0, 1] and let A, B, S, T, P and Q be mapping from X into itself satisfying the following conditions:

(I). P(X) ⊂ ST(X), Q(X) ⊂ AB(X);

(II). AB =BA, ST = TS, PB = BP, QT = TQ;

(III). either P or AB is continuous;

(IV). (P, AB) is compatible and (Q, ST) is occasionally weakly compatible or (Q, ST) is compatible and (P, AB) is occasionally weakly compatible.

(V). There exists k ∈ (0, 1) such that ∀ $x, y \in X$ and t > 0,

M (Px, Qy, kt) ≥ Min$\{M(Qy, STy, t), M(ABx, STy, t), M(Px, ABx, t)\}$,

N (Px, Qy, kt) ≤ Max$\{M(Qy, STy, t), M(ABx, STy, t), M(Px, ABx, t)\}$.

Then A, B, S, T, P, and Q have unique common fixed point in X.

**Proof.** Let $x_0 \in X$, from (I) there exists $x_1, x_2 \in X$ such that P $x_0 = STx_1$ and $Qx_1 = ABx_2$. Inductively, we can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that $Px_{2n-2} = STx_{2n-1} = y_{2n-1}$ and $Qx_{2n-1} = ABx_{2n} = y_{2n}$, for n = 1, 2, 3, … .

**Step 1:** Put x $=x_{2n}$ and $y = x_{2n+1}$ in (V) we get

$M(Px_{2n}, Qx_{2n+1}, kt) \ \geq \ Min\{M(Qx_{2n+1}, STx_{2n+1}, t), M(ABx_{2n}, STx_{2n+1}, t), M(Px_{2n}, ABx_{2n}, t)\}$,

$N(Px_{2n}, Qx_{2n+1}, kt) \ \leq \ Max\{M(Qx_{2n+1}, STx_{2n+1}, t), M(ABx_{2n}, STx_{2n+1}, t), M(Px_{2n}, ABx_{2n}, t)\}$

It follows that

$M(Px_{2n}, Qx_{2n+1}, kt) \geq Min\{M(y_{2n+2}, y_{2n+1}, t), M(y_{2n}, y_{2n+1}, t), M(y_{2n+1}, y_{2n}, t)\} \geq M(y_{2n}, y_{2n+1}, t)$

and $N(Px_{2n}, Qx_{2n+1}, kt) \leq Max\{N(y_{2n+2}, y_{2n+1}, t), N(y_{2n}, y_{2n+1}, t), N(y_{2n+1}, y_{2n}, t)\} \leq N(y_{2n}, y_{2n+1}, t)$

i.e. $M(y_{2n+1}, y_{2n+2}, kt) \ \geq M(y_{2n}, y_{2n+1}, t)$

and $N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t)$

Similarly, we have

$M(y_{2n+2}, y_{2n+3}, kt) \ \geq M(y_{2n+1}, y_{2n+2}, t)$

and $N(y_{2n+2}, y_{2n+3}, kt) \leq N(y_{2n+1}, y_{2n+2}, t)$

Thus, we have in general,

$M(y_{n+1}, y_{n+2}, kt) \ \geq \ M(y_n, y_{n+1}, t)$, for n = 1, 2, 3...

$N(y_{n+1}, y_{n+2}, kt) \ \leq \ N(y_n, y_{n+1}, t)$, for n = 1, 2,3,...

also it follows that

$$\begin{aligned} M(y_n, y_{n+1}, t) &\geq M(y_{n-1}, y_n, t/k) \\ &\geq M(y_{n-2}, y_{n-1}, t/k^2) \ \ldots \\ &\geq M(y_0, y_1, t/k^n) \rightarrow 1 \ as\ n \rightarrow \infty, \end{aligned}$$

and
$$\begin{aligned} N(y_n, y_{n+1}, t) &\leq N(y_n, y_{n+1}, t/k) \\ &\leq N(y_{n-2}, y_{n-1}, t/k^2) \ldots \\ &\leq N(y_0, y_1, t/k^n) \rightarrow 0 \ as\ n \rightarrow \infty, \end{aligned}$$

Hence $M(y_n, y_{n+1}, t) \rightarrow 1 \ as\ n \rightarrow \infty \ for\ any\ t > 0$,

and $N(y_n, y_{n+1}, t) \rightarrow 0 \ \ as\ n \rightarrow \infty \ for\ any\ t > 0$.

