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RESEARCH ARTICLE

SOME TRANSLATORS IN BIPOLAR VALUED FUZZY SUBSEMIRINGS OF A SEMIRING

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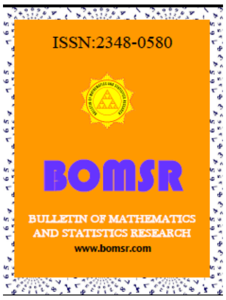

ABSTRACT

In this paper, we study some of translators of bipolar valued fuzzy subsemiring of a semiring and prove some results on these.

KEY WORDS: Bipolar valued fuzzy subset, bipolar valued fuzzy subsemiring, pseudo bipolar valued fuzzy coset, bipolar valued normal fuzzy subset, translators.

INTRODUCTION

In 1965, Zadeh [17] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc [9]. Lee [11] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [11, 12]. Anitha.M.S., Muruganantha Prasaid & K.Arjunan[2] defined as bipolar valued fuzzy subgroups of a group. We introduce the some important translators of the bipolar valued fuzzy subsemiring of the semiring and established some results.

1.PRELIMINARIES

1.1 Definition: A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^{+}(x), A^{-}(x) > / x \in X\}$, where $A^{+} : X \to [0, 1]$ and $A^{-} : X \to [-1, 0]$. The positive membership degree $A^{+}(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^{-}(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to the bipolar valued fuzzy set A.

1.2 Example: A = { < a, 0.5, –0.3 >, < b, 0.1, –0.7 >, < c, 0.5, –0.4 >} is a bipolar valued fuzzy subset of X = {a, b, c }.

1.3 Definition: Let R be a semiring. A bipolar valued fuzzy subset A of R is said to be a bipolar valued fuzzy subsemiring of R (BVFSSR) if the following conditions are satisfied,

(i) $A^{+}(x+y) \geq \min\{ A^{+}(x), A^{+}(y) \}$

(ii) $A^{+}(xy) \geq \min\{ A^{+}(x), A^{+}(y) \}$

(iii) $A^{-}(x+y) \leq \max\{ A^{-}(x), A^{-}(y) \}$

(iv) $A^{-}(xy) \leq \max\{ A^{-}(x), A^{-}(y) \}$ for all x and y in R.

1.4 Example: Let R = Z_3 = { 0, 1, 2 } be a semiring with respect to the ordinary addition and multiplication. Then A = { < 0, 0.5, – 0.6 >, < 1, 0.4, – 0.5 >, < 2, 0.4, – 0.5 > } is a bipolar valued fuzzy subsemiring of R.

1.5 Definition: Let $A = \langle A^{+}, A^{-} \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R and a in R. Then the pseudo bipolar valued fuzzy coset $(aA)^{p} = \langle (aA^{+})^{p}, (aA^{-})^{p} \rangle$ is defined by $(aA^{+})^{p}(x) = p^{+}(a)A^{+}(x)$ and $(aA^{-})^{p}(x) = -p^{-}(a)A^{-}(x)$, for every x in R and for some p in P.

1.6 Definition: Let A be a bipolar valued fuzzy subset of X. Then the following translators are defined as

(i) $?(A) = \{ \langle x, \min\{ ½, A^{+}(x) \}, \max\{ -½, A^{-}(x) \} \rangle$ / for all $x \in X$ }.

(ii) $!(A) = \{ \langle x, \max\{ ½, A^{+}(x) \}, \min\{ -½, A^{-}(x) \} \rangle$ / for all $x \in X$ }.

(iii) $Q_{\alpha, \beta}(A) = \{ \langle x, \min\{ \alpha, A^{+}(x) \}, \max\{ \beta, A^{-}(x) \} \rangle$ / for all $x \in X$ and α in [0, 1] and β in [–1, 0] }.

(iv) $P_{\alpha, \beta}(A) = \{ \langle x, \max\{ \alpha, A^{+}(x) \}, \min\{ \beta, A^{-}(x) \} \rangle$ / for all $x \in X$ and α in [0, 1] and β in [–1, 0] }.

(v) $G_{\alpha, \beta}(A) = \{ \langle x, \alpha A^{+}(x), -\beta A^{-}(x) \} \rangle$ / for all $x \in X$ and α in [0, 1] and β in [–1, 0] }.

