Vol.4.Issue.4.2016 (October -December)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

RESEARCH ARTICLE

ISSN:2348-0580

PGRW-OPEN MAP IN A TOPOLOGICAL SPACE

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ABSTRACT

The aim of this paper is to introduce pgrw-open map, pgrw*-open map and to obtain some of their properties. In section 3 a pgrw-open map is defined and compared with other open maps. In section 4 compositions of two pgrw-open maps is considered. In section 5 pgrw*-open map is defined and compared with pgrw-open map.

Keywords: pgrw-open set, pgrw-open map, pgrw*-open map.

1. Introduction

Different mathematicians worked on different versions of generalized open maps and related topological properties. δ-open[23], gpr-open [10], gp-open[14] rw-open [3]mappings were introduced and studied by Raja Mohammad Latif, Gnanambal Y, T. Noiri, H. Maki and J. Umehara, Benchalli S.S. and Wali R.S. respectively.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) represent the topological spaces. For a subset A of a space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A or A^c denotes the complement of A in X.

We recall the following definitions and results.

Definition 2.1

A subset A of a topological space (X, τ) is called

1. a semi-open set[13] if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A.
2. a pre-open set[5] if A ⊆ int(cl(A)) and a pre-closed set if cl(int(A)) ⊆ A.
3. an α-open set [16]if A ⊆ int(cl(int(A))) and an α -closed set if cl(int(cl(A)))⊆ A.
4. a semi-pre open set [1,18](β-open)[10] if A⊆cl(int(cl(A))) and a semi-pre closed set =β-closed) int(cl(int(A)))⊆A.
5. a regular open set [15] if A = int(clA)) and a regular closed set if A = cl(int(A)).
6. δ-closed [23] if A = cl δ(A), where clδ(A) = {xϵ X : int(cl(U))∩A≠ Φ, U ϵ τ and x ϵ U}

7. a regular α-open set (briefly, rα-open)[28] if there is a regular open set U such that U ⊂ A ⊂αcl(U).
8. a generalized closed set (briefly g-closed)[4] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
9. a generalized pre regular closed set(briefly gpr-closed)[10] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
10. a generalized semi-pre closed set(briefly gsp-closed)[9] if spcl(A) ⊆ U whenever A⊆U and U is open in X.
11. a w-closed set [26]if cl(A) ⊆ U whenever A⊆ U and U is semi-open in X.
12. a pre generalized pre regular closed set[2] (briefly pgpr-closed) if pcl(A)⊆ U whenever A⊆U and U is rg- open in X.
13. a generalized semi pre regular closed **(**briefly gspr-closed) set [11] if spcl(A)⊆U whenever A⊆U and U is regular open in X.
14. a generalized pre closed (briefly gp-closed) set[20] if pcl(A)⊆U whenever A⊆U and U is open in X.
15. a **#**regular generalized closed (briefly #rg-closed) set [27] if cl(A)⊆U whenever A ⊆ U and U is rw-open.
16. a g*s-closed [22]set if scl (A)⊆ U whenever A⊆ U, U is gs open.
17. a rw-closed [6] if cl(A)⊆U whenever A ⊆U and U is regular semi-open in X.
18. αg-closed[16] if αcl(A)⊆U whenever A ⊆U and U is open. In X.
19. a ωα-closed set[7] if αcl(A) ⊆ U whenever A⊆ U and U is ω-open in X.
20. an α-regular w closed set(briefly αrw -closed)[29] if αcl(A)⊆U whenever A⊆U and U is rw-open in X.
21. a rg-closed set[19] if cl(A) ⊆ U whenever A⊆ U and U is regular-open in X.

The complements of the above mentioned closed sets are the respective open sets.

Definition 2.2: A map f : (X, τ) → (Y, σ) is said to be

1. α-open[16] if f(F) is α–open in Y for every open subset F of X.
2. αg-open[16] if f(F) is αg-open in Y for every open subset F of X.
3. ωα-open[7] if f(V) is ωα-open in Y for every open subset V of X.
4. ω-open[26] if f(V) is ω-open in Y for every closed subset V of X.
5. rω-open[6] if f(V) is rw-open in Y for every closed subset V of X.
6. g*s -open [22] if for each open set F in X,f(F) is a g*s- open in y.
7. Contra open [3] if f(F) is closed in Y for every open set F of X.
8. Contra regular- open[5] if f(F) is r-closed in Y for every open set F of X.
9. Contra semi-open[24] if f(F) is s-closed in Y for every open set F of X.
10. Semi pre- open [18] if f(V) is semi-pre- open in Y for every open subset V of X.
11. g-open[4,21] if f(V) is g-opend in Y for every open subset V of X.
12. rα-open(28) if f(V) is rα-open in Y for every open subset V of X.
13. gpr-open [10] if f(U) is gpr-open in (Y, σ) for every open set U of (X, τ).
13. regular open [15] if f(U) is open in (Y, σ) for every regular open set U of (X, τ).
14. pre-open [17,24 if f (V) is pre-open in Y for every open set V of X.
15. gp-open[20] if the image of each open sets of X is gp-open in Y.
17. gspr–open[11] if f(V) is gspr-open in Y for every open subset V of X.
18. αrω-open [29] if the image of every open set in (X, τ) is αrω-open in Y, σ).
19. δ-open[23] if for every open set G in X, f (G) is a δ-open set in Y.
20. #rg-open [27] if f(F) is #rg-open in (Y,σ) for every #rg-open set F of X,τ.
21. gsp-open[9] if f(V) is gsp-open in (Y, σ) for every open set V of (X, τ).