For each $\varepsilon > 0\ and\ t > 0, we\ can\ choose\ n_0\ \epsilon\ N\ such\ that$

$M(y_n, y_{n+1}, t) > 1 - \varepsilon\ \ for\ all\ n > \ n_0$ and $N(y_n, y_{n+1}, t) < \ \varepsilon\ \ for\ all\ n > \ n_0$

For m, n $\in N, we\ suppose\ m \geq n$. Then we have

$$\begin{aligned} M(y_n, y_m, t) &\geq M(y_n, y_{n+1}, t/(m-n)) * M(y_{n+1}, y_{n+2}, t/(m-n)) * \ldots \\ &\quad * \ M(y_{m-1}, y_m, t/(m-n)) \\ &\geq (1 - \varepsilon) * (1-\varepsilon) * \ldots\ldots * (1-\varepsilon) \ \text{(m-n) times} \geq (1 - \varepsilon) \end{aligned}$$

and $N(y_n, y_m, t) \ \leq \ N(y_n, y_{n+1}, t/(m-n)) \lozenge N(y_{n+1}, y_{n+2}, t/(m-n)) \lozenge \ldots \lozenge \ N(y_{m-1}, y_m, t/(m-n)$

$\leq \ \varepsilon \lozenge \varepsilon \lozenge \ldots\ldots \lozenge \varepsilon$ (m-n) times $\leq \ \varepsilon$.

Hence $\{y_n\}$ is a cauchy sequence in X.Since (X, M, N, *, ◊) is complete, sequence $\{y_n\}$ converge to some point z ∈ X. Also its subsequence converges to the same point

i.e. $\{Qx_{2n+1}\} \rightarrow z$ and $\{STx_{2n+1}\} \rightarrow z$ (3.1.1)

and $\{Px_{2n}\} \rightarrow z$ and $\{ABx_{2n}\} \rightarrow z$ (3.1.2)

**Now** Suppose AB is continuous,

**Step 1-** Since AB is continuous, we have

$(AB)^2 x_{2n} \rightarrow ABz$ and

$ABPx_{2n} \rightarrow ABz$.

As (P, AB) is compatible, so by definition of compatible P (AB)$x_{2n} \rightarrow ABz$.

**Step 2-** $Put\ x = ABx_{2n}$ and $y = y_{2n+1}$ in (V), we get

M (PAB$x_{2n}$, Q$x_{2n+1}$, kt) ≥

Min $\{M(Qx_{2n}, STx_{2n+1}, t), M(ABABx_{2n}, STx_{2n+1}, t), M(PABx_{2n}, ABABx_{2n}, t)\}$

and ,

N(P$ABx_{2n}$, Q$x_{2n+1}$, kt) ≤

Max $\{N(Qx_{2n+1}, STx_{2n+1}, t), N(ABABx_{2n}, STx_{2n+1}, t), N(PABx_{2n}, ABABx_{2n}, t)\}$.

Taking limit n→ ∞, and using(3.1.1) (3.1.2),we get

i.e. $M(ABz, z, kt) \geq Min\{M(z, z, t), M(ABz, z, t), M(ABz, ABz, t)\}$

and $N(ABz, z, kt) \leq Max\{N(z, z, t), N(ABz, z, t), N(ABz, ABz, t)\}$

Thus $M(ABz, z, kt) \geq M(ABz, z, t)$ and $N(ABz, z, kt) \leq N(ABz, z, t)$

Therefore using lemma 2.1, we get ABz = z (3.1.3)