1.7 Definition: Let A be a bipolar valued fuzzy subset of X. Then the **height H(A) =** $\langle H(A^{+}), H(A^{-}) \rangle$ is defined as $H(A^{+}) = \sup A^{+}(x)$ for all x in X and $H(A^{-}) = \inf A^{+}(x)$ for all x in X.

1.8 Definition: Let A be a bipolar valued fuzzy subset of X. Then $^{0}A = \langle {^{0}A^{+}}, {^{0}A^{-}} \rangle$ is defined as $^{0}A^{+}(x)=A^{+}(x)H(A^{+})$ for all x in X and $^{0}A^{-}(x) = -A^{-}(x)H(A_{i}^{-})$ for all x in X.

1.9 Definition: Let A be a bipolar valued fuzzy subset of X. Then $^{\Delta}A = \langle {^{\Delta}A^{+}}, {^{\Delta}A^{-}} \rangle$ is defined as $^{\Delta}A^{+}(x) = A^{+}(x)/ H(A^{+})$ for all x in X and $^{\Delta}A^{-}(x) = -A^{-}(x) / H(A_{i}^{-})$ for all x in X.

1.10 Definition: Let A be a bipolar valued fuzzy subset of X. Then $^{\oplus}A = \langle {^{\oplus}A^{+}}, {^{\oplus}A^{-}} \rangle$ is defined as $^{\oplus}A^{+}(x) = A^{+}(x) + 1 - H(A^{+})$ for all x in X and $^{\oplus}A^{-}(x) = A^{-}(x) - 1 - H(A^{-})$ for all x in X.

1.11 Definition: Let A be a bipolar valued fuzzy subset of X. Then A is called bipolar valued normal fuzzy subset of X if $H(A^{+}) = 1$ and $H(A^{-}) = -1$.

2. SOME PROPERTIES

2.1 Theorem: If $A = \langle A^{+}, A^{-} \rangle$ and $B = \langle B^{+}, B^{-} \rangle$ are two bipolar valued fuzzy subsemirings of a semiring R, then their intersection $A \cap B$ is a bipolar valued fuzzy subsemiring of R.

2.2 Theorem: Let $A = \langle A^{+}, A^{-} \rangle$ be a bipolar valued fuzzy subsemiring of a semiring R. Then the pseudo bipolar valued fuzzy coset $(aA)^{p} = \langle (a A^{+})^{p}, (a A^{-})^{p} \rangle$ is a bipolar valued fuzzy subsemiring of the semiring R, for every a in R and p in P.

Proof: Let A be a bipolar valued fuzzy subsemiring of the semiring R. For every x and y in R, we have $(aA^{+})^{p}(x+y) = p^{+}(a)A^{+}(x+y) \geq p^{+}(a) \min\{ A^{+}(x), A^{+}(y) \} = \min\{p^{+}(a)A^{+}(x), p^{+}(a)A^{+}(y) \} = \min\{ (aA^{+})^{p}(x), (aA^{+})^{p}(y) \}$. Therefore $(aA^{+})^{p}(x+y) \geq \min\{(aA^{+})^{p}(x), (aA^{+})^{p}(y) \}$ for x and y in R. And $(aA^{+})^{p}(xy) = p^{+}(a)A^{+}(xy) \geq p^{+}(a) \min\{A^{+}(x), A^{+}(y) \} = \min\{ p^{+}(a)A^{+}(x), p^{+}(a)A^{+}(y) \} = \min\{ (aA^{+})^{p}(x), (aA^{+})^{p}(y) \}$. Therefore $(aA^{+})^{p}(xy) \geq \min\{(aA^{+})^{p}(x), (aA^{+})^{p}(y) \}$ for x and y in R. Also $(aA^{-})^{p}(x+y) = -p^{-}(a)A^{-}(x+y) \leq$

$-p^-(a)$ max { $A^-(x)$, $A^-(y)$ } = max { $-p^-(a)A^-(x)$, $-p^-(a)A^-(y)$ } = max {$(aA^-)^p(x)$, $(aA^-)^p(y)$ }. Therefore $(aA^-)^p(x+y) \leq$ max { $(aA^-)^p(x)$, $(aA^-)^p(y)$ } for x and y in R. And $(aA^-)^p(xy) = -p^-(a)A^-(xy) \leq -p^-(a)$ max { $A^-(x)$, $A^-(y)$ } = max {$-p^-(a)A^-(x)$, $-p^-(a)A^-(y)$ }= max { $(aA^-)^p(x)$, $(aA^-)^p(y)$ }. Therefore $(aA^-)^p(xy) \leq$ max {$(aA^-)^p(x)$, $(aA^-)^p(y)$ } for x and y in R. Hence $(aA)^p$ is a bipolar valued fuzzy subsemiring of the semiring R.