The following results are from [31]

Theorem: Every pgpr- open set is pgrw- open.

Theorem: If A is regular closed and pgrw-open, then A is pre-open.

Theorem: If A is closed and gp-open, then A is pgrw-open.

Theorem: If A is both closed and g-open, then A is pgrw-open.

Theorem: If A is regular- closed and gpr-open, then it is pgrw-open.

Theorem : If A is both semi- closed and w-open, then it is pgrw-open.

Theorem: If A is closed and αg-open, then it is pgrw-open**.**

The following results are from [31]

Theorem: For any topological space X,

i. Every open (α-open, regular-open, αrω-open, **#**rg-open, pgpr-open) set is pgrw-open.

ii. Every pgrw-open set is gp–open (gspr-open, gsp-open and gpr-open) .

Definition 2.3: pgrw-irresolute map[32]**:** A map f: (X, τ)→(Y, σ) is called a pre generalized regular weakly irresolute (pgrw–irresolute) map if $f^{-1}(V)$ is a pgrw–closed set in X for every pgrw–closed set V in Y.

Definition 2.3: pgrw-closed map[30,31]: A map f: (X, τ) → (Y, σ) is called a pgrw-closed map if for every closed set A in (X, τ) the image f(A) is a pgrw-closed set in (Y, σ).

3. pgrw-open maps:

Definition 3.1: A map f: (X, τ) → (Y, σ) is called a pgrw-open map if for every open set A in (X, τ) the image f(A) is a pgrw-open set in (Y, σ).

Example 3.2 :X={a,b,c,d} , τ ={X, φ, {a}, {b}, {a,b}, {a,b,c}}.

Y={a,b,c} , σ ={Y, φ, {a}, {b}, {a,b}}. pgrw-open sets in Y are Y, φ, {a}, {b}, {a,b}. A map f: (X, τ) → (Y, σ) is defined by f(a)=b, f(b)=a, f(c)=c, f(d)=a. Then f is a pgrw-open map.

Theorem 3.3: Every open map is a pgrw-open map.

Proof: f: (X, τ) → (Y, σ) is an open map.

⇒ ∀ open set A of X f(A) is open in Y.

⇒ ∀ open set A of X f(A) is pgrw-open in Y.

⇒ f:(X, τ) → (Y, σ) is a pgrw- open map.

The converse is not true.

Example 3.4: X={a,b,c} , τ ={X, φ, {a}, {b,c}} and Y={a,b,c,d} , σ ={Y, φ, {a}, {b}, {a,b}, {a,b,c}}. pgrw-open sets in Y are Y,φ,{a},{b},{c},{a,b}, {a,c}, {b,c}, {a,d}, {a,b,c},{a,b,d}. A map f: (X, τ) → (Y, σ) is defined by f(a)=c,f(b)=a,f(c)=a. f is a pgrw-open map, but not an open map.

Theorem 3.5: Every pre-open (regular-open, α-open,δ-open, #rg- open, pgpr-open, αrw-open) map is pgrw-open.

Proof: f: (X, τ) → (Y, σ) is a pre-open map.

⇒ ∀ open set A in X f(A) is pre- open in Y.

⇒ ∀ open set A in X f(A) is pgrw- open in Y.

⇒ f is a pgrw-open map.

Similarly remaining statements may be proved.

The converse is not true.

Example 3.6:X={a,b,c } , τ ={X, φ, {a}, {b}, {a,b}}.

Y={a,b,c,d} , σ ={Y, φ, {a}, {b}, {a,b}, {a,b,c}}.

pgrw-open sets in Y are Y, φ, {a,b,d} ,{a,b,c},{a,d}, {a,b} , {b,c}, {a,c},{a}, {b}, {c}.

pre-open sets in Y are Y,φ,{a},{b},{a,b},{a,b,c},{a,b,d}.

Regular open sets in Y are Y, φ,{a}, {b}.

δ-open sets in Y are Y, φ,{a}, {b},{a,b}.

α-open sets in Y are Y, φ,{a}, {b},{a,b},{a,b,c},{a,b,d}.

pgpr-open sets in Y are Y, φ,{a}, {b},{b,c},},{a,b,c},{a,b,d}. #rg-open sets in Y are Y, φ,{a}, {b},{a,b},{b,c},{a,c},{b,d},{a,b,c}. A map f: (X, τ)→(Y, σ) is defined by f(a)=b, f(b)=c, f(c)= d. f is pgrw-open. f{a,b} ={b,c} is neither pre-open nor regular open nor δ-open nor α-open and f({b})={c} is neither pgpr-open nor #rg-open. So f is neither pre-open nor regular-open nor α-open nor δ-open nor #rg-open nor pgpr-open.

Example 3.7: X={a,b,c }, τ ={X,φ,{a}, {a,c}}.Y={a,b,c,d} , σ ={Y, φ, {a}, {b}, {a,b}, {a,b,c}}. pgrw-open sets in Y are Y, φ, {a,b,d},{a,b,c},{a,d}, {a,b}, {b,c}, {a,c},{a}, {b}, {c}. αrw-open sets in Y are Y, φ,{a}, {b}, {c} . A map f: (X, τ) → (Y, σ) is defined by f(a)=a, f(b)=c, f(c)= d. f is pgrw-open, but not αrw-open.

