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RESEARCH ARTICLE

AN ALGORITHMIC APPROACH TO OBTAIN AN EDGE H-DOMINATING SET OF THE GRAPH

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ABSTRACT

In this paper, the algorithm is developed to find an edge H-dominating set of the graph. It is also proved that the edge H-dominating set obtained by the algorithm is minimal.

Key words: edge dominating set, edge H-dominating set, minimal edge H-dominating set.

AMS Subject Classification (2010)-05C69

1. INTRODUCTION

The concept of edge domination is well-known. Several papers have been appeared related to edge domination [1, 3, 5, 6]. A new concept called edge H-domination in graph is introduced in [7]. We consider the definitions of edge domination and edge H-domination of the graph. We explain the relation between these both concepts with the help of an example of the graph. An algorithm to _nd an edge dominating set is given in [4]. We have developed an algorithm to find an edge H-dominating set of the graph.

The edge H-dominating set of the Petersen graph is obtained by applying an algorithm on the graph. It is also proved that the edge H-dominating set obtained by an algorithm is a minimal edge H-dominating set.

Definition 1.1 [2] Let G be a graph. A subset F of an edge set $E(G)$ is said to be an *edge dominating set* of G if for every edge e not in F is adjacent to some edge in F. An edge dominating set F of G is a *minimal* edge dominating set if F does not have a proper subset which is an edge dominating set.

Definition 1.2 [7] Let G be a graph. A set $F \subseteq E(G)$ is said to be an edge H-dominating set of G if the following conditions are satisfied by any edge $e = uv \in E(G)$.

1) If e is an isolated edge then $e \in F$.

2) If e is a pendent edge with v as a pendent vertex and u is not a pendent vertex. If $uv \notin F$ then all the edges incident at u (except e) are in F.

3) If e is a pendent edge with u as a pendent vertex and v is not a pendent vertex. If $uv \notin F$ then all the edges incident at v (except e) are in F.

4) If neither u nor v is a pendent vertex and $uv \notin F$ then all the edges incident at u (except e) are in F or all the edges incident at v (except e) are in F.

Example 1.3 Consider the following graph with vertices 1, 2, 3, 4, 5, 6 and edges 12, 23, 34, 45, 51 and 56.

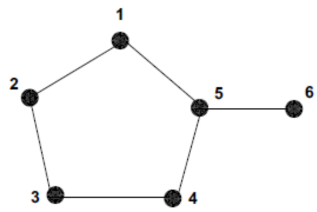


Figure 1 The graph G

The edge sets $F1$ = {23; 45; 15} and $F2$ = {23; 45; 56} are examples of edge H-dominating sets of the graph G. The sets $F1$ and $F2$ are also an edge dominating sets of G. The edge set F ={23; 45} is an edge dominating set but F is not an edge H-dominating set because for an edge 51 $\notin$ F, the edge incident at vertex 1 is 12 $\notin F$ also the edges incident at vertex 5 are 45 $\in$ F but 56 $\notin F$. However, we have the following remarks.

Remarks 1.4(1) Every edge H-dominating set of the graph G is edge dominating set but the converse need not be true.

(2) From the definition of edge H-dominating set, if an edge set F of the graph G is an edge H-dominating set then F contains all isolated edges of G.

(3) If G is a graph with $\Delta(G) \leq 2$ then every edge dominating set of G is an edge H-dominating set, where $\Delta(G)$ = maximum degree of a vertex in G. In particular, if the graph G is a cycle graph or path graph then every edge dominating set of G is an edge H-dominating set.

In [4], the edge dominating set of the graph is obtained by considering the following greedy approach.

1. Let $F = \phi$.
2. Take an edge $e = uv$, and let $F = F \cup \{e\}$.
3. Delete e from the graph (along with end vertices u and v).
4. If there are no edges left, stop, else go back to step 2.

Clearly, the resulting set F is an edge dominating set.

Definition 1.5 (adjacent pendent edges) Two pendent edges e_1 and e_2 are adjacent if they have a common end vertex.

2. An algorithm to find an edge H-dominating set

Apply the following steps on the graph G.

Let $F = \phi$.

Step 1. If the graph has isolated edges then

F = Collection of all isolated edges of the graph.

Delete the end vertices of all isolated edges from the graph.

Step 2. If the graph has pendent edges then

2(I) If $u_1v_1, u_2v_2, \ldots, u_nv_n$ are non adjacent pendent edges of G such that $deg(u_i) = 1$ and $deg(v_i) \geq 3$ for $I = 1, 2, \ldots, n$ then

$$F = F \bigcup_{i=1}^{n} \Big(\bigcup_{w \in N(v_i),\, w \neq u_i} v_i w \Big)$$

Delete the vertices ui, vi (for $i = 1, 2, \ldots, n$) from the graph

2(II) If $u_1v, u_2v, \ldots, u_nv$ are adjacent pendent edges of G incident at vertex v such that $\deg(u_i) = 1$ (for $i = 1, 2, \ldots, n$) and $\deg(v) \geq n$ then

$$F = F \bigcup_{w \in N(v),\, w \neq u_i \text{ for exactly one } i \in \{1,2,\ldots,n\}} vw$$

Delete the vertices ui (for $i = 1, 2, \ldots, n$), v from the graph.