2.3 Theorem: If A = 〈 A^+, A^- 〉 is a bipolar valued fuzzy subsemiring of a semiring R, then ?(*A*) = 〈 ?A^+, ?A^- 〉 is a bipolar valued fuzzy subsemiring of R.

Proof: For every x and y in R, we have ?$A^+(x+y)$ = min { ½, $A^+(x+y)$ } ≥ min { ½, min { $A^+(x)$, $A^+(y)$ } } = min { min { ½, $A^+(x)$ }, min { ½, $A^+(y)$ } } = min { ?$A^+(x)$, ?$A^+(y)$ }. Therefore ?$A^+(x+y)$ ≥ min { ?$A^+(x)$, ?$A^+(y)$ } for all x and y in R. Also ?$A^+(xy)$ = min { ½, $A^+(xy)$ }≥ min { ½, min { $A^+(x)$, $A^+(y)$ } } = min { min { ½, $A^+(x)$ }, min { ½, $A^+(y)$ } } = min { ?$A^+(x)$, ?$A^+(y)$ }. Therefore ?$A^+(xy)$ ≥ min {?$A^+(x)$, ?$A^+(y)$ } for all x and y in R. And ?$A^-(x+y)$ = max { –½, $A^-(x+y)$ } ≤ max {–½, max { $A^-(x)$, $A^-(y)$ } }= max { max {–½, $A^-(x)$ }, max { –½, $A^-(y)$ } } = max {?$A^-(x)$, ?$A^-(y)$ }. Therefore ?$A^-(x+y)$ ≤ max { ?$A^-(x)$, ?$A^-(y)$ } for all x and y in R. Also ?$A^-(xy)$ = max { –½, $A^-(xy)$ } ≤ max { –½, max { $A^-(x)$, $A^-(y)$ } } = max { max { –½, $A^-(x)$ }, max { –½, $A^-(y)$ } } = max { ?$A^-(x)$, ?$A^-(y)$ }. Therefore ?$A^-(xy)$ ≤ max { ?$A^-(x)$, ?$A^-(y)$ } for all x and y in R. Hence ?A is a bipolar valued fuzzy subsemiring of R.

2.4 Theorem: If A = 〈 A^+, A^- 〉 is a bipolar valued fuzzy subsemiring of a semiring R, then !(*A*) = 〈 !A^+, !A^- 〉 is a bipolar valued fuzzy subsemiring of R.

Proof: For every x and y in R, we have !$A^+(x+y)$ = max { ½, $A^+(x+y)$ } ≥ max { ½, min { $A^+(x)$, $A^+(y)$ }}= min { max { ½, $A^+(x)$ }, max { ½, $A^+(y)$ }} = min { !$A^+(x)$, !$A^+(y)$ }. Therefore !$A^+(x+y)$ ≥ min { !$A^+(x)$, !$A^+(y)$ } for all x and y in R. And !$A^+(xy)$ = max { ½, $A^+(xy)$ } ≥ max { ½, min { $A^+(x)$, $A^+(y)$ }} = min { max { ½, $A^+(x)$ }, max { ½, $A^+(y)$ }} = min { !$A^+(x)$, !$A^+(y)$ }. Therefore !$A^+(xy)$ ≥ min {!$A^+(x)$, !$A^+(y)$ } for all x and y in R. Also !$A^-(x+y)$ = min { –½, $A^-(x+y)$ } ≤ min {–½, max { $A^-(x)$, $A^-(y)$ }} = max { min { –½, $A^-(x)$ }, min { –½, $A^-(y)$ }} = max {!$A^-(x)$, !$A^-(y)$ }. Therefore !$A^-(x+y)$ ≤ max { !$A^-(x)$, !$A^-(y)$ } for all x and y in R. And !$A^-(xy)$ = min { –½, $A^-(xy)$ } ≤ min { –½, max { $A^-(x)$, $A^-(y)$ }} = max { min {–½, $A^-(x)$ }, min { –½, $A^-(y)$ }} = max { !$A^-(x)$, !$A^-(y)$ }. Therefore !$A^-(xy)$ ≤ max { !$A^-(x)$, !$A^-(y)$ } for all x and y in R. Hence !A is a bipolar valued fuzzy subsemiring of R.