Theorem 3.8: If f: (X, τ) → (Y, σ) is a contra-r-open and pgrw-open map, then f is pre-open.

Proof: f: (X, τ) → (Y, σ) is a contra-r-open and pgrw-open map.

⇒ ∀ open set A in X f(A) is regular closed and pgrw-open in Y.

⇒ ∀ open set A in X f(A) is pre-open in Y.

⇒ f is a pre-open map.

Theorem 3.9: If f: (X, τ) → (Y, σ)is a pgrw-open map, then f is a gp-open(gsp-open, gspr-open, gpr-open) map.

Proof: f: (X, τ) → (Y, σ) is a pgrw-open map.

⇒ ∀ open set A in X f(A) is a pgrw-open set in Y.

⇒ ∀ open set A in X f(A) is a gp-open set in Y.

⇒ f is a gp-open map.

Similarly the other results follow.

The converse is not true.

Example 3.10:X={a,b,c}, τ **={**X,φ,{a},{b,c}}. Y={a,b,c}, σ={Y, φ ,{a}}.

pgrw-open sets in Y are Y, φ, {a,c}, {a,b},{a}. gp-open sets in Y are Y,φ,{a,c},{a,b} ,{c},{a},{b}. gpr-open sets in Y are all subsets of Y. A map f: (X, τ) → (Y, σ) is defined by f :X→Y as f(a)=b,f(b)=a,f(c)=c. f is gp-open and gpr-open but, f is not pgrw-open because f({a})={b} is not pgrw-open.

Example 3.11:X={a,b,c,d} , τ ={X, φ, {a},{b},{a,b},{a,b,c}}.

Y={a,b,c } , σ ={Y, φ, {a}, {b}, {a,b}}. pgrw-open sets in Y are Y,φ, {a,b}, {b}, {a}. gsp-open sets in Y are all subsets of Y. gspr-open sets in Y are all subsets of Y. A map f: (X, τ) → (Y, σ) is defined by f(a)=c, f(b)=a, f(c)=b, f(d)=c. f is a gsp-open and gspr-open map, but f is not a pgrw-open map.

Theorem 3.12: If a map f: (X, τ) → (Y, σ) is pgrw-open and A is an open set of X, then f_A: (A, τ_A) → (Y, σ) is pgrw-open.

Proof: A is an open set of X. Let F be an open set of (A, τ_A). Then F = A ∩ E for some open set E of (X, τ) and so F is an open set of (X, τ). Since f is a pgrw–open map, f (F) is pgrw-open set in (Y, σ). But for every F in A , f_A(F)= f(F) and ∴ f_A: (A, τ_A) → (Y, σ) is pgrw–open.

Theorem 3.13: For any bijective map f: (X, τ) → (Y, σ), the following statements are equivalent:

i) f^{-1}: (Y, σ) → (X, τ) is pgrw-continuous.

ii) f is pgrw-closed.

iii) f is pgrw-open.

Proof:(i) ⇒ (ii):f: (X, τ) → (Y, σ) is bijective.⇒ f^{-1}:(Y, σ)→ (X, τ) exists and$(f^{-1})^{-1}$=f and so f is bijective and f^{-1} is pgrw-continuous.

⇒ $(f^{-1})^{-1}$=f and ∀closed set F in X,$(f^{-1})^{-1}$(F) is pgrw-closed in Y.

⇒∀closed set F in X,f (F) is pgrw-closed in Y.

⇒f is a pgrw-closed map.

(ii) ⇒(iii): U is an open set in X, f: (X, τ) → (Y, σ) is a pgrw-closed map and f is bijective.

⇒f(U^c) is pgrw-closed in Y and f(U^c)=$(f(U))^c$.
⇒$(f(U))^c$ is pgrw-closed in Y.
⇒f(U) is pgrw-open in Y. Hence f is a pgrw-open map.
iii) ⇒(i): A map f: (X, τ) → (Y, σ) is bijective and pgrw-open.
⇒inverse map f^{-1} exists,$(f^{-1})^{-1}$=f and ∀ open set U in X, f(U) is pgrw-open.
⇒∀ open set U in X, $(f^{-1})^{-1}$(U) is pgrw-open in Y.
⇒f^{-1}: (Y, σ) → (X, τ) is a pgrw-continuous map.

Theorem 3.14:A map f: (X, τ) → (Y, σ) is pgrw-open if and only if for any subset S of (Y, σ) and for any closed set F containing f^{-1}(S) in X, ∃ a pgrw-closed set K in Y such that S⊆K and f^{-1}(K) ⊆F.

Proof: i)f: X → Y is a map, S⊆Y and f^{-1}(S)⊆F, a subset of X.
⇒S∩ f(X-F) =φ⇒S⊆Y-f(X-F).
ii) f: (X, τ) → (Y, σ) is a pgrw-open map and F is a closed set in X.
⇒f(X-F) is a pgrw-open set in Y. ⇒Y-f(X-F)=K(say) is a pgrw-closed set in Y.
⇒ f^{-1}(K)=X-f^{-1}(f(X-F)) ⊆X-(X-F) =F. From (i) and (ii) if f: (X, τ) → (Y, σ) is a pgrw-open map, then ∀S ⊆Y and ∀ closed set F containing f^{-1}(S) in X, ∃ a pgrw-closed set K=Y-f(X-F) such that S⊆K and f^{-1}(K) ⊆F.