If there are m vertices $v_1, v_2, \ldots, v_m$ such that each vertex vj (for $j = 1, 2, \ldots, m$) incident with adjacent pendent edges $v_j u_{ji}$ (for $i = 1, 2, \ldots, t_j$) then

$$F = F \bigcup_{j=1}^{m} \Big(\bigcup_{w \in N(v_j),\, w \neq u_{ji} \text{ for exactly one } i \in \{1,2,\ldots,t_j\}} v_j w \Big)$$

where $t_1, t_2, \ldots, tm$ are the number of adjacent pendent edges incident to $v_1, v_2, \ldots, v_m$ respectively. Delete the vertices v_j (for $j = 1, 2, \ldots, m$) and u_{ji} (for $j = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, t_j$) from the graph.

Step 3. After applying Step 1 and Step 2.

Select an edge uv from the graph such that $deg(uv) \geq deg(xy)$ for all other edges xy of the graph. Also confirm that $deg(u) \geq deg(v)$.

$$F = F \bigcup_{w \in N(u),\, w \neq v} uw$$

and let $w_1, w_2, \ldots, w_n$ are neighbors of v other than u then

$$F = F \bigcup_{i=1}^{n} \Big(\bigcup_{t \in N(w_i),\, t \neq v} w_i t \Big)$$

Delete the vertices u, v, w_i (for $i = 1, 2, \ldots, n$) from the graph.

After deleting the vertices, apply step 3 continuously on the vertices deleted sub-graph till the graph has an edge uv such that $deg(u) \geq 2$ and $deg(v) \geq 2$.

At the end, the set F denotes an edge H-dominating set of the graph.

Examples 2.1 (1) We are applying the algorithm to find an edge H -dominating set of the following graph with 10 vertices and 15 edges.

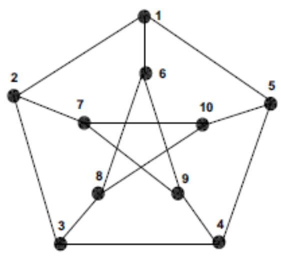


Figure 2 Petersen Graph *G*

Initially, assume that $F = \phi$

Step 1. does not execute because the graph has no isolated edges.

Step 2. does not execute because the graph has no pendent edges.

Step 3. Since the graph is 3-regular, we consider any edge uv from the graph such that $deg(uv) \geq deg(xy)$ for all other edges xy of the graph, also $deg(u) = deg(v)$ in this case.

Let uv = 12 with u = 1 and v = 2 then

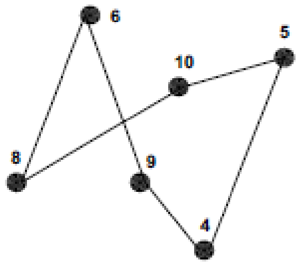
$F = F \cup \{15,16\}$ since $F = F \bigcup_{w \in N(u), w \neq v} uw$

Also, $w_1 = 3$ and $w_2 = 7$ are neighbors of $v = 2$ other than $u = 1$ therefore,

$F = F \cup \{34,38,79,710\}$ since $F = F \bigcup_{i=1}^{2} \left(\bigcup_{t \in N(w_i), t \neq v} w_i t \right)$

Thus, we get $F = \{15,16, 34,38,79,710\}$

Delete the vertices u, v, w_1 *and* w_2 from the graph. After deleting these vertices, we get the following graph.



G1 = G $\{u, v, w_1, w_2\}$

We again applying step 3 on vertices deleted subgraph since there is an edge uv such that $deg(u) \geq 2$ and $deg(v) \geq 2$.

Let uv = 45 with u = 4 and v = 5 then

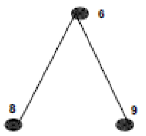
$F = F \cup \{49\}$

Also, w_1 = 10 is a neighbor of v = 5 other than u = 4 therefore,

$F = F \cup \{180\}$

Thus, we get $F =$ {15,16, 34,38,79,710}∪ {49,810}

Delete the vertices u, v, w_1 from the graph. After deleting these vertices, we get the following graph



$G_2 = G_1 - \{u, v, w_1\}$

Now, we stop the process because there is no edge uv in $G2$ such that $deg(u) \geq 2$ and $deg(v) \geq 2$.

Finally, the edge set v {15,16,34,38,79,710,49,810} is an edge H-dominating set of the Petersen graph *G*.

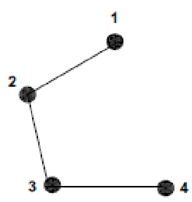
(2) Consider the graph given in FIgure 1. Apply the algorithm to find an edge H -dominating set of the graph.

Initially, assume that $F = \phi$.

