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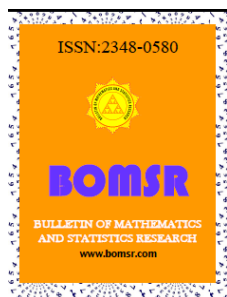


**ON THE STRUCTURE EQUATION  $F^3 + F^2 + F = 0$**

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**ABSTRACT**

In this paper, we have studied various properties of the structure equation  $F^3 + F^2 + F = 0$ . Nijenhuis tensor and Metric F-structure have also been discussed.

**Key words:** Differentiable manifold, projection operators, Nijenhuis tensor and metric.

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**INTRODUCTION**

Let  $M^n$  be a differentiable manifold of class  $C^\infty$  and  $F$  be a (1,1) tensor of class  $C^\infty$ , satisfying

(1.1)  $F^3 + F^2 + F = 0$

we define the operators  $l$  and  $m$  on  $M^n$  by

(1.2)  $l = F^3, \quad m = I - F^3$

where  $I$  is the identity operator.

from (1.1) and (1.2), we have

(1.3)  $l + m = I, \quad l^2 = l, \quad m^2 = m, \quad lm = ml = 0$   
 $Fl = lF = F, \quad Fm = mF = 0,$

**Theorem (1.1):** Let the (1,1) tensors  $p$  and  $q$  be defined by

(1.4)  $p = m + F^2, \quad q = m - F^2,$

Then

(1.5)  $pq = m - F, \quad p^3 = I = q^6 = pq^2, \quad p^2 = q^2$

**Proof:** Using (1.2), (1.3) and (1.4) we get the results

**Theorem (1.2):** Let the (1,1) tensors  $\alpha$  and  $\beta$  be defined by

$$(1.6) \quad \alpha = m + F, \quad \beta = m - F, \text{ then}$$

$$(1.7) \quad \alpha^2 = \beta^2, \quad \alpha^3 = I = \beta^6$$

**Proof:** Using (1.2), (1.3) and (1.6), we get  $\alpha^2 = m + F^2 = \beta^2$ . The other results follow similarly.

**Theorem (1.3):** Let the (1,1) tensors  $\gamma$  and  $\delta$  be defined by

$$(1.8) \quad \gamma = l - F^3, \quad \delta = l + F^3, \text{ then}$$

$$(1.9) \quad \gamma = 0, \quad \delta = 2l, \quad \delta^n = 2^n l$$

**Proof:** From (1.2), (1.3) and (1.8),  $\delta = 2l, \delta^2 = 4l, \dots, \delta^n = 2^n l$  etc.

**Theorem (1.4):** Let the (1,1) tensors  $a$  and  $b$  be defined by

$$(1.10) \quad a = l - F^2, \quad b = l + F^2, \text{ then}$$

$$(1.11) \quad ab = l - F, \quad a^2 + 3b^2 = 0$$

**Proof:** Using (1.1), (1.3), and (1.10)

$$(1.12) \quad a^2 = l + F - 2F^2 = F^3 + F - 2F^2 = F^3 + F + F^2 - 3F^2 \\ = -3F^2, \text{ similarly } b^2 = F^2 \text{ thus } a^2 + 3b^2 = 0, \text{ etc.}$$

## 2. Nijenhuis tensors:

The Nijenhuis tensors corresponding to the operators  $F, l, m$  be defined as

$$(2.1) \quad N(X, Y) = [FX, FY] + F^2[X, Y] - F[FX, Y] - F[X, FY]$$

$$(2.2) \quad N_l(X, Y) = [lX, lY] + l^2[X, Y] - l[lX, Y] - l[X, lY]$$

$$(2.3) \quad N_m(X, Y) = [mX, mY] + m^2[X, Y] - m[mX, Y] - m[X, mY]$$

**Theorem (2.1):** Let  $F, l, m$  satisfy (1.1) and (1.2), then

$$(2.4) \quad (i) \quad N(mX, mY) = F^2[mX, mY]$$

$$(ii) \quad mN(mX, mY) = 0$$

$$(iii) \quad N_l(mX, mY) = l[mX, mY]$$

$$(iv) \quad N_m(lX, lY) = m[lX, lY]$$

$$(v) \quad N_l(lX, mY) = 0$$

$$(vi) \quad N_m(mX, lY) = 0$$

**Proof:** With proper replacements of  $X$  and  $Y$  in (2.1), (2.2) and (2.3), and using (1.3) we get the results.

## 3. metric f-structure

Let the Riemannian metric  $g$  satisfies

(3.1)  $\nabla F(X, Y) = g(FX, Y)$  is symmetric, then

$$(3.2) \quad g(FX, Y) = G(X, FY)$$

and  $\{F, g\}$  is called metric  $F$ -structure

**Theorem (3.1):** Let  $F$  satisfies (1.1), then

$$(3.3) \quad g(FX, F^2Y) = g(X, Y) - \nabla m(X, Y), \text{ then}$$

$$(3.4) \quad \nabla m(X, Y) = g(mX, Y) = g(X, mY)$$

Proof: From (1.2), (1.3) and (3.2)

$$\begin{aligned} g(FX, F^2Y) &= g(X, F^3Y) \\ &= g(X, lY) \\ &= g(X, (I - m)Y) \\ &= g(X, Y) - g(X, mY) \\ &= g(X, Y) - \nabla m(X, Y) \end{aligned}$$

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