Conversely

Suppose that f: (X, τ) → (Y, σ) is a map such that ∀ S⊆Y and for every closed set F containing f^{-1}(S) in X, ∃ a pgrw-closed set K in Y such that S ⊆K and f^{-1}(K) ⊆F.
∀ U⊆X, f^{-1}($(f(U))^c$) ⊆U^c. If U is an open subset of X, then U^c is closed in X. Take S =$(f(U))^c$ and F=U^c. Then by the hypothesis ∃ a pgrw- closed set K in Y such that $(f(U))^c$ ⊆K and f^{-1}(K) ⊆U^c. ⇒K^c⊆f(U) and U⊆$(f^{-1}(K))^c$⇒ K^c⊆f(U) and f(U)⊆f($(f^{-1}(K))^c$) ⊆K^c.
⇒ K^c⊆f(U) ⊆K^c⇒K^c =f(U). As K is pgrw-closed , K^c is pgrw-open. Thus ∀ open set U in X, f(U) is pgrw-open in Y. Hence f is a pgrw-open map.

Theorem 3.15: If a map f: (X, τ) → (Y, σ) is pgrw-open, then f^{-1}(pgrwcl(B)) ⊆cl(f^{-1}(B)) ∀ subset B of (Y, σ).

Proof: For any map f: X → Y and for any subset B of Y, f^{-1}(B)⊆cl(f^{-1}(B)) and cl(f^{-1}(B)) is closed in X. So f: (X, τ) → (Y, σ) is a pgrw-open map , B is any subset of Y and cl(f^{-1}(B)) is a closed set containing f^{-1}(B) in X. ⇒ ∃ a pgrw-closed set K of Y such that B ⊆K and f^{-1}(K) ⊆cl(f^{-1}(B)). ⇒pgrwcl(B) ⊆ pgrwcl(K)=K and f^{-1}(K) ⊆cl(f^{-1}(B)).
⇒f^{-1}(pgrw-cl(B)) ⊆cl(f^{-1}(B)).

Theorem 3.16: If a map f: (X, τ) → (Y, σ) is pgrw-open, then f(int(A)) ⊆ pgrw-int(f(A)) for every subset A of (X, τ).

Proof: Let f: (X, τ) → (Y, σ) be a pgrw- open map and A be any subset of (X, τ). Then int(A) is open in (X, τ) and so f(int(A))is pgrw-open in (Y, σ). As int(A)⊆ A, f(int(A)) ⊆ f(A). ∴by Theorem in [32], f(int(A)) ⊆pgrw-int(f(A)).

Theorem 3.17: If a map f : (X, τ) → (Y, σ) is pgrw-open, then for each neighbourhood U of x in (X, τ), there exists a pgrw-neighbourhood[32] W of f(x) inY such that W ⊆f(U).

Proof: Let f : (X, τ) → (Y, σ) be a pgrw–open map. Let x ϵ Xand U be a neighbourhood of x in (X, τ). Then ∃ an open set G in (X, τ) such that x ϵG ⊆U. Now f(x) ϵ f(G) ⊆f(U) and f(G) is a pgrw-open set in (Y, σ), because f is a pgrw-open map. By Theorem 6.7 in [33], f(G) is a pgrw-neighbourhood of each of its points. Taking f(G) = W, W is a pgrw-neighbourhood of f(x) in (Y, σ) such that W ⊆f(U).

Theorem 3.18: If f: (X, τ) → (Y, σ) is a contra open and gp-open map, then f is pgrw-open.

Proof : f: (X, τ) → (Y, σ) is a contra open and gp-open map.

⇒∀ open set V in X f(V) is a closed and gp-open set in Y .
⇒ ∀ open set V in X f(V) is pgrw-open in Y.
⇒ f is a pgrw-open map.

Theorem 3.19: If f: (X, τ) → (Y, σ) is a contra-open and αg-open map, then f is pgrw-open.

Proof: f: (X, τ) → (Y, σ) is a contra-open and αg-open map.
⇒∀ open set V in X f(V) is a closed and αg- open set in Y .
⇒ ∀ open set V in X f(V) is pgrw-open in Y.
⇒ f is a pgrw-open map.

Theorem 3.20: If f: (X, τ) → (Y, σ) is a contra regular- open and gpr-open map, then f is a pgrw-open map.

Proof: f: (X, τ) → (Y, σ) is a contra regular- open and gpr-open map.
⇒∀ open set V in X f(V) is a regular-closed and gpr- open set in Y .
⇒ ∀ open set V in X f(V) is pgrw-open in Y.
⇒ f is a pgrw-open map.

Theorem 3.21: If f: (X, τ) → (Y, σ) is a contra semi-open and w-open map, then f is a pgrw-open map.

Proof: f: (X, τ) → (Y, σ) is a contra semi-open and w-open map.
⇒∀ open set V in X f(V) is a semi-closed and w-open set in Y .
⇒ ∀ open set V in X f(V) is pgrw-open in Y.
⇒ f is a pgrw-open map.

Theorem 3.22: If f: (X, τ) → (Y, σ) is a contra open and g- open map, then f is pgrw-open.