Step 1. does not execute because the graph has no isolated edges.

Step 2. Since 56 is a pendent edge of the graph, by step **2(I)** of the algorithm, F ={15, 45}.

Delete the vertices 5, 6 from the graph. After deleting these vertices, we get the following graph.



$G - \{5, 6\}$

Step 3. Since there is an edge 23 such that deg (2) = 2 and deg (3) = 2, F = F ∪ {34} or $F = F \cup \{12\}$

Thus, Finally the edge set F = {15, 45, 34} or F = {15, 45,12} is an edge H-dominating set of the graph *G*.

Question 2.2 Is the set F obtained by the algorithm edge H-dominating set ?

Answer: Yes.

Proof: Consider an edge $e = xy \in E(G) - F$. By step 1 of the algorithm, F contains all the isolated edges of the graph G, therefore $e = xy$ is not an isolated edge of G.

To prove that the set F obtained by the algorithm is an edge H-dominating set, it is enough to prove for any edge $e = xy \in E(G) - F.$, either all the edges incident at x are in F or all the edges incident at y are in F.

Case 1. Suppose that $xy \in E(G) - F.$ is a pendent edge of G with $deg(x) = 1$ and $deg(y) \geq 2$.

1(i). If xy does not have any adjacent pendent edge(s) in G then by **(I)** of step 2 of algorithm, all the edges incident at y are in F (except xy).

1(ii). If xy has adjacent pendent edge(s) in G then by (II) of step 2 of algorithm, all the edges incident at y are in F if xy 62 F.

Similarly, if xy 62 F is a pendent edge with $deg(y) = 1$ and $deg(x) \geq 2$ then also

by similar argument, we prove that all the edges incident at x are in F if $xy \notin F$.

Case 2. Suppose that $e = xy \in E(G) - F$ is not a pendent edge of G. Therefore $deg(x) \geq 2$ and $deg(y)$ ≥ 2.

Since $xy \notin F$ and xy is not a pendent edge, by step 3 of algorithm, either $xy = uv$ or $xy = vw$ for some $w \in N(v)$, $w \neq u$.

If $xy = uv$ then by step 3 of algorithm, all the edges incident at x = u are in F (except $uv = xy$).

If $xy = vw$ for some $w \in N(v)$, w 6= u then also by step 3 of algorithm, all the edges incident at $y = w$ are in F (except $vw = xy$).

Thus, F is an edge H-dominating set of G.

Definition 2.3 Let G be a graph and F be an edge H-dominating set of G. The set F is said to be a minimal edge H-dominating set if $F - \{e\}$ is not an edge H-dominating set for every edge $e \in F$

Example 2.4 The edge sets {12, 45, 56}, {34,15, 56} are minimal edge H-dominating sets of the graph given in figure 1.

Question 2.5 Is the set F obtained by the algorithm minimal ?

Answer: Yes.

Proof: To prove F is minimal, it is enough to prove F – {xy} is not an edge H-dominating set, for any edge $xy \in F$.

Since xy is an edge of G which belongs to F and F is an edge H-dominating set obtained by applying the algorithm on G, there are following three possibilities about xy of F.

Case 1. $xy \in F$ is an isolated edge of G.

Since xy is an isolated edge of G and $xy \in F$. $F - \{xy\}$ is not an edge H-dominating set of G.

Case 2. $xy \in$ F is a pendent edge of G.

Since xy is a pendent edge of G and $xy \in F$. By (II) of step 2 of algorithm, there is at least one pendent edge e of the graph G such that e is adjacent to xy and $e \notin F$.If xy is removed from F then we get two adjacent pendent edges e and xy outside F. It

means that $F - \{xy\}$ g is not an edge H-dominating set of G.

Case 3. $xy \in F$ is neither isolated nor pendent edge of G. By step 3 of algorithm, either xy is incident to a vertex u of an edge $uv \notin F$ or xy is incident to a vertex w of an edge $vw \notin F$ for *some* $w \in N(v)$, $w \neq u$.

3(i). If xy is incident to a vertex u of an edge $uv \notin F$ also by step 3 of algorithm, $vw \notin F$ for all $w \in N(v)$, $w \neq u$.. Therefore, in case of $F - \{xy\}$, there is an edge uv$\notin F$ such that an edge xy incident to u and $xy \notin F$ -$\{xy\}$ also there is an edge vw incident to v with w $\neq u$ and $vw \notin F - \{xy\}$. Thus, uv does not edge H-dominated by the edges of $F - \{xy\}$. Hence F $- \{xy\}$is not an edge H-dominating set of G.

3(ii). If xy is incident to a vertex w of an edge vw $\notin$ F for some $w \in N(v)$, $w \neq u$ then by similar argument, we prove that an edge vw ($w \neq u$, $w \in N(v)$) does not edge H-dominated by edges of $F - \{xy\}$. Hence $F - \{xy\}$is not an edge H-dominating set of G.

Hence F is a minimal edge H-dominating set of G.

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