2.5 Theorem: If A = 〈 A^+, A^- 〉 is a bipolar valued fuzzy subsemiring of a semiring R, then $Q_{\alpha,\beta}(A)$ = 〈 $Q_{\alpha,\beta}(A)^+$, $Q_{\alpha,\beta}(A)^-$ 〉 is a bipolar valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1] and β in [–1, 0], we have $Q_{\alpha,\beta}(A)^+(x+y)$ = min { α, $A^+(x+y)$ } ≥ min { α, min { $A^+(x)$, $A^+(y)$ }} = min { min { α, $A^+(x)$ }, min {α, $A^+(y)$ }} = min { $Q_{\alpha,\beta}(A)^+(x)$, $Q_{\alpha,\beta}(A)^+(y)$ }. Therefore $Q_{\alpha,\beta}(A)^+(x+y)$ ≥ min{ $Q_{\alpha,\beta}(A)^+(x)$, $Q_{\alpha,\beta}(A)^+(y)$} for all x and y in R. And $Q_{\alpha,\beta}(A)^+(xy)$ = min {α, $A^+(xy)$ } ≥ min { α, min { $A^+(x)$, $A^+(y)$ }} = min { min { α, $A^+(x)$ }, min { α, $A^+(y)$ }} = min {$Q_{\alpha,\beta}(A)^+(x)$, $Q_{\alpha,\beta}(A)^+(y)$}. Therefore $Q_{\alpha,\beta}(A)^+(xy)$ ≥ min{ $Q_{\alpha,\beta}(A)^+(x)$, $Q_{\alpha,\beta}(A)^+(y)$ } for all x and y in R. Also $Q_{\alpha,\beta}(A)^-(x+y)$ = max { β, $A^-(x+y)$ } ≤ max { β, max {$A^-(x)$, $A^-(y)$ }} = max { max { β, $A^-(x)$ }, max { β, $A^-(y)$ } } = max { $Q_{\alpha,\beta}(A)^-(x)$, $Q_{\alpha,\beta}(A)^-(y)$ }. Therefore $Q_{\alpha,\beta}(A)^-(x+y)$ ≤ max { $Q_{\alpha,\beta}(A)^-(x)$, $Q_{\alpha,\beta}(A)^-(y)$ } for all x, y in R. And $Q_{\alpha,\beta}(A)^-(xy)$ = max { β, $A^-(xy)$ } ≤ max { β, max { $A^-(x)$, $A^-(y)$ }} = max { max { β, $A^-(x)$ }, max { β, $A^-(y)$ }} = max { $Q_{\alpha,\beta}(A)^-(x)$, $Q_{\alpha,\beta}(A)^-(y)$ }. Therefore $Q_{\alpha,\beta}(A)^-(xy)$ ≤ max { $Q_{\alpha,\beta}(A)^-(x)$, $Q_{\alpha,\beta}(A)^-(y)$} for all x and y in R. Hence $Q_{\alpha,\beta}(A)$ is a bipolar valued fuzzy subsemiring of R.

2.6 Theorem: If A = 〈 A^+, A^- 〉 is a bipolar valued fuzzy subsemiring of a semiring R, then $P_{\alpha,\beta}(A)$ = 〈 $P_{\alpha,\beta}(A)^+$, $P_{\alpha,\beta}(A)^-$〉 is a bipolar valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1] and β in [–1, 0], we have $P_{\alpha,\beta}(A)^+(x+y)$ = max { α, $A^+(x+y)$ } ≥ max { α, min { $A^+(x)$, $A^+(y)$ }} = min { max { α, $A^+(x)$ }, max{ α, $A^+(y)$ }} = min{ $P_{\alpha,\beta}(A)^+(x)$, $P_{\alpha,\beta}(A)^+(y)$ }. Therefore $P_{\alpha,\beta}(A)^+(x+y)$ ≥ min{ $P_{\alpha,\beta}(A)^+(x)$, $P_{\alpha,\beta}(A)^+(y)$ } for all x and y in R. And $P_{\alpha,\beta}(A)^+(xy)$ = max { α,