Proof: f: (X, τ) → (Y, σ) is a contra open and g- open map.
⇒∀ open set V in X f(V) is a closed and g-open set in Y .
⇒ ∀ open set V in X f(V) is pgrw-open in Y.
⇒ f is a pgrw-open map.

The following examples illustrate that the pgrw-open map and rw-open map (g^*s-open map , rα-open map and wα- open map, β-open map,semi- open map) are independent.

Example 3.23: To show that pgrw- open map and rw- open map are independent.

i) X={a,b,c} , τ ={X, φ, {a}, {a,c}}. Y={a,b,c,d} , σ ={Y, φ, {a,b},{c,d}}.

pgrw-open sets in Y are all subsets of Y. rw-open sets in Y are Y,φ,{a,b},{c,d}. A map f: (X, τ) → (Y, σ) is defined by by f(a)=b, f(b)=c, f(c)=a. f is pgrw- open , but f is not rw- open.

ii) X={a,b,c},τ ={X, φ, {a}},Y={a,b,c} , σ ={Y, φ, {a}, {b}, {a,b}}.

pgrw-open sets in Y are Y, φ , {a,b}, {a}, {b}. rw-open sets in Y areY,φ,{a},{b},{c},{a,b}. A map f: (X, τ) → (Y, σ) is defined by f:(X, τ) → (Y, σ) by f(a)=c, f(b)=a, f(c)=b. f is not pgrw-open, but f is rw-open.

Example 3.24: To show that pgrw-open map and g^*s-open map are independent.

i)X={a,b,c,d} , τ ={X, φ, {a}, {a,c}}. Y={a,b,c} , σ ={Y, φ, {a},{b,c}}.

pgrw-open sets in Y are all subsets of Y. g*s-open sets in Y are Y,φ,{a}, {b,c}. A map f: (X, τ) → (Y, σ) is defined by f(a)=c, f(b)=a, f(c)=b,f(d)=b. f is pgrw-open , but f is not g*s-open.

ii)X={a,b,c,d} , τ ={X, φ, {b,c},{b,c,d} {a,b,c }}. Y={a,b,c,d} , σ ={Y, φ, {a}, {b},{a,b}, {a,b,c}}. pgrw-open sets in Y are Y,φ,{a,b,d}, {a,b,c}, {a,d}, {a,b},{b,c}, {a,c}, { a}, {b}, {c}. g^*s-open sets in Y are Y, φ,{b,c,d},{a,c,d},{a,b,d},{a,b,c}, {a,d}, {a,b}, {b,c}, {b,d}, {a},{b}. A map f: (X, τ) → (Y, σ) is defined by f(a)=c, f(b)=d, f(c)=b, f(d)=a. f is not pgrw-open , but f is g^*s-open.

Example 3.25: To show that pgrw-open map and rα-open map are independent.

i)X={a,b,c}, τ={X,φ,{a},{a,c}}and Y={a,b,c},σ ={Y, φ, {a},{b,c}}. Pgrw-open sets in Y are all subsets of Y. rα-open sets in Y are Y,φ ,{a},{b,c}. A map f: (X, τ) → (Y, σ) is defined by f(a)=c, f(b)=a, f(c)=b. f is a pgrw-open map, but f is not rα-open .

ii)X={a,b,c,d} , τ ={X, φ, {a}, {c,d}, {a,c,d}}. Y={a,b,c,d}, σ={Y,φ,{a}, {b}, {a,b}, {a,b,c}}. Pgrw-open sets in Y are Y, φ ,{a,b,d},{a,b,c},{a,d},{a,b},{b,c},{a,c},{a},{b},{c}.

rα-open sets in Y are Y,φ,{b,c,d},{a,c.d}, {a},{b},{b,c},{a,c},{b,d},{a,d}.

A map f: (X, τ) → (Y, σ) is defined by f(a)=d, f(b)=a, f(c)=c , f(d)=d. f is not a pgrw-open map, but f is rα-open.

Example 3.26: To show that pgrw- open map and wα- open map are independent.

i)X={a,b,c,d}, τ={X,φ,{a,b}, {c,d}}. Y={a,b,c,d},σ ={Y, φ,{b,c},{b,c,d},{a,b,c}}

Pgrw-open sets in Y are Y, φ , {b,c,d}, {a,c,d}, {a,b,d}, {a,b,c}, {c,d}, {b,d}, {a,b}, {b,c}, {a,c}, {b}, {c}. wα-open sets in Y are Y, φ, {b,c,d}, {a,b,c}, {b,c}, {b}, {c}. A map f: (X, τ) → (Y, σ) is defined by f(a)=c, f(b)=d, f(c)=a,f(d)=b. f is pgrw-open , but f is not wα-open.

ii) X={a,b,c}, τ ={X,φ,{a},{b}, {a,b}}. Y={a,b,c,d} σ ={Y, φ,{ a},{b}, {a,b},{a,b,c}}

Pgrw-open sets in Y are Y,φ,{a,b,d},{a,b,c},{a,d}, {a,b},{b,c},{a,c}, {a},{b,},{c}.

wα-open sets in Y are Y,φ , {b,c,d}, {a,c,d}, {a,b,d}, {c,d}, {a,d}, {b,d}, {d}, {c}.

A map f: (X, τ) → (Y, σ) is defined by f(a)=d, f(b)=c, f(c)=b. f is not pgrw-open ,but it is wα-open.