**Step 3-** $Put\ x = z$ and $y = x_{2n+1}$ in (V), we have

$$M(Pz, Qx_{2n+1}, kt) \geq Min\{M(Qx_{2n+1}, STx_{2n+1}, t), M(ABz, STx_{2n+1}, t), M(z, z, t)\}$$

and

$$N(Pz, Qx_{2n+1}, kt) \leq Max\{N(Qx_{2n+1}, STx_{2n+1}, t), N(ABz, STx_{2n+1}, t), N(z, z, t)\}.$$

Taking n→ ∞ and using equation (3.1.1), (3.1.2), we get

$M(Pz, z, kt) \geq Min\ \{M(z, z, t), M(z, z, t), M(Pz, z, t)\}$

and $N(Pz, z, kt) \leq Max\ \{N(z, z, t), N(z, z, t), N(Pz, z, t)\}$.

i.e. $M(Pz, z, kt) \geq M(Pz, z, t)$ and $N(Pz, z, kt) \leq N(Pz, z, t)$.

Therefore, by using lemma 2.1, we get Pz = z. Therefore, ABz = Pz = z.

**Step 4-** $Putting\ x = Bz$ and $y = x_{2n+1}$ in condition (V), we get

$M(PBz, Qx_{2n+1}, kt) \geq Min\{M(Qx_{2n+1}, STx_{2n+1}, t), M(ABBz, STx_{2n+1}, t), M(PBz, ABBz, t)\}$

and $N(PBz, Qx_{2n+1}, kt) \leq Max\{N(Qx_{2n+1}, STx_{2n+1}, t), N(ABBz, STx_{2n+1}, t), N(PBz, ABBz, t)\}$

Taking n→ ∞ and using (3.1.1), (3.1.2),(ii) and lemma 2.1

BP = PB, AB = BA, so we have P(Bz) = B(Pz) = Bz and (AB)(Bz) = (BA)(Bz) = B(ABz) = Bz.

Taking n→ ∞ and using (1) we get Bz = z

and also we have ABz = z this implies $Az = z$.

Therefore, Az = Bz = Pz = z. (3.1.4)

**Step 5-** As P(X) ⊂ ST(X), there exists u ∈ X such that z = Pz = STu,

Putting $x = x_{2n}$ $and\ y = u\ in\ (V)$, we get

$$M(Px_{2n}, Qu, kt) \geq Min\{M(Qu, STu, t), M(ABx_{2n}, STu, t), M(Px_{2n}, ABx_{2n}, t)\}$$

and $N(Px_{2n}, Qu, kt) \leq Max\{N(Qu, STu, t), N(ABx_{2n}, STu, t), N(Px_{2n}, ABx_{2n}, t)\}$.

Taking n → ∞ and using (3.1.1) and (3.1.2), we get

$M(z, Qu, kt) \geq Min\{M(Qu, z, t), M(z, z, t), M(z, z, t)\}$

and $N(z, Qu, kt) \leq Max\{N(Qu, z, t), N(z, z, t), N(z, z, t)\}$

i.e. $M(z, Qu, kt) \geq M(z, Qu, t)$ and $N(z, Qu, kt) \leq N(z, Qu, t)$

Therefore, by using lemma 2.1, we get Qu = z. Hence STu = z = Qu.

Since (Q, ST) is occasionally weakly compatible, so we have QSTu = STQu. Thus, Qz = STz.

**Step 6-** Putting $x = x_{2n}$ $and\ y = z\ in\ (V)$, we get

$$M(Px_{2n}, Qz, kt) \geq Min\{M(Qz, STz, t), M(ABx_{2n}, STz, t), M(Px_{2n}, ABx_{2n}, t)\}$$

And $N(Px_{2n}, Qz, kt) \leq Max\{N(Qz, STz, t), N(ABx_{2n}, STz, t), N(Px_{2n}, ABx_{2n}, t)\}$.