$A^+(xy)$ } $\geq$ max { α, min { $A^+(x)$, $A^+(y)$ }} = min { max { α, $A^+(x)$ }, max { α, $A^+(y)$ }} = min { $P_{\alpha,\beta}(A)^+(x)$, $P_{\alpha,\beta}(A)^+(y)$ }. Therefore $P_{\alpha,\beta}(A)^+(xy) \geq$ min { $P_{\alpha,\beta}(A)^+(x)$, $P_{\alpha,\beta}(A)^+(y)$ } for all x and y in R. Also $P_{\alpha,\beta}(A)^-(x+y)$ = min { β, $A^-(x+y)$ } $\leq$ min { β, max { $A^-(x)$, $A^-(y)$ }} = max{ min{ β, $A^-(x)$ }, min{ β, $A^-(y)$}} = max{ $P_{\alpha,\beta}(A)^-(x)$, $P_{\alpha,\beta}(A)^-(y)$ }. Therefore $P_{\alpha,\beta}(A)^-(x+y) \leq$ max{ $P_{\alpha,\beta}(A)^-(x)$, $P_{\alpha,\beta}(A)^-(y)$ } for all x and y in R. And $P_{\alpha,\beta}(A)^-(xy)$ = min { β, $A^-(xy)$ }$\leq$ min { β, max { $A^-(x)$, $A^-(y)$ }} = max { min { β, $A^-(x)$ }, min { β, $A^-(y)$ }} = max { $P_{\alpha,\beta}(A)^-(x)$, $P_{\alpha,\beta}(A)^-(y)$ }. Therefore $P_{\alpha,\beta}(A)^-(xy) \leq$ max { $P_{\alpha,\beta}(A)^-(x)$, $P_{\alpha,\beta}(A)^-(y)$ } for all x and y in R. Hence $P_{\alpha,\beta}$ (A) is a bipolar valued fuzzy subsemiring of R.

2.7 Theorem: If A = ⟨ A^+, A^- ⟩ is a bipolar valued fuzzy subsemiring of a semiring R, then $G_{\alpha,\beta}(A)$ = ⟨ $G_{\alpha,\beta}(A)^+$, $G_{\alpha,\beta}(A)^-$ ⟩ is a bipolar valued fuzzy subsemiring of R.

Proof: For every x and y in R, α in [0, 1] and β in [–1, 0], we have $G_{\alpha,\beta}(A)^+(x+y)$ = α $A^+(x+y) \geq$ α (min{ $A^+(x)$, $A^+(y)$ }) = min { α$A^+(x)$, α$A^+(y)$ } = min { $G_{\alpha,\beta}(A)^+(x)$, $G_{\alpha,\beta}(A)^+(y)$ }. Therefore $G_{\alpha,\beta}(A)^+(x+y) \geq$ min{$G_{\alpha,\beta}(A)^+(x)$, $G_{\alpha,\beta}(A)^+(y)$ } for all x and y in R. And $G_{\alpha,\beta}(A)^+(xy)$ = α $A^+(xy) \geq$ α (min { $A^+(x)$, $A^+(y)$ }) = min { α $A^+(x)$, α $A^+(y)$ } = min{$G_{\alpha,\beta}(A)^+(x)$, $G_{\alpha,\beta}(A)^+(y)$}. Therefore $G_{\alpha,\beta}(A)^+(xy) \geq$ min{$G_{\alpha,\beta}(A)^+(x)$, $G_{\alpha,\beta}(A)^+(y)$ } for all x and y in R. Also $G_{\alpha,\beta}(A)^-(x+y)$ = –β $A^-(x+y) \leq$ –β (max {$A^-(x)$, $A^-(y)$ }) = max { –β $A^-(x)$, –β $A^-(y)$ } = max { $G_{\alpha,\beta}(A)^-(x)$, $G_{\alpha,\beta}(A)^-(y)$ }. Therefore $G_{\alpha,\beta}(A)^-(x+y) \leq$ max{$G_{\alpha,\beta}(A)^-(x)$, $G_{\alpha,\beta}(A)^-(y)$} for all x and y in R. And $G_{\alpha,\beta}(A)^-(xy)$ = –β $A^-(xy) \leq$ –β (max { $A^-(x)$, $A^-(y)$ }) = max{ –β $A^-(x)$, –β $A^-(y)$ } = max{ $G_{\alpha,\beta}(A)^-(x)$, $G_{\alpha,\beta}(A)^-(y)$ }. Therefore $G_{\alpha,\beta}(A)^-(xy) \leq$ max{ $G_{\alpha,\beta}(A)^-(x)$, $G_{\alpha,\beta}(A)^-(y)$ } for all x and y in R. Hence $G_{\alpha,\beta}$ (A) is a bipolar valued fuzzy subsemiring of R.