Example 3.27: To show that pgrw-open map and β-open map are independent.

i) X={a,b,c,d},τ={X,φ,{a},{b},{a,b},{a,b,c}} and

Y={a,b,c,d}, σ={Y,φ,{a},{b},{a,b},{a,b,c}} pgrw-open sets in Y are Y, φ, {a,b.d}, {a,b,c}, {a,d}, {a,b}, {b,c}, {a,c}, {a}, {b}, {c}. β-open sets in Y are Y, φ,{b,c,d},{a,c,d}, {a,b,d}, {a,b,c}, {a,d}, {b,d}, {a,c}, {a,b}, {b,c}, {a},{b}. A map f: (X, τ) → (Y, σ) is defined by f(a)=c,f(b)=b,f(c)=a,f(d)=d. f is pgrw-open map, but not β-open.

ii) A map f: (X, τ) → (Y, σ) is defined by f(a)=b, f(b)=b, f(c)=d, f(d)=c in the above example. f is β-open, but not pgrw-open.

Example 3.28: To show that pgrw-open map and semi-open map are independent.

i) X={a,b,c}, τ ={X,φ,{a}} and Y={a,b,c,d}, σ={Y,φ,{a},{b},{a,b},{a,b,c}}

Pgrw-open sets in Y are Y,φ,{a,b,d},{a,b,c},{a,d}, ,{a,b},{b,c},{a,c}, {a},{b},{c}.

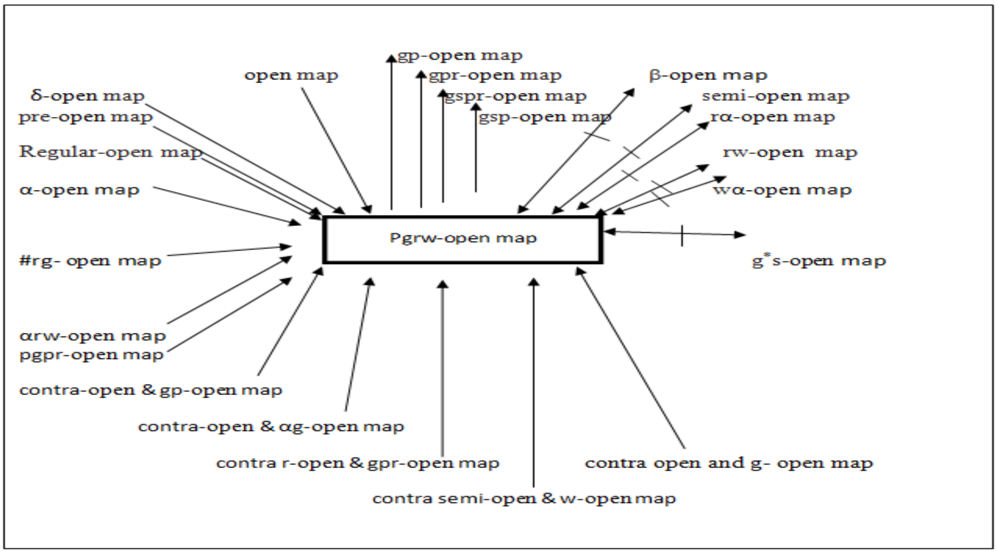
Semi-open sets in Y are Y, φ, {b,c,d},{a,c,d},{a,b,d},{a,b,c},{a,c},{b,c},{a,d},{a,b}, {b,c},{a},{b}. A map f: (X, τ) → (Y, σ) is defined by f(a)=c f(b)=a,f(c)=b. f is pgrw-open, but f is not semi-open.

ii) A map f: (X, τ) → (Y, σ) is defined by f(a)=b, f(b)=c, f(c)=d in the above example. f is semi-open, but not pgrw-open.

In the diagram,

A ⟶ B means'If A, then B.'

A ↮ B means ' A and B are independent.'



4. Composition of maps:

Remark 4.1: The composition of two pgrw-open maps need not be a pgrw-open map.

Example 4.2:X={a,b,c},τ ={X,φ,{a},{a,b}} . Y={a,b,c}, σ={Y, φ ,{a}}. pgrw-open sets in Y are Y,φ, {a,c},{a,b},{a}. Z={a,b,c}, η ={Z,,φ,{a},{c},{a,c}}. pgrw-open sets in Z are Z,φ,{a,c},{c},{a}. Let f: (X, τ) → (Y, σ) and g: (Y,σ) → (Z, η) be the identity maps. Then f and g are pgrw-open maps. The composition g∘f:(X, τ) → (Z, η) is not a pgrw-open map, because {a,b} is open in X and g∘f({a,b})= {a,b} is not pgrw-open in Z.

Theorem 4.3: If f: (X, τ) → (Y, σ) is an open map and g: (Y,σ) → (Z, η) is a pgrw-open map, then the composition g∘f:(X, τ) → (Z, η) is a pgrw-open map.

Proof: f: (X, τ) → (Y, σ) is an open map and g: (Y,σ) → (Z, η) is a pgrw-open map.

⇒∀ open set F in X f(F) is an open set in (Y, σ) and g(f(F)) is a pgrw-open set in (Z, η).

⇒ ∀ open set F in X g∘f(F) =g(f(F)) is a pgrw-open set in (Z, η).

⇒ g∘f:(X, τ) → (Z, η) is a pgrw-open map.

Remark 4.4: If f: (X, τ) → (Y, σ) is a pgrw-open map and g:(Y, σ) → (Z, η) is an open map, then the composition gof need not be a pgrw-open map.

