Vol.4.Issue.4.2016 (October -December)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

RESEARCH ARTICLE

ISSN:2348-0580

ON THE STRUCTURE EQUATION $F^3+F^2+F=0$

LAKHAN SINGH[1], SHAILENDRA KUMAR GAUTAM[2]
[1]Department of Mathematics, D.J. College, Baraut, Baghpat (U.P.),India
email: lakhansingh.077@gmail.com
[2]Eshan College of Engineering, Mathura(UP),India
email:skgautamrbs@yahoo.com

ABSTRACT

In this paper, we have studied various properties of the sturcture equation $F^3+F^2+F=0$. Nijenhuis tensor and Metric F-structure have also been discussed.

Key words: Differnetiable manifold, projection operators, Nijenhuis tensor and metric.

INTRODUCTION

Let M^n be a differentiable manifold of class C^∞ and F be a (1,1) tensor of class C^∞, satisfying

(1.1) $F^3+F^2+F=0$

we define the operators l and m on M^n by

(1.2) $l=F^3, \qquad m=I-F^3$

where I is the identity operator.

from (1.1) and (1.2), we have

(1.3) $l+m=I, \qquad l^2=l,\ m^2=m, \quad lm=ml=0$

$Fl=lF=F, \quad Fm=mF=0,$

Theorem (1.1): Let the (1,1) tensors p and q be defined by

(1.4) $p=m+F^2, \quad q=m-F^2,$

Then

(1.5) $pq=m-F,\ p^3=I=q^6=pq^2,\ p^2=q^2$

Proof: Using (1.2), (1.3) and (1.4) we get the results

Theorem (1.2): Let the (1,1) tensors α and β be defined by

(1.6) $\alpha = m + F, \quad \beta = m - F,$ then

(1.7) $\alpha^2 = \beta^2, \quad \alpha^3 = I = \beta^6$

Proof: Using (1.2), (1.3) and (1.6), we get $\alpha^2 = m + F^2 = \beta^2$. The other results follow similarly.

Theorem (1.3): Let the (1,1) tensors γ and δ be defined by

(1.8) $\gamma = l - F^3, \ \delta = l + F^3,$ then

(1.9) $\gamma = 0, \quad \delta = 2l, \ \delta^n = 2^n l$

Proof: From (1.2), (1.3) and (1.8), $\delta = 2l, \ \delta^2 = 4l, ... \ \delta^n = 2^n l$ etc.

Theorem (1.4): Let the (1,1) tensors *a* and *b* be defined by

(1.10) $a = l - F^2, \quad b = l + F^2,$ then

(1.11) $ab = l - F, \quad a^2 + 3b^2 = 0$

Proof: Using (1.1), (1.3), and (1.10)

(1.12) $a^2 = l + F - 2F^2 = F^3 + F - 2F^2 = F^3 + F + F^2 - 3F^2$

$= -3F^2,$ similarly $b^2 = F^2$ thus $a^2 + 3b^2 = 0,$ etc.

2. **Nijenhuis tensors**:

The Nijenhuis tensors corresponding to the operators *F, l, m* be defined as

(2.1) $N(X,Y) = [FX,FY] + F^2[X,Y] - F[FX,Y] - F[X,FY]$

(2.2) $\underset{l}{N}(X,Y) = [lX,lY] + l^2[X,Y] - l[lX,Y] - l[X,lY]$

(2.3) $\underset{m}{N}(X,Y) = [mX,mY] + m^2[X,Y] - m[mX,Y] - m[X,mY]$

Theorem (2.1): Let *F, l, m* satisfy (1.1) and (1.2), then

(2.4) (i) $N(mX,mY) = F^2[mX,mY]$

(ii) $mN(mX,mY) = 0$

(iii) $\underset{l}{N}(mX,mY) = l[mX,mY]$

(iv) $\underset{m}{N}(lX,lY) = m[lX,lY]$

(v) $\underset{l}{N}(lX,mY) = 0$

(vi) $\underset{m}{N}(mX,lY) = 0$

Proof: With proper replacements of *X* and *Y* in (2.1), (2.2) and (2.3), and using (1.3) we get the reuslts.

3. **metric f-structrue**

Let the Riemannian metric *g* satisfies

(3.1) $`F(X,Y) = g(FX,Y)$ is symmetric, then

(3.2) $g(FX,Y) = G(X,FY)$

and $\{F, g\}$ is called metric *F*-structure

Theorem (3.1): Let *F* satisfies (1.1), then

(3.3) $g(FX,F^2Y) = g(X,Y) - `m(X,Y),$ then

(3.4) $`m(X,Y) = g(mX,Y) = g(X,mY)$

Proof: From (1.2), (1.3) and (3.2)

$$\begin{aligned} g(FX,F^2Y) &= g(X,F^3Y) \\ &= g(X,lY) \\ &= g(X,(I-m)Y) \\ &= g(X,Y) - g(X,mY) \\ &= g(X,Y) - `m(X,Y) \end{aligned}$$

REFERENCES

[1]. A Bejancu, On semi-invariant submanifolds of an almost contact metric manifold. An Stiint Univ., "A.I.I. Cuza" Lasi Sec. Ia Mat. (Supplement) 1981, 17-21.

[2]. B. Prasad, Semi-invariant submanifolds of a Lorentzian Para-sasakian manifold, Bull Malaysian Math. Soc. (Second Series) 21 (1988), 21-26.

[3]. F. Careres,Linear invairant of Riemannian product manifold, Math Proc. Cambridge Phil. Soc. 91 (1982), 99-106.

[4]. Endo Hiroshi,On invariant submanifolds of connect metric manifolds, Indian J. Pure Appl. Math 22 (6) (June-1991), 449-453.

[5]. H.B. Pandey & A. Kumar,Anti-invariant submanifold of almost para contact manifold. Prog. of Maths Volume 21(1): 1987.

[6]. K. Yano,On a structure defined by a tensor field *f* of the type (1,1) satisfying $f^3+f=0$. Tensor N.S., 14 (1963), 99-109.

[7]. R. Nivas & S. Yadav,On CR-structures and $F_\lambda(2\nu+3,2)$ - HSU - structure satisfying $F^{2\nu+3} + \lambda^r F^2 = 0$, Acta Ciencia Indica, Vol. XXXVII M, No. 4, 645 (2012).

[8]. Abhisek Singh, On horizontal and complete lifts Ramesh Kumar Pandey of (1,1) tensor fields F satisfying & Sachin Khare, the structure equation $F(2k+S,S)$=0. International Journal of Mathematics and soft computing. Vol. 6, No. 1 (2016), 143-152