2.8 Theorem: If *A* = ⟨ A^+, A^- ⟩ and *B* = ⟨ B^+, B^- ⟩ are bipolar valued fuzzy subsemirings of a semiring R, then !(*A*∩*B*) = !(*A*) ∩!(*B*) is also a bipolar valued fuzzy subsemiring of R.

Proof: By Theorem 2.1 and 2.4, it is true.

2.9 Theorem: If *A* = ⟨ A^+, A^- ⟩ and *B* = ⟨ B^+, B^- ⟩ are bipolar valued fuzzy subsemirings of a semiring R, then ?(*A*∩*B*) = ?(*A*) ∩ ?(*B*) is also a bipolar valued fuzzy subsemiring of R.

Proof: By Theorem 2.1 and 2.3, it is true.

2.10 Theorem: If A = ⟨ A^+, A^- ⟩ is a bipolar valued fuzzy subsemiring of a semiring R, then !(?(*A*)) = ?(!(*A*)) is also a bipolar valued fuzzy subsemiring of R.

Proof: By Theorem 2.3 and 2.4, it is true.

2.11 Theorem: If *A* = ⟨ A^+, A^- ⟩ and *B* = ⟨ B^+, B^- ⟩ are bipolar valued fuzzy subsemirings of a semiring R, then $P_{\alpha,\beta}(A\cap B) = P_{\alpha,\beta}(A) \cap P_{\alpha,\beta}(B)$ is also a bipolar valued fuzzy subsemiring of R.

Proof: By Theorem 2.1 and 2.6, it is true.

2.12 Theorem: If *A* = ⟨ A^+, A^- ⟩ and *B* = ⟨ B^+, B^- ⟩ are bipolar valued fuzzy subsemirings of a semiring R, then $Q_{\alpha,\beta}(A\cap B) = Q_{\alpha,\beta}(A) \cap Q_{\alpha,\beta}(B)$ is also a bipolar valued fuzzy subsemiring of R.

Proof: By Theorem 2.1 and 2.5, it is true.

2.13 Theorem: If A = ⟨ A^+, A^- ⟩ is a bipolar valued fuzzy subsemiring of a semiring R, then $P_{\alpha,\beta}(Q_{\alpha,\beta}(A)) = Q_{\alpha,\beta}(P_{\alpha,\beta}(A))$ is also a bipolar valued fuzzy subsemiring of R.

Proof: By Theorem 2.5 and 2.6, it is true.

2.14 Theorem: If *A* = ⟨ A^+, A^- ⟩ and *B* = ⟨ B^+, B^- ⟩ are bipolar valued fuzzy subsemirings of a semiring R, then $G_{\alpha,\beta}(A\cap B) = G_{\alpha,\beta}(A) \cap G_{\alpha,\beta}(B)$ is also a bipolar valued fuzzy subsemiring of R.

Proof: By Theorem 2.1 and 2.7, it is true.

2.15 Theorem: If A is a bipolar valued fuzzy subsemiring of a semiring R, then $^{\oplus}A$ is a bipolar valued fuzzy subsemiring of the semiring R.