Example 4.5:X={a,b,c}, τ={X,φ,{a},{b},{a,b}}, Y={a,b,c}, σ={Y, φ ,{a},{b,c}}, Z={a,b,c}, η ={Z,φ, {b},{c},{b,c}}. pgrw-open sets in Y are all subsets of Y. pgrw-open sets in Z are Z, φ,{b,c},{c},{b}. Let f :X→Y be the identity map. Then f is a pgrw-open map. A map g :Y→Z is defined by g(a)=a, g(b)=a, g(c)=b , then g is open. {a} is open in X and (g∘f)({a})={a} is not pgrw-open in Z. g∘f: (X, τ) →(Z, η) is not a pgrw-open map.

5. pgrw*-open map:

Definition 5.1: A map f: (X, τ) → (Y, σ) is said to be a pgrw*-open map if for every pgrw-open set A in (X, τ) the image f(A) is a pgrw-open set in (Y, σ).

Example 5.2: X={a,b,c} , τ ={X, φ, {a}, {a,,c}}. pgrw-open sets in X are X, φ,{a,c}, {a,b},{a}.Y={a,b,c} , σ ={Y, φ, {a}, {b,c}}. pgrw-open sets in Y are Y, φ, {b,c},{a,c},{a,b}, {a},{b},{c}. A map f: (X, τ) → (Y, σ) is defined by f(a)=b, f(b)=c, f(c)=a. f is a pgrw*-open map.

Theorem 5.3: Every pgrw*-open map is a pgrw-open map.

Proof : Let f: (X, τ) → (Y, σ) be a pgrw*-open map. Let A be an open set in X. Then A is pgrw-open. As f is a pgrw*-open map, f(A) is pgrw-open in Y. Hence f is a pgrw-open map.

The converse is not true.

Example 5.4: X={a,b,c} , τ ={X, φ, {a}, {b,c}}. pgrw-open sets in X are X, φ,{b,c}, {a,c}, {a,b,},{a},{b},{c}. Y={a,b,c} , σ ={Y, φ, {a}, {a,,c}}. pgrw-open sets in Y are Y, φ ,{a,c}, {a,b}, {a}. A map f: (X, τ) → (Y, σ) is defined by f(a)=a, f(b)=a, f(c)=c. f is a pgrw-open map, but not pgrw*-open.

Theorem 5.5: If f: (X, τ) → (Y, σ) is a pgrw-open map and g: (Y, σ) → (Z, η) is a pgrw*-open map, then the composition g ∘ f: (X, τ) →(Z, η) is pgrw-open.

Proof:f:(X, τ) → (Y, σ) is a pgrw-open map and g: (Y, σ) → (Z, η) is a pgrw*-open map.

⇒∀ open set A in X, f(A) is pgrw-open in Y and g(f(A)) is pgrw-open in Z.

⇒∀ open set A in X, g∘f(A) is pgrw-open in Z.

⇒g ∘ f: (X, τ) →(Z, η) is a pgrw-open map.

Theorem 5.6: If f: (X, τ)→(Y, σ) and g: (Y, σ) → (Z, η) are pgrw*-open maps, then the composition g∘f:(X, τ) →(Z, η) is also pgrw*-open.

Proof: f:(X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are pgrw*-open maps.

⇒∀ pgrw-open set A in X, f(A) is pgrw-open in Y and g(f(A)) is pgrw-open in Z.

⇒∀ pgrw- open set A in X, g∘f(A) is pgrw-open in Z.

⇒g ∘ f: (X, τ) →(Z, η) is a pgrw*-open map.

Theorem 5.7: For any bijective map f :(X, τ)→(Y, σ) the following statements are equivalent:
i) f^{-1}: (Y, σ) → (X, τ) is pgrw-irresolute.
ii) f is a pgrw*-closed map.
iii) f is a pgrw* -open map.

Proof: (i) ⇒ (ii):f :(X, τ)→(Y, σ) is a bijective map. ⇒f^{-1} exists and $(f^{-1})^{-1}$=f.
Hence f is bijective and f^{-1}: (Y, σ) → (X, τ) is pgrw-irresolute.
⇒ $(f^{-1})^{-1}$=f and∀ pgrw-closed set U in X ,$(f^{-1})^{-1}$(U) is pgrw-closed in Y.
⇒∀pgrw-closed set U in X , f(U) is pgrw-closed in Y. ⇒ f is pgrw* -closed map.
(ii) ⇒(iii): f :(X, τ)→(Y, σ) is a bijective and pgrw*-closed map.
⇒∀ set A in X , $f(A^c)=[f(A)]^c$ and ∀ pgrw-open set U in X, $f(U^c)$ is pgrw-closed in Y.
⇒∀ pgrw-open set U in X, $(f(U))^c$ is pgrw-closed in Y. ⇒∀ pgrw-open set U in X, f(U) is pgrw-open in Y.⇒ f is a pgrw*-open map.
(iii) ⇒(i): f :(X, τ)→(Y, σ) is a bijective and pgrw*-open map.
⇒ f^{-1}: (Y, σ) → (X, τ) exists and $(f^{-1})^{-1}$=f and ∀ pgrw-open set U in X, f(U) is pgrw-open in Y.
⇒∀ pgrw-open set U in X, $((f)^{-1})^{-1}$(U) is pgrw-open in Y.
⇒f^{-1}: (Y, σ) → (X, τ) is pgrw-irresolute.