Taking n → ∞ and using (3.1.2) and step 5, we get

$$M\ (z, Qz, kt) \geq Min\{M(Qz, Qz, t), M(z, Qz, t), M(z, z, t)\}$$

$and$ $N\ (z, Qz, kt) \leq Max\{N(Qz, Qz, t), N(z, Qz, t), N(z, z, t)\}$

i.e. $M\ (z, Qz, kt) \geq M(z, Qz, t)$ and $N\ (z, Qz, kt) \leq N(z, Qz, t)$

Therefore by using lemma 2.1, we get Qz = z.

**Step 7-** Putting $x = x_{2n}$ $and\ y = Tz\ in\ (V)$, we get

$M(Px_{2n}, QTz, kt) \geq Min\{M(QTz, STTz, t), M(ABx_{2n}, STTz, t), M(Px_{2n}, ABx_{2n}, t)\}$

and $N(Px_{2n}, Qz, kt) \leq Max\{N(QTz, STTz, t), N(ABx_{2n}, STTz, t), N(Px_{2n}, ABx_{2n}, t)\}$.

As QT = TQ and ST = TS, we have QTz = TQz = Tz and ST(Tz) = T(STz) = TQz = Tz.

Taking limit n → ∞, we get $M(z, Tz, kt) \geq Min\{M(Tz, Tz, t), M(z, Tz, t), M(z, z, t)\}$

and $N(z, Tz, kt) \leq Max\{M(Tz, Tz, t), M(z, Tz, t), M(z, z, t)\}$

i.e. $M(z, Tz, kt) \geq M(z, Tz, t)$ and $N(z, Tz, kt) \leq N(z, Tz, t)$.

Therefore, by using lemma 2.1, we get Tz = z.

Now STz = Tz = z implies Sz = z. Hence Sz = Tz = Qz = z. (3.1.5)

combining (3.14) and (3.15), we get Az = Bz = Pz = Qz = Tz = Sz = z.

Hence, z is the common fixed point of A, B, S, T, P and Q.

**Uniqueness-** Let u be another common fixed point of A, B, S, T, P and Q.

Then Au = Bu = Pu = Qu = Su = Tu = u.Put x = z and y =u in (V),

We get $M(Pz, Qu, kt) \geq Min\{M(Qu, STu, t), M(ABz, STu, t), M(Pz, ABz, t)\}$,

and $N(Pz, Qu, kt) \leq Max\{N(Qu, STu, t), N(ABz, STu, t), N(Pz, ABz, t)\}$.

Thus, we get $M(z, u, kt) \geq Min\{M(u, u, t), M(z, u, t), M(z, z, t)\}$

and $N(z, u, kt) \leq Max\{N(u, u, t), N(z, u, t), N(z, z, t)\}$.

Therefore $M(z, u, kt) \geq M(z, u, t)$ and $N(z, u, kt) \leq N(z, u, t)$.

Therefore using lemma 2.1, we ge z = u.

Therefore, z is the common fixed point of self maps A, B, S, T, P and Q.

Note that the case is similar when we take the pair (Q, ST) is occasionally weakly compatible and (P, AB) is compatible.

**Remark 3.1:** If we take B = T = I in theorem 3.1, we get following corollary

**Corollary 3.1:** Let (X, M, N, *, ◊) be a complete intuitionistic fuzzy metric space with continuous t-norm * and t * t $\geq t$ and continuous t-conorm ◊ and t ◊ t $\leq t$, for all t ∈ [0, 1] and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:

(I). P(X) ⊂ S(X), Q(X) ⊂ A(X);

(II). either P or A is continuous;

(III). (P, A) is compatible and (Q, S) is occasionally weakly compatible;

(IV). There exists k ∈ (0, 1) such that ∀ $x, y \in X$ and t > 0,

M (Px, Qy, kt) ≥ Min$\{M(Qy, Sy, t), M(Ax, Sy, t), M(Px, Ax, t)\}$,

N (Px, Qy, kt) ≤ Max$\{N(Qy, Sy, t), N(Ax, Sy, t), N(Px, Ax, t)\}$.

Then A, S, P, and Q have unique common fixed point in X.

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