Proof: Let x and y in R. We have $^{\oplus}A^+(x+y) = A^+(x+y) +1 – H(A^+) \geq$ min { $A^+(x)$, $A^+(y)$ }+1– $H(A^+)$ = min { $A^+(x)$ +1– $H(A^+)$, $A^+(y)$ + 1– $H(A^+)$ } = min { $^{\oplus}A^+(x)$, $^{\oplus}A^+(y)$ } which implies $^{\oplus}A^+(x+y) \geq$ min { $^{\oplus}A^+(x)$, $^{\oplus}A^+(y)$ } for all x and y in R. And $^{\oplus}A^+(xy) = A^+(xy) +1 – H(A^+) \geq$ min { $A^+(x)$, $A^+(y)$ }+1– $H(A^+)$ = min { $A^+(x)$ +1– $H(A^+)$, $A^+(y)$ + 1– $H(A^+)$ } = min { $^{\oplus}A^+(x)$, $^{\oplus}A^+(y)$ } which implies $^{\oplus}A^+(xy) \geq$ min {$^{\oplus}A^+(x)$, $^{\oplus}A^+(y)$ } for all x and y in R. Also ${}^{\oplus}A^{-}(x+y) = A^{-}(x+y) - 1 - H(A^{-}) \leq \max\{A^{-}(x), A^{-}(y)\} - 1 - H(A^{-}) = \max\{A^{-}(x) - 1 - H(A^{-}), A^{-}(y) - 1 - H(A^{-})\} = \max\{{}^{\oplus}A^{-}(x), {}^{\oplus}A^{-}(y)\}$ which implies ${}^{\oplus}A^{-}(x+y) \leq \max\{{}^{\oplus}A^{-}(x), {}^{\oplus}A^{-}(y)\}$ for all x and y in R. And ${}^{\oplus}A^{-}(xy) = A^{-}(xy) - 1 - H(A^{-}) \leq \max\{A^{-}(x), A^{-}(y)\} - 1 - H(A^{-}) = \max\{A^{-}(x) - 1 - H(A^{-}), A^{-}(y) - 1 - H(A^{-})\} = \max\{{}^{\oplus}A^{-}(x), {}^{\oplus}A^{-}(y)\}$ which implies ${}^{\oplus}A^{-}(xy) \leq \max\{{}^{\oplus}A^{-}(x), {}^{\oplus}A^{-}(y)\}$ for all x and y in R. Hence ${}^{\oplus}A$ is a bipolar valued fuzzy subsemiring of R.

2.16 Theorem: Let A be a bipolar valued fuzzy subsemiring of a semiring R. Then

(i) $H(A^{+}) = 1$ if and only if ${}^{\oplus}A^{+}(x) = A^{+}(x)$ for all x in R

(ii) $H(A^{-}) = -1$ if and only if ${}^{\oplus}A^{-}(x) = A^{-}(x)$ for all x in R.

(iii) ${}^{\oplus}A^{+}(x) = 1$ if and only if $H(A^{+}) = A^{+}(x)$ for all x in R

(iv) ${}^{\oplus}A^{-}(x) = -1$ if and only if $H(A^{-}) = A^{-}(x)$ for all x in R.

(v) ${}^{\oplus}({}^{\oplus}A) = {}^{\oplus}A$.

Proof: It is trivial.

2.17 Theorem: If A is a bipolar valued fuzzy subsemiring of a semiring R, then ${}^{0}A$ is a bipolar valued fuzzy subsemiring of the semiring R.

Proof: For any x in R, we have ${}^{0}A^{+}(x+y) = A^{+}(x+y)H(A^{+}) \geq \min\{A^{+}(x), A^{+}(y)\}H(A^{+}) = \min\{A^{+}(x)H(A^{+}), A^{+}(y)H(A^{+})\} = \min\{{}^{0}A^{+}(x), {}^{0}A^{+}(y)\}$ which implies that ${}^{0}A^{+}(x+y) \geq \min\{{}^{0}A^{+}(x), {}^{0}A^{+}(y)\}$ for all x and y in R. And ${}^{0}A^{+}(xy) = A^{+}(xy)H(A^{+}) \geq \min\{A^{+}(x), A^{+}(y)\}H(A^{+}) = \min\{A^{+}(x)H(A^{+}), A^{+}(y)H(A^{+})\} = \min\{{}^{0}A^{+}(x), {}^{0}A^{+}(y)\}$. Therefore ${}^{0}A^{+}(xy) \geq \min\{{}^{0}A^{+}(x), {}^{0}A^{+}(y)\}$ for all x and y in R. Also ${}^{0}A^{-}(x+y) = -A^{-}(x+y)H(A^{-}) \leq (-)\max\{A^{-}(x), A^{-}(y)\}H(A^{-}) = \max\{-A^{-}(x)H(A^{-}), -A^{-}(y)H(A^{-})\} = \max\{{}^{0}A^{-}(x), {}^{0}A^{-}(y)\}$ which implies that ${}^{0}A^{-}(x+y) \leq \max\{{}^{0}A^{-}(x), {}^{0}A^{-}(y)\}$ for all x and y in R. And ${}^{0}A^{-}(xy) = -A^{-}(xy)H(A^{-}) \leq (-)\max\{A^{-}(x), A^{-}(y)\}H(A^{-}) = \max\{-A^{-}(x)H(A^{-}), -A^{-}(y)H(A^{-})\} = \max\{{}^{0}A^{-}(x), {}^{0}A^{-}(y)\}$. Therefore ${}^{0}A^{-}(xy) \leq \max\{{}^{0}A^{-}(x), {}^{0}A^{-}(y)\}$ for all x and y in R. Hence ${}^{0}A$ is a bipolar valued fuzzy subsemiring of the semiring R.