References

[1]. Abd El-Monsef ME, El-Deeb S N, Mahmoud RA., β-open sets and β-continuous mappings, Bull. Fac. Sci.Assiut Univ. 1983; 12:77-90.
[2]. Anitha M and P. Thangavelu, On pre generalized pre regular closed sets (pgpr), Acta ciencia Indian, (2005), 31 M(4) , 1035-1040.
[3]. Baker C.W., Contra open and Contra closed functions,Math. Sci1994; 17:413-415.
[4]. Balachandran K, Sundaram P, Maki H. On Generalized Continuous Maps in Topological Spaces, Mem. I Ac. Sci.Kochi Univ. Math., 1991; 12:5-13.
[5]. Balasubramanian, S, T. K. Krishnamurthy, P. Aruna Swati Vyjayanti and Sandhya., Contra regular pre closed mappings, IJMA-4(90,2013,184-191
[6]. Benchalli S.S. and Wali R.S., on rw- Closed sets is Topological Spaces, Bull, Malays, Math, sci, soc. 30, 2007, 99-110.
[7]. Benchalli S.S., Patil P.G. and Rayanagaudar T.D. ωα-Closedsets is Topological Spaces, The Global. J Appl. Mathand Math.Sci., 2009; 2:53-63.
[8]. Cameron DE., Properties of s-closed spaces, Proc. Amer. Math, soc 1978; 72: 581-586
[9]. Dontchev. J, On generalizing semi-pre open sets. Mem. Fac. Sci. Kochi Uni. Ser. A Math., (1995), 35-48.
[10]. Gnanambal Y., On generalized pre regular closed sets in topological spaces, Indian J Pure. Appl. Math.1997; 28(3):351-360.
[11]. Govindappa Navalgi, Chandrashekharappa A.S and S.V.Gurushantanavar., Gspr –open functions in Toplogical Spaces. Int. J. of Mathematical Sciences & Applications, IJMSA Vol.2 Nr.1 , Jan.2012 ,pp. 231-237 ISSN : 2230-9888
[12]. Jastad ON. On some classes of nearly open sets, Pacific J. Math., 1965; 15 : 961-970
[13]. Levine N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41
[14]. Levine N. Generalized closed sets in topology, Rend. Circ. Mat. Palermo 1970; 19(2):89-96.
[15]. Long P.E., Herington LL. Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 1978; 27:20-28.

[16]. Maki H, Devi R, Balachandran K. Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math1994;15:51-63.

[17]. Mashhour AS, Abd El-Monsef ME, El-Deeb S N. On pre-continuous and weak pre continuous mappings,Proc.Math. Phys.Soc. Egypt, 1982; 53:47-53.

[18]. Navalagi G B., "On Semi-pre Continuous Functions and Properties of Generalized Semi-pre Closed in Topology", IJMS, 2002; 29(2):85-98.

[19]. N. Palaniappan and K. Rao, Regular generalized closed sets, Kyungpook Math. J., 33 (1993), 211-219.

[20]. Noiri, T., H. Maki and J. Umehara, "Generalized Preclosed Functions".Mem. Fac. Sci. Kochi. Univ. (Math)19, (1998), 13-20

[21]. Pushpalatha A. Studies on generalizations of mapping in topological spaces, PhD Thesis, Bharathiar University, Coimbatore, 2000.

[22]. Pushpalatha, g*s-Closed Sets in Topological Spaces, Int. J. Contemp. Math. Sciences, Vol. 6, 2011, no. 19, 917 - 929

[23]. Raja Mohammad Latif, Characterizations of delta-open sets and mappings in topological spaces may 12, 2014

[24]. Sen A.K and P.Bhattacharya, On pre closed mappings, Bull.Cal.Math.Soc., 85(1983),409-412.

[25]. Stone M. Application of the theory of Boolean rings to general topology, Trans. Amer. Math.Soc.1937; 41 : 374-481.

[26]. Sundaram P, Sheik John M. On w-closed sets in topology, Acta Ciencia Indica, 2000; 4:389-39

[27]. Syed Ali Fathima and S and Mariasingam. M, "On #RG-Continuous and #RG-irresolute functions", Journal of Advanced Studies in Topology, 3(4),2012, 28-33

[28]. Vadivel A, K Vairamamanickam. rgα-Closed sets & rgα-open sets in Topological Spaces, Int J of math, Analysis.2009; 3(37):1803-1819

[29]. Wali, RS., Prabhavati S Mandalageri., On αrω-closed and αrω-open maps in Topological Spaces, International Journal of Applied Research, 2015; 1(11): 511-518.

[30]. Wali R.S and Vijayakumari T. Chilakwad, On Pre Generalized Regular Weakly [pgrw]-Closed sets in a Topological Space, International Journal of Mathematical Archive-6(1), 2015, 76-85,

[31]. Wali R.S and Vijayakumari T. Chilakwad, On pre generalized regular weakly-open sets in a topological space, International Journal of Mathematical Archive-6(11), 2015, 163-170,

[32]. Wali R.S and Vijayakumari T. Chilakwad, Pgrw-Continuous and Pgrw-Irresolute Maps in Topological Spaces IOSR Journal of Mathematics, (Volume 12, Issue 1 Ver. II (Jan. - Feb. 2016), PP 01-10