2.18 Theorem: If A is a bipolar valued fuzzy subsemiring of a semiring R, then ${}^{\Delta}A$ is a bipolar valued fuzzy subsemiring of R.

Proof: For any x in R, we have ${}^{\Delta}A^{+}(x+y) = A^{+}(x+y) / H(A^{+}) \geq \min\{A^{+}(x), A^{+}(y)\} / H(A^{+}) = \min\{A^{+}(x) / H(A^{+}), A^{+}(y) / H(A^{+})\} = \min\{{}^{\Delta}A^{+}(x), {}^{\Delta}A^{+}(y)\}$ which implies that ${}^{\Delta}A^{+}(x+y) \geq \min\{{}^{\Delta}A^{+}(x), {}^{\Delta}A^{+}(y)\}$ for all x and y in R. And ${}^{\Delta}A^{+}(xy) = A^{+}(xy) / H(A^{+}) \geq \min\{A^{+}(x), A^{+}(y)\} / H(A^{+}) = \min\{A^{+}(x) / H(A^{+}), A^{+}(y) / H(A^{+})\} = \min\{{}^{\Delta}A^{+}(x), {}^{\Delta}A^{+}(y)\}$. Therefore ${}^{\Delta}A^{+}(xy) \geq \min\{{}^{\Delta}A^{+}(x), {}^{\Delta}A^{+}(y)\}$ for all x and y in R. Also ${}^{\Delta}A^{-}(x+y) = -A^{-}(x+y) / H(A^{-}) \leq (-)\max\{A^{-}(x), A^{-}(y)\} / H(A^{-}) = \max\{-A^{-}(x) / H(A^{-}), -A^{-}(y) / H(A^{-})\} = \max\{{}^{\Delta}A^{-}(x), {}^{\Delta}A^{-}(y)\}$ which implies that ${}^{\Delta}A^{-}(x+y) \leq \max\{{}^{\Delta}A^{-}(x), {}^{\Delta}A^{-}(y)\}$ for all x and y in R. And ${}^{\Delta}A^{-}(xy) = -A^{-}(xy) / H(A^{-}) \leq (-)\max\{A^{-}(x), A^{-}(y)\} / H(A^{-}) = \max\{-A^{-}(x) / H(A^{-}), -A^{-}(y) / H(A^{-})\} = \max\{{}^{\Delta}A^{-}(x), {}^{\Delta}A^{-}(y)\}$. Therefore ${}^{\Delta}A^{-}(xy) \leq \max\{{}^{\Delta}A^{-}(x), {}^{\Delta}A^{-}(y)\}$ for all x and y in R. Hence ${}^{\Delta}A$ is a bipolar valued fuzzy subsemiring of the semiring R.

2.19 Theorem: If A is a bipolar valued normal fuzzy subsemiring of a semiring R, then ${}^{0}A = A$.

Proof: It is trivial.

2.20 Theorem: If A is a bipolar valued normal fuzzy subsemiring of a semiring R, then ${}^{\Delta}A = A$.

Proof: It is trivial.

2.21 Theorem: Let A be a bipolar valued fuzzy subsemiring of a semiring R. (i) If $H(A^{+}) < 1$, then ${}^{0}A^{+} < A^{+}$.

(ii) $H(A^{-}) > -1$, then ${}^{0}A^{-} > A^{-}$.

(iii) $H(A^{+}) < 1$ and $H(A^{-}) > -1$, then ${}^{0}A < A$.

Proof: It is trivial.

2.22 Theorem: Let A be a bipolar valued fuzzy subsemiring of a semiring R. (i) If $H(A^{+}) < 1$, then ${}^{\Delta}A^{+} > A^{+}$.

(ii) $H(A^-) > -1$, then $^{\Delta}A^- < A^-$.

(iii) $H(A^+) < 1$ and $H(A^-) > -1$, then $^{\Delta}A > A$.

(iii) $H(A^+) < 1$ and $H(A^-) > -1$, then $^{\Delta}A$ is a bipolar valued normal fuzzy subsemiring of R.

Proof: It is trivial.

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